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Energy loss and straggling of charged particles in plasmas of all degeneracies

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The electronic energy loss and straggling of swift charged particles in weakly interacting plasmas are described in a quantum-mechanical formalism that includes thermal effects. It applies to plasmas over wide ranges of densities and temperatures reaching from degenerate to nondegenerate systems, as they are found in solids, stars, and in dense and dilute fusion plasmas. Using approximations for the dielectric function, we calculate the stopping power S and the straggling Ω for slow particles in plasmas of varying degree of degeneracy, and discuss the dependence of the results on particle velocity and plasma temperature T. For nondegenerate plasmas, one obtains the relation $\Omega^2 = 2kTS$ between these quantities.

I. INTRODUCTION

During the last five years interest has focused on the penetration of swift light and heavy atomic particles through media under conditions of extreme pressure and temperature, stimulated by the exploration of laser or heavy-ion-induced inertially confined fusion (ICF). The physical conditions during pellet implosion pose intriguing questions as to the energy transfer between moving particles and plasmas of rapidly changing density, and the concomitant changes of the range-energy relations that determine the retention of the particle kinetic energies within the samples. Previous analyses of this problem were based on the quantum-mechanical dielectric theory[1] or the classical binary-collision approximation.[2] The results were applied to the medium at zero temperature. Thermal as well as quantum-mechanical effects were taken into account by Skupsky,[3] who made use of the dielectric formalism to derive the energy loss of charged particles with velocities lower than those of the electrons in the plasma. These results were applied to the slowing down of the 3.5-MeV α particles produced in the dominant deuterium-tritium (DT) reaction in ICF.[4] Current feasibility studies of different inertial confinement fusion programs require a complete and accurate description of the energy-loss process for a variety of ionic species, over a wide range of nonrelativistic ion velocities in very dense and hot plasmas.

In this paper we present a quantum-mechanical treatment of the energy loss in plasmas under conditions which include those of cold, dense conduction electron fluids in metals, hot and dense plasmas of interest for ICF, and dilute plasmas in magnetically confined fusion (MCF). To clarify our ideas we have prepared a map of plasma conditions as shown in Fig. 1. The ordinates represent the electron plasma temperature T and the abscissa displays the one-electron radius r_s(a.u.), related to the electron density n(upper scale) as $(4\pi/3)\, r_s^3 = 1/na_0^3$. Degenerate or "cold" plasmas and nondegenerate or "hot" plasmas are separated by the line where the reduced temperature $\Theta \equiv kT/E_F$ has the value $\Theta = 1$; k denotes the Boltzmann constant and $E_F = 1.84/r_s^2$ the Fermi energy of the electrons in atomic units (1 a.u. = 27.2 eV). The domain marked M covers metals ranging from $r_s = 1.5$ for W to $r_s = 5.9$ for Cs; its upper bounds are given by the melting and boiling points. Another important plasma in our environment is the sun, indicated by the shaded band. The solar plasma stretches from $r_s \sim 0.3$ at the center to $r_s \sim 100$ near the surface, and becomes very dilute in the corona where $r_s > 10^4$. By comparison, conditions in the interior of Jupiter correspond to $r_s \sim 1$ at $T \sim 10^4$ K and in white dwarfs to $r_s \sim 0.01$ at $T \sim 10^7$ K.

A question of general interest is the distinction between weakly and strongly interacting plasmas. The parameter $\chi^2 \equiv e^2/\pi\hbar v_F$ measures the ratio between potential and kinetic energies of the electrons in a degenerate electron gas.[5] Nondegeneracy can be included in this parameter through the expression

$$\chi^2 = \frac{3}{10\pi}\left(\frac{9\pi}{4}\right)^{1/3}\frac{V}{K}, \tag{1}$$

where $V = e^2/r_s a_0$. The mean kinetic energy K will here be approximated by

$$K \equiv \tfrac{1}{2} m\, v_e^2 \simeq \tfrac{3}{5}\, E_F + \tfrac{3}{2}\, kT, \tag{2}$$

where $\frac{3}{5}E_F$ is the mean kinetic energy of a fully degenerate electron gas, and $\frac{3}{2}kT$ is that of a nondegenerate plasma. With this simple extension, Eq. (1) becomes $\chi^2 = 0.166 r_s(1 + \frac{5}{2}\Theta)^{-1}$. Our range of interest in *weakly interacting plasmas* ($\chi^2 < 1$) covers the whole area above the line drawn for $\chi^2 = 1$ in Fig. 1.

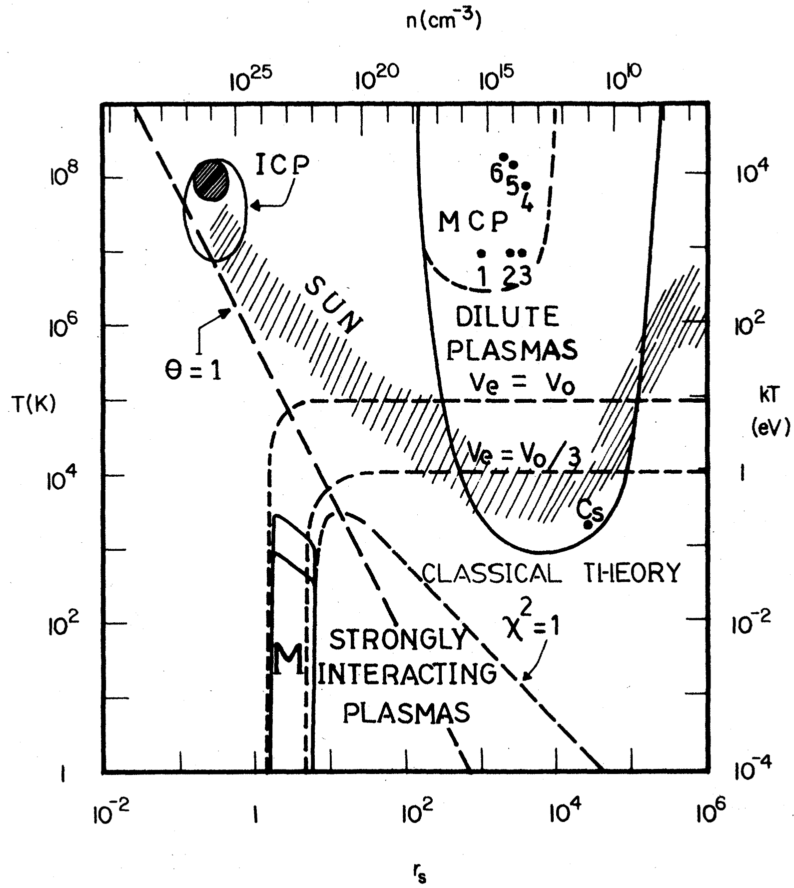

FIG. 1. Plasma conditions in metals (M), the sun, and man-made plasmas of interest for nuclear fusion in the context of inertial confinement (ICP) and magnetic confinement of plasmas (MCP). The line denoted $\chi^2=1$ separates the conditions in strongly interacting plasmas from those in weakly interacting plasmas. The lines $v_e=v_0$ and $v_e=v_0/3$ indicate the transition region between the lower right-hand quadrant, where classical theories describe the energy loss, and all other plasmas where quantum-mechanical descriptions are appropriate. Below the line $\Theta\equiv kT/E_F=1$, the plasmas are degenerate or "cold," above the line they are nondegenerate or "hot."

The extent to which a classical- or a quantum-mechanical description is appropriate to derive the rate of energy loss in a medium is gauged by the parameter[6,7] $\xi\equiv Z^*e^2/\hbar v$, where Z^*e is the effective charge[8] of the ion moving with velocity v. To indicate the transition between classical- ($\xi\gg1$) and quantum-mechanical ($\xi\ll1$) descriptions we replace the projectile velocity v by the mean electron velocity v_e estimated from Eq. (2).[9] The dashed lines in Fig. 1 correspond to $v_e=v_0\equiv e^2/\hbar$ and $v_e=v_0/3$. They give an indication for light ions, with $Z^*\sim1$, of the transition region from the classical description, applicable in the lower right-hand corner of the figure, to the quantum-mechanical treatment of the energy loss that is applicable to most plasma conditions.

Measurements[10] of the energy loss of light and heavy ions in a Cs plasma at $T\cong2000$ K and $n=10^{11}$ cm^{-3} have been performed in the classical domain indicated by the point Cs in Figs. 1. The conditions in prototypes of fusion reactors based on magnetically confined plasmas (MCP), such as in tokamak experiments, fall in the domain marked MCP. The numbered points refer to plasmas produced in (1) Alcator A, M.I.T., (2) Princeton Large Torus 1, (3) Kurachatov tokamak T-3, (4) Princeton Large Torus 2, (5) Princeton tokamak fusion test reactor (projected), and (6) International tokamak reactor (projected). The laser-induced or heavy-ion-induced implosion of fusion fuel pellets starts from normal solid-state conditions and leads to the extreme conditions of inertially confined plasmas in a distinct domain of high densities and temperatures marked ICP in Fig. 1. The shaded area indicates the region of interest for the ignition of DT fusion reactions. Clearly, the treatment of the stopping power of these manmade plasmas require a quantum-mechanical formulation in all ranges of Θ. In the following we generalize the quantum-mechanical description of the energy loss in a way that can be immediately applied to plasmas under these various conditions.

The second quantity of interest to characterize the slowing down process is the energy-loss straggling Ω, which describes the statistical fluctuations of the energy loss of the particle.[6] To be consistent with the usual definition of the stopping power $S = -\Delta E/\Delta x$, as the magnitude of the mean energy-loss per unit pathlength, we define Ω^2 as the square of the standard deviation of the energy-loss distribution per unit pathlength, i.e.,

$$\Omega^2 = [\langle(\Delta E)^2\rangle - \langle\Delta E\rangle^2]/\Delta x .$$

In Sec. II we adapt the dielectric function formalism for the description of the energy dissipation of a heavy particle in a thermalized medium, taking into account the contributions from both emission and absorption processes. As a result, the energy loss is given, to first order, by an integral over only spontaneous emission events. The straggling integral is obtained from contributions of all absorption and emission processes. In the high-temperature limit, however, both integrals converge, resulting in the simple relation $\Omega^2 = 2kTS$. Using analytical forms for the dielectric function, we illustrate in Sec. III the behavior of the energy loss and straggling for slow particles in electronic plasmas of all degeneracies over the whole range of Θ values.

In Sec. IV, we give an illustrative example for the straggling of 3.5-MeV alpha particles in inertially confined DT plasmas. The implications of our work are summarized and discussed Sec. V as they may be of interest for studies of plasmas under fusion conditions and in astrophysical problems.

II. ENERGY LOSS AT FINITE TEMPERATURES

A comprehensive treatment of the energy-loss problem, in terms of the equilibrium dielectric function $\epsilon(q, \omega)$, can be formulated by starting from the scattering rate

$$R(\vec{q}, \omega) = \left(\frac{4\pi Z e^2}{q^2}\right)^2 \frac{2\pi}{\hbar^2} S(\vec{q}, \omega) , \qquad (3)$$

for energy transfer $\hbar\omega = E(\vec{p}\,') - E(\vec{p})$ and momentum transfer $\hbar\vec{q} = \vec{p}\,' - \vec{p}$, which applies to the scattering of a particle of charge Ze, with initial momentum $\vec{p}$ and energy $E(\vec{p})$, to the final state given by $\vec{p}\,'$, $E(\vec{p}\,')$. The dynamical structure factor $S(\vec{q}, \omega)$ is related to the dielectric function $\epsilon(\vec{q}, \omega)$ through[11]

$$S(\vec{q}, \omega) = \frac{\hbar q^2}{4\pi^2 e^2} N(\omega) \,\mathrm{Im}\left(\frac{-1}{\epsilon(\vec{q}, \omega)}\right) , \qquad (4)$$

where $N(\omega) \equiv [\exp(\beta\hbar\omega) - 1]^{-1}$ and $\beta = 1/kT$.

The temperature dependence is contained in the dielectric function $\epsilon(\vec{q}, \omega)$ and in the Planck function $N(\omega)$. The energy-loss rate is given by

$$\frac{dE}{dt} = \int \frac{d^3\vec{p}\,'}{(2\pi\hbar)^3} \hbar\omega R(\vec{q}, \omega)$$

$$= \left(\frac{Ze}{\pi}\right)^2 \int d^3q \,\frac{\omega N(\omega)}{q^2} \,\mathrm{Im}\left(\frac{-1}{\epsilon(\vec{q}, \omega)}\right) , \qquad (5)$$

where $\omega \equiv \omega(\vec{p}, \vec{q})$ is determined from

$$\hbar\omega(\vec{p}, \vec{q}) \equiv E(\vec{p}\,') - E(\vec{p}) = \hbar\,\vec{q}\cdot\vec{v} + \frac{\hbar q^2}{2M} \qquad (6)$$

in terms of the incident velocity $\vec{v} = \vec{p}/M$ and the mass M of the projectile. For heavy particles $M \gg m$, recoil effects are small and we can expand Eq. (5) in terms of $\Delta\omega \equiv \hbar q^2/2M$ to obtain

$$\frac{dE}{dt} = \left(\frac{dE}{dt}\right)_0 + \left(\frac{dE}{dt}\right)_1 + \cdots , \qquad (7)$$

where the first two terms are

$$\left(\frac{dE}{dt}\right)_0 = \left(\frac{Ze}{\pi}\right)^2 \int d^3q \,\frac{\omega N(\omega)}{q^2} \,\mathrm{Im}\left(\frac{-1}{\epsilon(q, \omega)}\right) \Bigg|_{\omega = \vec{q}\cdot\vec{v}} , \qquad (8)$$

$$\left(\frac{dE}{dt}\right)_1 = \left(\frac{Ze}{\pi}\right)^2 \frac{\hbar}{2M} \times \int d^3q \,\frac{\partial}{\partial\omega} \left[\omega N(\omega) \,\mathrm{Im}\left(\frac{-1}{\epsilon(q, \omega)}\right)\right] \Bigg|_{\omega = \vec{q}\cdot\vec{v}} . \qquad (9)$$

The integrals range over both negative frequencies (*loss processes*) and positive frequencies (*gain processes*), but it is here more instructive to transform them into integrals over positive frequencies only.

We can simplify the expression for the main term $(dE/dt)_0$, Eq. (8), by splitting the integral into the $\omega > 0$ and $\omega < 0$ parts, and then making use of the relations $N(\omega) + N(-\omega) = -1$ and $\epsilon(\vec{q}, \omega) = \epsilon^*(\vec{q}, \omega)$; this leads to an expression of the form

$$\left(\frac{dE}{dt}\right)_0 = \int_{\omega > 0} d^3q \,N(\omega) f(\vec{q}, \omega) - \int_{\omega > 0} d^3q [N(\omega) + 1] f(\vec{q}, \omega) . \qquad (10)$$

The two terms in $N(\omega)$ cancel exactly, with the result for the stopping power S,

$$S \equiv -\frac{dE}{dx} \cong \frac{-1}{v}\left(\frac{dE}{dt}\right)_0$$

$$= \frac{2}{\pi}\left(\frac{Ze}{v}\right)^2 \int_0^\infty \frac{dq}{q}\int_0^{qv} d\omega\, \omega\, \mathrm{Im}\left(\frac{-1}{\epsilon(q,\omega)}\right). \quad (11)$$

The only temperature dependence is now contained in the energy-loss function $\mathrm{Im}[-1/\epsilon(q,\omega)]$, and arises from a thermal redistribution of the oscillator strengths in the medium. One can interpret this result as a cancellation between the processes of *stimulated absorption* and *stimulated emission* of energy $\hbar\omega$ by the projectile,[12] since both processes are proportional to the Planck distribution $N(\omega)$ that characterizes the thermal equilibrium of excitation quanta in the medium. Thus, the energy-loss rate is only determined by *spontaneous emission* processes, which are independent of $N(\omega)$.

A similar analysis can be made for the energy-loss straggling Ω,

$$\Omega^2 = \int \frac{d^3\vec{p}\,'}{(2\pi\hbar)^3}\,(\hbar\omega)^2 R(\vec{q},\omega)\,, \quad (12)$$

which can be expanded as

$$\Omega^2 = \Omega_0^2 + \Omega_1^2 + \cdots,$$

with

$$\Omega_0^2 = \frac{Z^2e^2\hbar}{\pi^2 v}\int d^3q \frac{\omega^2}{q^2} N(\omega)\, \mathrm{Im}\left(\frac{-1}{\epsilon(q,\omega)}\right)\Bigg|_{\omega=\vec{q}\cdot\vec{v}}, \quad (13)$$

$$\Omega_1^2 = \frac{Z^2e^2\hbar^2}{2\pi^2 Mv}\int d^3q \frac{\partial}{\partial\omega}\times\left[\omega^2 N(\omega)\, \mathrm{Im}\left(\frac{-1}{\epsilon(q,\omega)}\right)\right]\Bigg|_{\omega=\vec{q}\cdot\vec{v}}. \quad (14)$$

For the balance between positive and negative frequencies in the Ω_0^2 term, all the contributions from stimulated absorption ($\omega > 0$), proportional to $N(\omega)$, and those from stimulated and spontaneous emission ($\omega < 0$), proportional to $[N(\omega)+1]$, are collected, and one obtains

$$\Omega_0^2 = \frac{2Z^2e^2\hbar}{\pi v^2}\int_0^\infty \frac{dq}{q} \times \int_0^{qv} d\omega\, \omega^2[2N(\omega)+1]\, \mathrm{Im}\left(\frac{-1}{\epsilon(q,\omega)}\right). \quad (15)$$

The temperature dependence of Ω_0^2 is contained in $N(\omega)$ and $\epsilon(q,\omega)$.

We discuss now our results for low and high temperatures. When $kT \ll \hbar\omega$, $N(\omega)\to O$, and we retrieve the expression for the energy straggling in a degenerate electron gas.[5] Explicit integrations of S and Ω for $T=0$ exist already in the literature.[13,14] In the opposite limit $kT \gg \hbar\omega$, we can approximate $[2N(\omega)+1] \cong 2kT/\hbar\omega$. The straggling integral Eq. (15) then becomes identical to the stopping integral Eq. (11) multiplied by $2kT$, i.e., straggling Ω and stopping power S are related as

$$\Omega^2(v,n,T) \cong 2kT\, S(v,n,T)\,, \quad (16)$$

for all values of v, n, and T such that the condition $\hbar\omega \ll kT$ is fulfilled. Since the frequencies of interest fall in the integration range from zero to $\omega_{\max} = 2mv(v+v_e)/\hbar$, Eq. (16) will apply when

$$2mv(v+v_e) \lesssim kT\,. \quad (17)$$

In the limit $\Theta \equiv kT/E_F \gg 1$ one approaches $\frac{1}{2}mv_e^2 \cong \frac{3}{2}kT$, and Eq. (17) defines the domain $v \lesssim 0.15v_e$, corresponding to projectiles much slower than the thermal electrons in the plasma. The velocity dependence of Ω^2 is the same as that of S, viz., $\Omega^2 \propto v$. By contrast, in a degenerate electron gas at low velocities, Ω^2 is a quadratic function of v.[14]

The applicability of Eq. (16) to a hot plasma $kT \gg E_F$ accords with a classical description, in terms of the Fokker-Planck equation, for the fluctuations in the energy of a slow particle in a thermalized medium. It pertains, moreover, to a general quantum-mechanical relation between the generalized resistance and voltage fluctuations in linear dissipative systems.[15]

III. APPROXIMATIONS AND RESULTS FOR LOW VELOCITIES

The simultaneous action of thermal and quantum-mechanical effects in the processes of energy exchange between a slowly moving particle $v \lesssim v_e$, and a plasma of varying degree of degeneracy, give rise to important effects in the dissipation of the particle's kinetic energy. The slowing down phenomenon for $v \ll v_e$ can be described in terms of the low-frequency approximation for the real and imaginary parts of the dielectric function[16] $\epsilon(q,\omega) = \epsilon_1(q,\omega) + i\epsilon_2(q,\omega)$. The temperature-dependent function[3] becomes

$$\epsilon_1(q,\omega) \cong \epsilon_s(q) = 1 + q_s^2/q^2\,, \quad (18)$$

$$\epsilon_2(q,\omega) \cong \frac{2m^2e^2\omega}{(\hbar q)^3}\left[\exp\left(\frac{\hbar^2q^2}{8mkT} - \eta\right) + 1\right]^{-1}. \quad (19)$$

Here q_s is a wave vector that determines the screening of a nearly static charge in a plasma. The degeneracy parameter $\eta = \mu/kT$ depends on the chemical potential μ and, at all degeneracies, is related to Θ as $F_{1/2}(\eta) = \frac{2}{3}\Theta^{-3/2}$ through the Fermi integral $F_{1/2}(\eta)$. The value of q_s is given in terms of the logarithmic derivative of $F_{1/2}(\eta)$ as $q_s^2 = q_D^2 \times F'_{1/2}(\eta)/F_{1/2}(\eta)$. A good approximation[17] (5%) is provided by the interpolation between the low- and high-temperature limits

$$q_s^{-2} = q_{TF}^{-2} + q_D^{-2} , \tag{20}$$

where $q_{TF} = (3\omega_p^2/v_F^2)^{1/2}$ is the Thomas-Fermi (TF) wave number, $q_D = (m\omega_p^2/kT)^{1/2}$ the Debye screening wave number, $\omega_p = (4\pi n e^2/m)^{1/2}$ the plasma frequency, and $v_F = (3\pi^2 n)^{1/3}\,\hbar/m$ the Fermi velocity of the electron gas.

With the approximations Eqs. (18)–(20), we have performed numerical integrations of Eqs. (11) and (15) for a fixed electron density $n = 10^{25}$ cm^{-3} ($r_s = 0.54$) and for various plasma temperatures in the velocity range $v \lesssim v_e$. The results for $v/v_F = 0.1$, 0.5, and 1.0 are shown in Fig. 2 as a function of the reduced temperature $\theta = kT/E_F$. In Fig. 2(a), the values of the energy straggling Ω and of the stopping power S were scaled to make them independent of Z, as Ω^2/Ω_B^2 (solid lines) and as $2kTS/\Omega_B^2$ (dashed lines) where $\Omega_B^2 = 4\pi n Z^2 e^4$ is the Bohr value for the straggling of a high-velocity particle (i.e., $v \gg v_e$). Figure 2 (b) shows the straggling-to-stopping relation $\Omega^2/2E_F S$ versus Θ. This illustrates how the asymptotic behavior, Eq. (16), is approached when the condition imposed by Eq. (17) is fulfilled.

An analytical result for the low-velocity stopping power and energy straggling, valid for $kT \gg E_F$, can be obtained by further approximating Eq. (19) for a nondegenerate electron gas, viz.,

$$\epsilon_2(q,\omega) \cong \frac{2m^2e^2\omega}{(\hbar q)^3}\,\frac{n\hbar}{2}\left(\frac{2\pi}{mkT}\right)^{3/2}\exp\left(\frac{-\hbar^2q^2}{8mkT}\right). \tag{21}$$

With Eq. (21), Eqs. (11) and (15) can be integrated simply, and yield

$$\Omega^2 \cong 2kTS \cong \frac{4}{3}\left(\frac{2\pi m}{kT}\right)^{1/2} nZ^2e^4 v[(1+y)e^y E_1(y) - 1] , \tag{22}$$

where $E_1(y)$ is the exponential integral of the argument $y = \hbar^2 q_D^2/8mkT = (\hbar\omega_p/kT)^2/8$. For $y \ll 1$, $E_1(y) = \ln(1/y) - 0.577$, and we finally obtain

$$\Omega^2 \cong 2kTS \cong \frac{8}{3}\left(\frac{2\pi m}{kT}\right)^{1/2} nZ^2e^4 v\left[\ln\left(\frac{kT}{\hbar\omega_p}\right) + \frac{1}{4}\right]. \tag{23}$$

This result is shown with dash-dot curves in Fig. 2(a). It quickly approaches the numerical results when Θ becomes larger than one.

We can now make a few comments on the physical meaning of our results. For $\Theta = 0$, only electrons close to the Fermi surface can participate in the energy loss of a slow particle. The stopping power starts from a value close to the one calculated by Fermi and Teller[18] at $T = 0$, and then decreases monotonically with increasing temperature; when $kT \gg E_F$, it becomes $\propto vT^{-3/2}\ln(kT/\hbar\omega_p)$. This asymptotic behavior is analogous to the well-known Bethe-Bolch dependence $v^{-2}\ln(2mv^2/\hbar\omega_p)$ at high velocities $v \gg v_e$, applicable to both cold and hot plasmas. In fact, they are two manifestations of the decrease in the energy-loss rate with increasing *relative* velocity between the particle and the electrons in the medium.

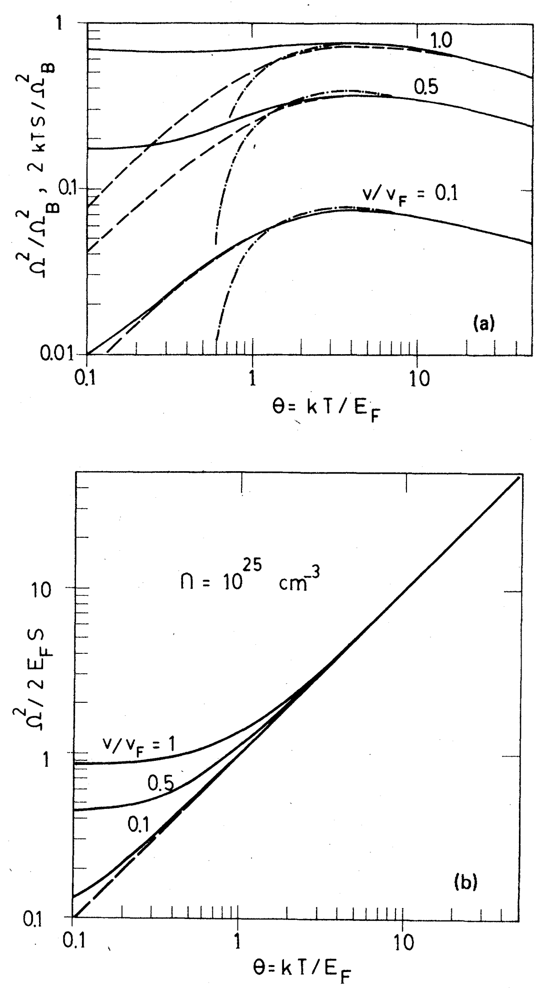

FIG. 2. (a) Values of the stopping power S and energy straggling Ω for slow heavy ($M \gg m$) particles, $v/v_F = 0.1$, 0.5, and 1.0, in an electron gas of density $n = 10^{25}$ cm^{-3}, as a function of the reduced temperature $\Theta \equiv kT/E_F$. The values are scaled according to Ω^2/Ω_B^2 (solid lines) and $2kTS/\Omega_B^2$ (dashed lines), where $\Omega_B^2 = 4\pi nZ^2e^4$ is the straggling for a swift particle of charge Ze. The high-temperature approximation of Eq. (23) is shown with dash-dot lines. (b) Results for the relation $\Omega^2/2E_F S$ as a function of the electron gas temperature. For $\theta > 1$ thermal fluctuations dominate the energy straggling and the relation Ω^2/S becomes independent of density and velocity, as predicted by Eq. (16).

The situation is different for the energy straggling. With increasing temperature there is an initial increase in the energy straggling, as more electrons are able to absorb energy. This is followed by a maximum and then a decline proportional to $vT^{-1/2}\ln(kT/\hbar\omega_P)$. The energy straggling for a slow particle in a hot electronic plasma ($kT \gg E_F$) is a decreasing function of the relative velocity $v_{rel} \simeq v_e \simeq (kT/m)^{1/2}$. By contrast, the high-velocity dependence of the straggling, in both cold and hot plasmas, is simply given by $\Omega_B^2 = 4\pi n Z^2 e^4$. These two limiting conditions are, for the straggling, qualitatively different. At high velocities $v \gg v_e$ *spontaneous* processes are dominant [$N(\omega) \ll 1$ for large energy transfers $\hbar\omega$] and the medium temperature is unimportant, just as in the behavior of the stopping power. But for a slow particle $v \ll v_e$ in a hot plasma, the occurrence of *induced* processes, i.e., successive events of emission and absorption, stimulated by the existence of a large number of thermally excited quanta in the medium [$N(\omega) \gg 1$ for all the frequencies of interest], becomes the dominant mode for the fluctuations in the particle energy. Curiously, the straggling in this high-temperature limit becomes essentially equal to the stopping power, Eq. (16), which depends at all degeneracies on spontaneous processes alone.

Figure 3 shows the velocity dependence of the

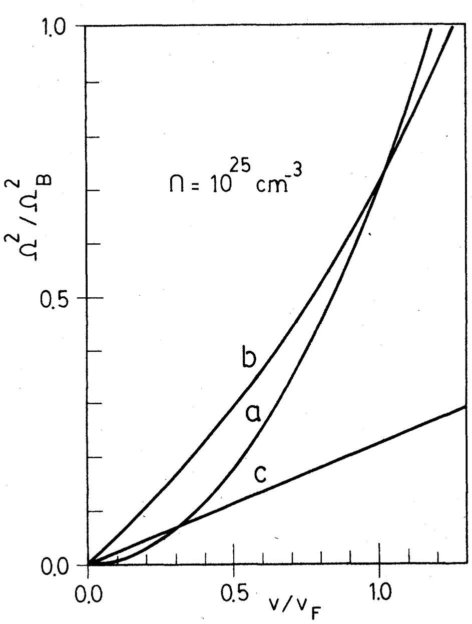

FIG. 3. Velocity dependence of the energy of heavy charged particles in an electron gas at finite straggling temperature. Curve (a), for $\Theta = 10^{-2}$, corresponds to a nearly degenerate plasma, where $\Omega^2 \propto v^2$; curve (b), $\Theta = 1.1$, corresponds to the transition region $kT \sim E_F$; curve (c), $\Theta = 510$, is for a hot plasma where $\Omega^2 \propto v$, as given by Eq. (23).

straggling for $v \lesssim v_F$ and three temperatures of the plasma. Curve (a) is for a nearly degenerate electron gas where $\Omega^2 \propto v^2$, curve (b) is for the transition regime $kT \sim E_F$, and case (c) corresponds to a hot plasma $kT \gg E_F$ where $\Omega^2 \propto v$. As found by Skupsky,[3] the stopping power S in the low-velocity region considered in this paper is, at all plasma temperatures, proportional to the particle velocity v.

IV. AN ILLUSTRATIVE EXAMPLE

Consider an α particle of kinetic energy $E_\alpha =$ 3.5 MeV as produced in the dominant DT reaction of a fusing plasma. This situation pertains to the energy transport in controlled fusion or in stellar matter. The accumulated straggling in energy, δE, after the particle has slowed down to some energy $E < E_\alpha$ over a trajectory of length x becomes

$$(\delta E)^2 = \int_0^x \Omega^2 dx = \int_E^{E_\alpha} \frac{\Omega^2(E')}{S(E')}\, dE' . \tag{24}$$

We discuss two limits of $\Theta \equiv kT/E_F$.

1. Hot plasma $\Theta \gg 1$. With Eq. (16), we obtain immediately

$$(\delta E)^2 = 2kT(E_\alpha - E). \tag{25}$$

Over the range of the particle $E \ll E_\alpha$ the energy straggling due to electronic process becomes

$$\delta E \simeq (2kT\,E_\alpha)^{1/2}, \tag{26}$$

independent of the plasma density. At the ICF and MCP ignition temperatures, $T \sim 10^8$ K. This is comparable to the width of the initial distribution of E_α caused by the thermal motion of the reactants. We find $\delta E \simeq 260$ keV. In stars, where $kT \sim 1$ keV, this value becomes $\delta E \simeq 80$ keV. If one adds nuclear straggling and stopping in Eq. (24), Ω^2 becomes larger by a factor ~1.4 while S does not change perceptively, and so, δE increases by at most 20% due to nuclear scattering.

2. Cold, dense plasma $\Theta \ll 1$, $\chi^2 = v_0/\pi v_F \ll 1$, and $v < v_F$. The stopping power takes the form[13,18,19]

$$S \simeq \frac{2}{3\pi}\,\frac{Z^2 e^4 m^2}{\hbar^3}\, v \ln\frac{\pi v_F}{v_0} \tag{27}$$

and the straggling[14]

$$\Omega^2 = \Omega_B^2\,(v/v_F)^2, \tag{28}$$

so that

$$\frac{\Omega^2}{S} = \frac{2mv_F v}{\ln(\pi v_F/v_0)}\,, \tag{29}$$

and Eq. (24) yields

$$(\delta E)^2 = \frac{8}{3} \frac{E_F E_\alpha}{\ln(\pi v_F/v_0)} \frac{v_\alpha}{v_F} \left(1 - \frac{v^3}{v_\alpha^3}\right) \quad (30)$$

in terms of $E_F = \frac{1}{2} m v_F^2$, $v_\alpha = (2E_\alpha/M_\alpha)^{1/2} = 6v_0$, $v = (2E/M_\alpha)^{1/2}$, and $M_\alpha = 4u$, the mass of the α particle. At electron densities $n \sim 10^{26}$ cm^{-3}, for which $r_s \simeq 0.25$ and $v_F \simeq 8v_0$, and at temperatures such that $kT \ll E_F \sim 1$ keV, the straggling, when $v \ll v_\alpha$, becomes $\delta E \simeq 40$ keV. These examples demonstrate the content of Fig. 2(b), viz., that straggling increases with temperature despite the slow change of Ω^2 with θ depicted in Figs. 2(a). Such effects influence the range straggling in ways that should be unimportant for confinement fusion.

V. CONCLUSIONS

The quantum-mechanical treatment of the energy loss in an electron gas in thermal equilibrium, as formulated here, applies to a wide range of nonrelativistic plasmas, which includes most of the cases found in nature. Equations (11) and (15) provide the basis for a unified description, where the temperature-dependent contributions from energy-gain and -loss processes to the stopping power and straggling integral have been accounted for explicitly. A comprehensive quantum-mechanical analysis of the energy loss and straggling of ions in plasmas calls now for a thorough study of the temperature dependence of the dielectric function $\epsilon(q,\omega)$.

In this paper we have concentrated on the main new effects, which occur for partially degenerate systems, $kT \sim E_F$, in the velocity range $v \lesssim v_e$, and can be studied in terms of low-frequency approximations to $\epsilon(q,\omega)$. This permits us to describe the transition from degenerate to nondegenerate plasmas, of much interest for studies of laser and ion-beam ignition of fusion reactions via the inertial confinement of DT pellets. In this velocity range, the temperature and velocity dependences of the stopping power S and of the energy straggling Ω have been studied.

A relation between S and Ω is obtained, namely, $\Omega^2 = 2\,kTS$, which applies to a slow particle moving in a hot plasma. A first analysis indicates that this relation could provide a method to evaluate the electron temperature of a plasma under working conditions of tokamak machines using, for instance, a proton beam probe. One should, however, be aware of other sources of energy-loss fluctuations in the case of heavy-ion beams[20] due to fluctuations in the charge state of the ions. For partially ionized plasmas, the contribution from excitations and ionizations of the ion cores should be added to the results of our treatment.

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