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Pair production in the field of atomic electrons

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The total cross section for pair production in the field of atomic electrons (triplet production) is considered in detail. A discussion of the relevant theoretical papers is presented; the more familiar cross section for pair production in the Coulomb field of the nucleus serves for comparison. The effect of exchange and γ-e interactions, atomic binding, and radiative corrections are all considered. Earlier expressions for the recoil distribution have been simplified considerably. Numerical values for screening corrections, exchange and γ-e contributions, and total cross sections are given. Comparison is made with experimental cross sections for pair production on hydrogen and deuterium.

I. INTRODUCTION

In recent years, precision measurements of the total absorption cross section for photons have been made in the energy range from 10–160 MeV.[1] The aim of these experiments was the determination of the nuclear absorption cross section. This is, however, only a fraction, of at most 5–7 percent, of the total absorption cross section. The main part comes from atomic processes. Thus, these measurements call for very accurate cross sections for the various electromagnetic processes: pair production, the Compton effect, triplet production, and the photoelectric effect. In connection with the analysis of these experiments it has been recognized[2,3] that small discrepancies exist between the atomic cross sections derived from these measurements and the current theoretical values, some of which have uncertainties of the same order of magnitude as the experimental uncertainties. In an attempt to locate this discrepancy between theory and experiment, we have been making a systematic investigation of the theoretical expressions for the various atomic cross sections, both to establish the most accurate cross section currently obtainable and to attempt to place error estimates on the theoretical expressions. In this paper we examine the best available expressions for pair production in the field of atomic electrons (triplet production) and evaluate the total triplet cross section as a function of energy in the range 10–350 MeV for a number of elements. We will make reference to a number of theoretical expressions for this cross section existing in the literature, specifically to the work of Wheeler and Lamb,[4] Borsellino,[5] Ghizzetti,[6] Suh and Bethe,[7] Mork,[8] and Haug.[9] Each of them has made significant contributions to the understanding of this process and from their work we will obtain what we believe to be the most accurate currently available expression for the triplet cross section. In Sec. II of this paper we present a comparison of triplet production with pair production in the field of the nucleus. A discussion of the diagrams, momentum distributions and total cross sections for each of these processes is given. A brief review of the essential elements contributed by the authors just mentioned in Refs. 4–9 is included. The Coulomb corrections and radiative corrections to the total triplet cross section are discussed at the end of this section. In Sec. III we give some details pertinent to the numerical calculations. A discussion of the results of this paper is given in Sec. IV. In Appendix A we give detailed expressions for the Borsellino recoil distribution and total cross section for triplet production. We show that certain integrals in the original work of Borsellino may be expressed in terms of dilogarithms. Finally, in Appendix B we give critical comments and an errata for the work of Mork and Olsen[16] on the radiative correction to the total-pair cross section.

II. PAIR AND TRIPLET PRODUCTION CROSS SECTIONS

Before entering into the details of these calculations, however, it would be well to compare pair production in the field of an electron with the more familiar process of pair production in the Coulomb field of the nucleus. If we allow for the possibility of nuclear recoil (i.e., consider the nucleus to have a finite mass) then there are (in 1st Born approximation) four diagrams in the case of pair production in the nuclear field, shown in Fig. 1. The diagrams I and II are generally called "Bethe-Heitler" diagrams. They account for recoil solely kinematically, in that the recoiling nu-

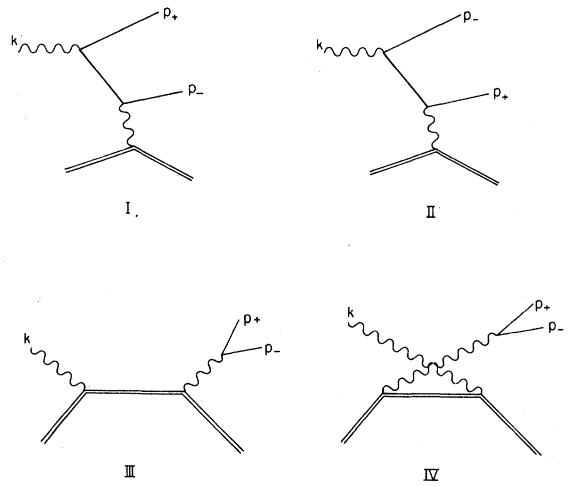

FIG. 1. Feynman diagrams for pair production in the field of the nucleus including recoil. Diagrams I and II are the "Bethe-Heitler" diagrams.

cleus may absorb energy as well as momentum. If the momentum transfers q which are important for the particular experimental situation are such that $q^2/2M \ll k$ (M being the nuclear mass, k the photon energy) then the nuclear field may be approximated by a static potential, and we have the diagrams shown in Fig. 2, which give the familiar Bethe-Heitler[10] cross section. The diagrams III and IV in Fig. 1 are generally called "Compton" diagrams in view of their obvious similarity to the diagrams for Compton scattering. It should be noted that the Bethe-Heitler diagrams I and II have only one photon exchanged with the nucleus, whereas the Compton diagrams III and IV are two-photon exchange diagrams.

In the case of pair production from a *free* electron, we again have the four diagrams shown in Fig. 1, but in addition four more are obtained by exchange of the final electrons. In the literature[8] the diagrams I and II of Fig. 3 are referred to as "Borsellino" diagrams (since they are the ones considered in his work, Ref. 5), and III and IV are called γ-e diagrams. These four are identical

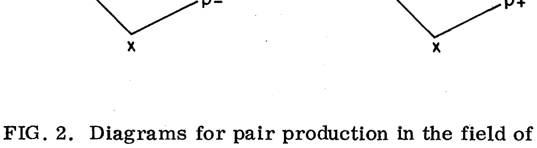

FIG. 2. Diagrams for pair production in the field of a static potential.

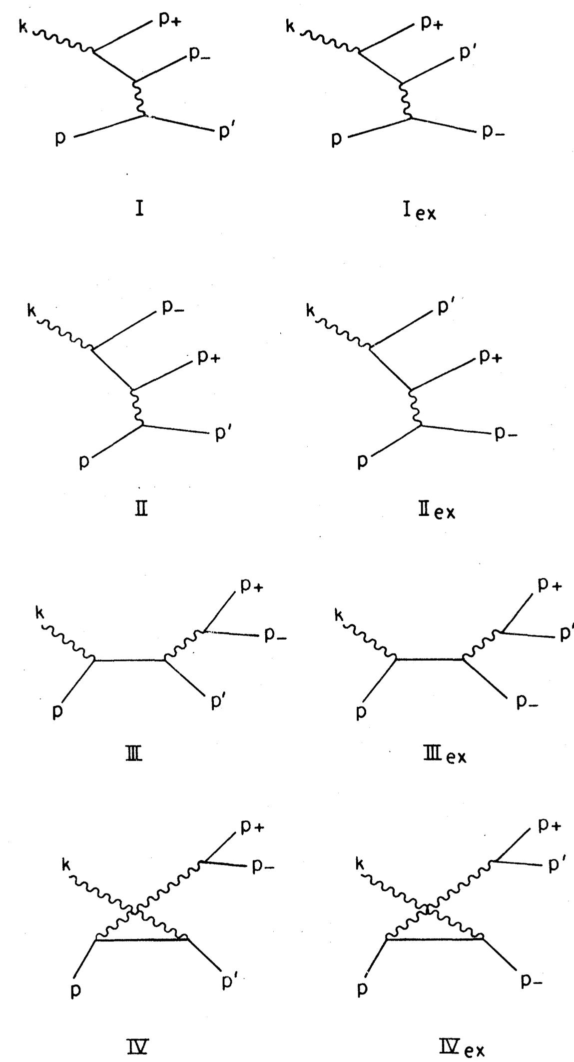

FIG. 3. Feynman diagrams for pair production in the field of a free electron. I and II are the "Borsellino" diagrams, III and IV are the γ-e diagrams, I_{ex} and II_{ex} are the Borsellino exchange diagrams, and III_{ex} and IV_{ex} are the γ-e exchange diagrams.

to the four diagrams shown in Fig. 1 provided that we neglect, in III and IV of Fig. 1, any possible contribution from excited states of the nucleus in the intermediate state. Returning to Fig. 3, I_{ex} and II_{ex} are referred to[8] as Borsellino exchange diagrams and III_{ex} and IV_{ex} are referred to as γ-e exchange diagrams.

In the eight diagrams shown in Fig. 3 a pair is produced in the field of a *free* electron. Thus they do not truly describe the situation in which the pair is created in the field of an electron bound in the atom. Likewise, the four diagrams in Fig. 1 do not truly represent the situation in which the pair is produced in the field of the atom (the nucleus together with bound electrons). For neither process is an "exact" solution possible. Under the most general conditions this would involve a complete solution of the many-body problem. However, under the assumption that the created pair (or, in the case of bremsstrahlung, the incident and scattered electron) have sufficiently high energy, we may follow the approach of Wheeler and Lamb.[4] More specifically, this requires (i) that the time of traversal of the atom by the created pair be small compared to the time τ associated with the atomic electrons ($\tau \sim h/E$) and (ii) that the important momentum transfers q are small enough compared to the momenta of the created particles that the motion of the atomic electrons may be neglected during the time that the process occurs. Under these assumptions, for pair production in the nuclear field the momentum is absorbed by the atom as a whole, without changing its state. Then for high energies and small momentum transfers ($q \ll mc$), the effects of the bound electrons may be accounted for by the atomic form factor $F(q)$ (The term "high energy" is generally taken to imply particle and photon energies much larger than the electron rest energy, mc^2. However, we believe that the model just described requires only that the created pair have energies much greater than the binding energy of the inner (K-shell) electrons. Even for high-Z elements, these are of the order of 100 keV, which is 1% of the lowest photon energy considered in this paper. We therefore neglect the atomic binding energy throughout, in both the dynamics and the kinematics.):

$$F(q)=\frac{1}{Z}\int \Psi_0^*(r_1,\ldots,r_Z)\sum_{i=1}^{Z} e^{i\mathbf{q}\cdot\mathbf{r}_i}\Psi_0(r_1,\ldots,r_Z)d\tau_1,\ldots,d\tau_Z\,. \tag{1}$$

For triplet production, the atomic state changes and we have instead the incoherent scattering function $S(q)$:

$$\begin{aligned}S(q)&=\frac{1}{Z}\sum_{f\neq 0}\left|\int \Psi_f^*(r_1,\ldots,r_Z)\sum_{i=1}^{Z} e^{i\mathbf{q}\cdot\mathbf{r}_i}\Psi_0(r_1,\ldots,r_Z)d\tau_1,\ldots,\ d\tau_Z\right|^2\\&=\frac{1}{Z}\sum_{i,j}\int \Psi_0^* e^{i\mathbf{q}\cdot(\mathbf{r}_i-\mathbf{r}_j)}\Psi_0 d\tau_1,\ldots,d\tau_Z - Z\,|F(q)|^2\,.\end{aligned} \tag{2}$$

Equations (1) and (2) were given in Ref. 4.

For large momentum transfers ($q \gtrsim mc$) the atomic electrons may be treated as free (in triplet production) and the nuclear field may be treated as a point Coulomb field in nuclear pair production. Thus,

$$F(q)\to 0\,,$$
$$S(q)\to 1\,,$$

for $q \gtrsim mc$. Following this approach, the cross section for nuclear pair production may then be written as

$$\sigma_n(k)\approx Z^2\int_{q_m}^{q_0}[1-F(q)]^2\frac{d\sigma_n}{dq}dq+Z^2\int_{q_0}^{q_M}\frac{d\sigma_n}{dq}dq \tag{3}$$

and that for triplet production as

$$\sigma_t(k)\approx Z\int_{q_m}^{q_0}S(q)\frac{d\sigma_t}{dq}dq+Z\int_{q_0}^{q_M}\frac{d\sigma_t}{dq}dq\,. \tag{4}$$

Here $d\sigma_n/dq$ is the cross section in a point Coulomb field $V=-e^2/r$, integrated over all variables other than the recoil momentum, $d\sigma_t/dq$ is the cross section for triplet production from a free electron, similarly integrated, and q_m and q_M are the kinematic limits for each of the processes:

$$\left.\begin{matrix}q_m\\q_M\end{matrix}\right\}=k\mp(k^2-4)^{1/2}\quad\text{for nuclear pair production}\,, \tag{3$'$}$$

$$\left.\begin{matrix}q_m\\q_M\end{matrix}\right\}=\frac{k(k-1)\mp(k+1)[k(k-4)]^{1/2}}{2k+1}\quad\text{for triplet production}\,. \tag{4$'$}$$

The momentum q_0 is of order mc, chosen so that

$$F(q_0)\approx 0\,,$$
$$S(q_0)\approx 1\,.$$

This is the procedure that was used by Bethe[11] to obtain the cross sections for bremsstrahlung and pair production in the nuclear field, and by Suh and Bethe[7] for pair production in the field of an electron, in both cases for high energies. Now for the nuclear field, the problem of the proper choice of q_0 and the small errors involved in this division of the integration region can be avoided. We are concerned here with the total cross section, for which the important contribution comes from momentum transfers $q \lesssim mc$. Thus, the contribution from the Compton diagrams, III and IV in Fig. 1, which are of order $Z(q^2/2M)$, as well as the contribution from the kinematic recoil terms in the Bethe-Heitler diagrams, I and II in Fig. 1, which are of order $q^2/2M$, can both be neglected, since the significant values of $q^2/2M$ are then small relative to the energy of the incident photon k (or of the incident electron, in the case of bremsstrahlung). Then in Eq. (3) we need only the recoil distribution for a fixed point Coulomb potential $d\sigma_{\text{Coul}}/dq$, and this has been given without further approximation by Jost, Luttinger, and Slotnick.[12] We may then write, in place of Eq. (3),

$$\sigma_n(k) = Z^2 \int_{q_m}^{q_M} [1 - F(q)]^2 \frac{d\sigma_{\text{Coul}}(k,q)}{dq} dq . \tag{5}$$

In the case of pair production in the electron field, the situation is somewhat more subtle. First we must deal with the problem of the γ-e and exchange diagrams—III, IV and I_{ex}, II_{ex}, III_{ex} and IV_{ex} in Fig. 3. None of these are truly properly treated in the "static" picture of the atom. On the other hand they are important only for large momentum transfers ($q \gg mc$), for which we do not invoke the static picture at all, but rather treat the atomic electrons as free. Thus, we neglect these diagrams in the recoil distribution used in the first integral in Eq. (4) ($q_m \leq q \leq q_0$), as was also done by Suh and Bethe,[7] retaining, in $q_m \leq q \leq q_0$, only the contribution from the Borsellino diagrams, I and II in Fig. 3:

$$\sigma_t(k) \approx Z \int_{q_m}^{q_0} S(q) \frac{d\sigma_{\text{Bors}}}{dq} dq + Z \int_{q_0}^{q_M} \frac{d\sigma_t}{dq} dq . \tag{6}$$

A simple rewriting of this expression gives

$$\sigma_t(k) = Z \int_{q_m}^{q_M} \frac{d\sigma_{\text{Bors}}}{dq} dq + Z \int_{q_m}^{q_M} \left(\frac{d\sigma_t}{dq} - \frac{d\sigma_{\text{Bors}}}{dq} \right) dq - Z \int_{q_m}^{q_M} [1 - S(q)] \frac{d\sigma_{\text{Bors}}}{dq} dq$$
$$- Z \int_{q_m}^{q_0} \left(\frac{d\sigma_t}{dq} - \frac{d\sigma_{\text{Bors}}}{dq} \right) dq + Z \int_{q_0}^{q_M} [1 - S(q)] \frac{d\sigma_{\text{Bors}}}{dq} dq . \tag{7}$$

Again, as discussed in arriving at Eq. (5), we may neglect the next to the last term in the expression above since $d\sigma_t/dq$ and $d\sigma_{\text{Bors}}/dq$ differ only in the contribution from the γ-e and exchange terms, which may be neglected for $q \leq q_0$. In addition, we may neglect the last term above since $S(q) \approx 1$ for $q \geq q_0$. We thus arrive at the expression

$$\sigma_t(k) = Z [\sigma_{\text{Bors}}(k) + \Delta\sigma_{\text{Haug}}(k) - \Delta S(k, Z)] , \tag{8}$$

where

$$\sigma_{\text{Bors}}(k) = \int_{q_m}^{q_M} \frac{d\sigma_{\text{Bors}}}{dq} dq , \tag{9}$$

$$\Delta\sigma_{\text{Haug}}(k) = \int_{q_m}^{q_M} \left(\frac{d\sigma_t}{dq} - \frac{d\sigma_{\text{Bors}}}{dq} \right) dq , \tag{10}$$

$$\Delta S(k, Z) = \int_{q_m}^{q_M} [1 - S(q)] \frac{d\sigma_{\text{Bors}}}{dq} dq . \tag{11}$$

A few observations are in order at this point. The largest term here, $\sigma_{\text{Bors}}(k)$, is the total triplet cross section in the field of a free electron, as given by Borsellino[5] and Ghizzetti[6] (thus including only diagrams I and II in Fig. 3). It has the great calculational advantage that it is given analytically as well as in the form of an expansion in successive powers of $1/k$, given here in Appendix A. This is of particular importance for the intermediate energy region (10–160 MeV) with which we are dealing here. As may be seen from this expansion, and is discussed in detail by Suh and Bethe,[7] the leading term in this expansion is identical to the high energy limit of the cross section for pair production in a pure Coulomb field

$$\sigma_{\text{Bors}}(k) \underset{k\to\infty}{=} \alpha r_0^2 [\tfrac{28}{9} \ln(2k) - \tfrac{218}{27}] , \tag{12}$$

where $\alpha = 1/137.036$ is the fine structure constant and $r_0 = 2.817\,77 \times 10^{-13}$ cm is the classical electron radius. However, in the intermediate energy region of concern to us here, the total cross section $\sigma_{\text{Bors}}(k)$ may differ by almost a factor of 2 from the total cross section for pair production in the point Coulomb field $\sigma_{\text{Coul}}(k)$ [given by Eq. (5) with $F(q) \equiv 0$, $Z = 1$]. This is shown graphically in Fig. 4, where we plot the percentage difference between $\sigma_{\text{Bors}}(k)$ and $\sigma_{\text{Coul}}(k)$ as a function of the photon energy. As is clear from that figure, although the two cross sections do indeed approach the same limit and become essentially identical for energies in the GeV region, they differ significantly for energies below 100 MeV. This point may also be seen from the successive terms in the high energy expansions for the two cross sections. For moderate photon energies the correction terms to $\sigma_{\text{Bors}}(k)$ are larger by a factor k than the corrections to $\sigma_{\text{Coul}}(k)$. For triplet production,

$$\sigma_{\text{Bors}}(k) = \alpha r_0^2 \left[\tfrac{28}{9} \ln(2k) - \tfrac{218}{27} + \frac{1}{k} \left(-\tfrac{4}{3} \ln^3(2k) + 3 \ln^2(2k) - \frac{60 + 16a}{3} \ln(2k) + \frac{123 + 12a + 16b}{3} \right) + O\left(\frac{\ln^3(2k)}{k^2} \right) \right]$$
$$(a = -2.467\,40, \quad b = 1.803\,10), \tag{13}$$

whereas for pair production in the point Coulomb field of the nucleus,[13]

$$\sigma_{\mathrm{Coul}}(k) = \alpha r_0^2 \left[\tfrac{28}{9} \ln(2k) - \tfrac{218}{27} + \left(\frac{2}{k}\right)^2 \left(\tfrac{2}{3} \ln^3(2k) - \ln^2(2k) + (6 - \tfrac{1}{3}\pi^2)\ln(2k) - \tfrac{7}{2} + 2\zeta(3) + \frac{\pi^2}{6} \right) + O\left(\frac{\ln 2k}{k^4}\right) \right],$$
$$[\zeta(3) = 1.202\,0569\cdots]. \quad (14)$$

It should be noted, however, that this equality in the limit of high energies, of the total cross section for pair production in the field of an electron with that for pair production in the field of the nucleus (apart from an obvious factor Z), holds only when screening is neglected. In fact the effects of screening dominate at extremely high energies. The total cross sections for these two processes are then constant (rather than increasing logarithmically with photon energy) and are moreover unequal. Thus, if the atomic screening is given by the Thomas-Fermi model, we have, in the limit of extremely high energies,[14]

$$\sigma_n(k) \underset{k\to\infty}{=} Z^2 \alpha r_0^2 [\tfrac{28}{9} \ln(183 Z^{-1/3}) - \tfrac{2}{27}], \quad (15)$$

whereas

$$\sigma_t(k) \underset{k\to\infty}{=} Z \alpha r_0^2 [\tfrac{28}{9} \ln(1271 Z^{-2/3}) - \tfrac{2}{27}]. \quad (16)$$

The difference between $\sigma_n(k)$ and $\sigma_t(k)$ as given in Eqs. (15) and (16) should be expected. The screening functions multiplying the respective recoil distributions $\{[1 - F(q)]^2$ in Eq. (3) and $S(q)$ in Eq. (6)$\}$ are very different in the region of momentum transfers that is strongly affected by the screening, $q_m \leq q \leq q_0 \ll 1$.

Let us now return to our expression for the triplet cross section, Eq. (8), and consider the second term there, $\Delta\sigma_{\mathrm{Haug}}(k)$. This term is the difference between the total triplet cross section as calculated by Haug[9] (including all eight diagrams of Fig. 3) and that of Borsellino (who includes only diagrams I and II in Fig. 3). This correction may be obtained directly from Table 1 in Ref. 9 as a function of the photon energy k and represents the contribution of the γ-e and exchange diagrams. It is to be noted that neither $\sigma_{\mathrm{Bors}}(k)$ nor $\Delta\sigma_{\mathrm{Haug}}(k)$ invoke the high-energy approximation. Haug finds that for incident photon energies $k > 7.5$ MeV, the correction $\Delta\sigma_{\mathrm{Haug}}(k)$ is always less than 1.2% of the total triplet cross section in the field of a free electron. The correction term $\Delta\sigma_{\mathrm{Haug}}(k)$ is not included in the calculation of Suh and Bethe; the first calculation of the contribution of the γ-e and exchange diagrams was performed some eight years after their work by Mork.[8] His calculation employed a Monte Carlo procedure for the integration of the differential cross section. In the most recent calculation of this process, however, Haug[9] has been able to perform the integration over the angles of the outgoing electrons analytically, thus improving considerably the accuracy of the numerical results. One should thus choose, for the numerical evaluation of the correction $\Delta\sigma_{\mathrm{Haug}}(k)$, the values given by Haug rather than those appearing in the earlier work of Mork. Mork's work is nonetheless extremely valuable in that it is the only one in which the contributions of the individual diagrams (and their interference) are calculated separately, integrated over all variables, as a function of the incident photon energy.

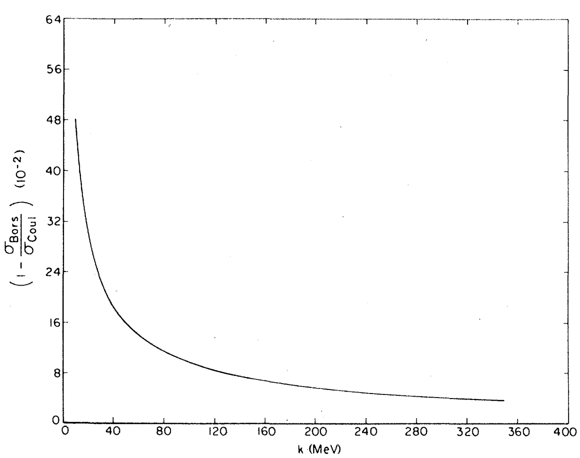

FIG. 4. Percentage difference between the total cross sections $\sigma_{\mathrm{Bors}}(k)$ and $\sigma_{\mathrm{Coul}}(k)$ as a function of photon energy k.

Finally we consider the last term in Eq. (8), the "screening correction" $\Delta S(k, Z)$. As we have already noted, the significant contribution to this correction, given by Eq. (11), comes only from small momentum transfers, $q_m \leq q \leq q_0 \ll 1$. Moreover, for this region of small momentum transfers the recoil distributions $d\sigma_{\mathrm{Bors}}/dq$ and $d\sigma_{\mathrm{Coul}}/dq$ approach the same limiting distribution for very high energies, $k \gg 1$. This was shown by Suh and Bethe,[7] who note that this may be understood from the observation that for these very small momentum transfers the recoil energy taken up by the field particle, regardless of whether it is an electron or a nucleus, is negligible compared to its rest mass. The field in which the pair is produced thus behaves as if it were a static one. In Ref. 7 an additional condition is imposed on q, viz., $\frac{1}{2}kq - 1 \gg 1/Mk$. While it is true, as noted there, that this restriction is unimportant, we would like to point out that this additional condition is in fact not needed for the analysis leading to Eq. (6) in Ref. 7. If throughout that equation one replaces the term $1 - 2/kq$ by $1 - q_m/q$, and uses the exact expression for q_m, then the resulting expression is valid for all q in the region $q_m \leq q \leq q_0 \ll 1$. However, the approach to the high energy limit of the recoil distribution is very slow. Just as we saw in Eq. (13) for the total cross section, so also for the recoil distribution the correction terms[7] to the high-energy limit of $d\sigma_{\text{Bors}}/dq$ are of order $1/k$. Thus the conclusion illustrated in Fig. 4, viz., that in the intermediate energy region $\sigma_{\text{Coul}}(k)$ is not a good approximation for $\sigma_{\text{Bors}}(k)$, is also true for the screening correction $\Delta S(k, Z)$. If in Eq. (11) we replace $d\sigma_{\text{Bors}}/dq$ by its small q high energy limit, thereby writing

$$\Delta S(k, Z) \approx \int_{q_m}^{q_M} dq[1 - S(q)] \left.\frac{d\sigma_{\text{Coul}}}{dq}\right|_{k \gg 1;\, q_m \leq q \leq q_0 \ll 1} \tag{17}$$

as was done by Wheeler and Lamb[4] and by Suh and Bethe,[7] then the results are indeed quite accurate for energies in the GeV region (errors $< 1\%$), but again may be in error by as much as a factor of 2 in the intermediate energy region considered here. This is shown in Fig. 5, where we plot the percentage difference between $\Delta S_{\text{Bors}}(k, Z)$ and $\Delta S_{\text{Coul}}(k, Z)$ as a function of photon energy for Pb. Here $\Delta S_{\text{Bors}}(k, Z) = \Delta S(k, Z)$ as calculated from Eq. (11) using the recoil distribution $d\sigma_{\text{Bors}}/dq$ [and integration limits given by (4′)], whereas $\Delta S_{\text{Coul}}(k, Z) = \Delta S(k, Z)$ as calculated from Eq. (11) using the recoil distribution $d\sigma_{\text{Coul}}/dq$ and integration limits given by (3′).

We thus see that the contributions of the "Borsellino" diagrams converge very slowly to their high-energy limit. They are within 1% of this limit only when we reach the GeV region. Thus, one

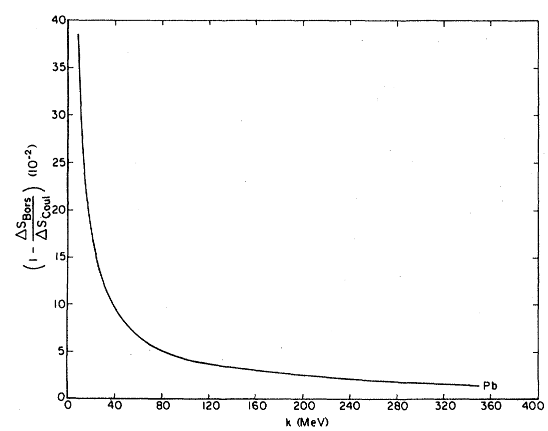

FIG. 5. Percentage difference between the screening corrections $\Delta S_{\text{Bors}}(k,Z)$ and $\Delta S_{\text{Coul}}(k,Z)$ as a function of the photon energy k for Pb.

should not make any high-energy approximations in the contribution from these diagrams in the intermediate energy region. This is in sharp contrast to the behavior of the contribution of the γ-e and exchange diagrams. As shown in Fig. 3 of Ref. 8 and in Table 1 of Ref. 9, the contribution of the γ-e and exchange diagrams is very large for photon energies between threshold ($4\,mc^2$) and a few MeV, but goes to zero rapidly with increasing k, being less than 1.2% of σ_t for $k > 7.5$ MeV. For this reason, as well as the fact that they give a small contribution for the small momentum transfers which are involved in the integral, Eq. (11) for $\Delta S(k, Z)$, we may neglect the γ-e and exchange diagrams in the screening correction $\Delta S(k, Z)$.

Finally, we mention two other corrections to the triplet cross section due to higher order interactions. The first is the Coulomb correction. As the effective Z of the target is unity, one may indeed expect that this correction will be small, except for energies very near threshold. If we use the Coulomb corrections to pair production in the nuclear field as a qualitative guide, then from the work of Øverbø[15] (with $Z = 1$) we see that although the Coulomb corrections are very significant for photon energies within a few keV of threshold, they are completely negligible for photon energies an MeV or more above threshold. They may therefore be neglected here. The second modification of σ_t as given in Eq. (8) is that due to radiative corrections. All existing calculations of the radiative correction to the total cross section for either pair production[16,17] or bremsstrahlung,[18] whether in the field of a nucleus or an electron, make use of the Weizsäcker-Williams method and the high-energy approximation. The only calculation specific to triplet production[17] gives the radiative correction to the positron spectrum, but not to the total cross section. We therefore follow here the suggestion of Mork[8] (see pages 1070 and 1071 of Ref. 8) and use the radiative correction to the total cross section for pair production in the nuclear field, calculated by Mork and Olsen[16] at high energies. They find (Table IV of Ref. 16) a radiative correction very close to 1%, essentially independent of energy and Z. We therefore include this by writing, as our final expression for the triplet cross section,

$$\sigma_t(k) = Zf_{\text{rad}}[\sigma_{\text{Bors}}(k) + \Delta\sigma_{\text{Haug}}(k) - \Delta S(k, Z)] , \tag{18}$$

where $f_{\text{rad}} \equiv 1.01$, and the other terms are given by our Eqs. (9), (10) and (11). To be sure, for the lowest part of the energy region considered here, this value for f_{rad} may be open to question.

Table I gives the unscreened triplet cross section $\sigma_t(k)$ for $Z = 1$ [Eq. (18) with $\Delta S(k, Z) \equiv 0$] as

TABLE I. The Borsellino total cross section $\sigma_{\mathrm{Bors}}(k)$, the correction due to exchange and γ-e diagrams, $\Delta\sigma_{\mathrm{Haug}}(k)$, and the unscreened triplet cross section $\sigma_t(k) = f_{\mathrm{rad}}[\sigma_{\mathrm{Bors}}(k) + \Delta\sigma_{\mathrm{Haug}}(k)]$, each in mb as a function of the photon energy k, in MeV. As in Eq. (18), we use $f_{\mathrm{rad}} \equiv 1.01$.

k (MeV)	$\sigma_{\mathrm{Bors}}(k)$ (mb)	$\Delta\sigma_{\mathrm{Haug}}(k)$ (mb)	$\sigma_t(k)$ unscreened (mb)
9.0	1.0063	−0.0028	1.0134
10.0	1.1560	0.0019	1.1695
15.0	1.8006	0.0171	1.8359
20.0	2.3103	0.0255	2.3592
25.0	2.7274	0.0307	2.7856
50.0	4.0962	0.0357	4.1733
75.0	4.9208	0.0363	5.0068
100.0	5.5078	0.0306	5.5938
125.0	5.9620	0.0251	6.0470
150.0	6.3318	0.0213	6.4166
175.0	6.6431	0.0190	6.7288
200.0	6.9118	0.0179	6.9990
250.0	7.3583	0.0181	7.4501
300.0	7.7207	0.0198	7.8179
350.0	8.0254	0.0210	8.1269

well as the values of $\sigma_{\mathrm{Bors}}(k)$ and $\Delta\sigma_{\mathrm{Haug}}(k)$.

III. COMPUTATIONS

The screening correction $\Delta S(k, Z)$ was computed by numerical integration of Eq. (11). The differential cross section

$$d\sigma_{\mathrm{Bors}}/dq \equiv d\sigma_{\mathrm{Bors}}(q,k)/dq$$

is given in Appendix A, where it is written in terms of the same functions as appear in the expression for the recoil distribution for pair production in a point Coulomb field, $d\sigma_{\mathrm{Coul}}/dq$, viz., logarithms and dilogarithms. Figure 6 shows the recoil distribution $d\sigma_{\mathrm{Bors}}(q,k)/dq$ for three different photon energies: $k = 10$, 50, and 100 MeV, and for comparison, the corresponding recoil distribution $d\sigma_{\mathrm{Coul}}(q,k)/dq$. The numerical integration of Eq. (11) was accomplished by using a Simpson integration routine with automatic step selection. For the incoherent scattering function $S(q) \equiv S(q,Z)$ we have used the tabulated values given in Ref. 19, interpolated for different momentum transfers q by a third order Spline function. All computations were carried out in double precision (18 significant digits). A check of the numerical procedure was made by integrating the recoil distribution $d\sigma_{\mathrm{Bors}}/dq$ over q, using precisely the same method as that used in the computation of the screening correction. The results of these calculations for several photon energies k were then compared with the cross sections obtained from the Borsellino-Ghizzetti expansion formula for triplet production in the field of a free electron, $\sigma_{\mathrm{Bors}}(k)$ in Eq. (A23) in Appendix A. The results were found to agree within the accuracy of the integration routine (10^{-4}).

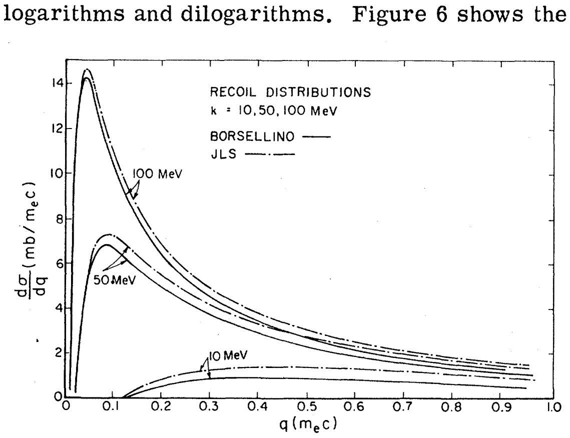

FIG. 6. The recoil distributions $d\sigma_{\mathrm{Bors}}/dq$ and $d\sigma_{\mathrm{Coul}}/dq$ (designated above as JLS; see Ref. 12) as a function of momentum transfer q for photon energies $k = 10$, 50, and 100 MeV.

IV. RESULTS AND DISCUSSION

The screening correction for pair production in the field of the atomic electrons, $Z\Delta S(k,Z)$, has been computed for several elements in the intermediate energy range of 9–350 MeV. The results of this calculation are given in Table II. Figure 7 displays $Z\Delta S(k,Z)$ as a percentage of the total triplet cross section for a free electron, $f_{\mathrm{rad}}[\sigma_{\mathrm{Bors}}(k) + \Delta\sigma_{\mathrm{Haug}}(k)]$.

Table III shows the screening correction to the cross section for hydrogen, obtained with the exact incoherent scattering function $S(q) = 1 - F^2(q)$. Here $F(q) = (1 + \pi^2 a_0^2 q^2)^{-2}$ is the hydrogen atomic form factor ($a_0 = 5.2918 \times 10^{-9}$ cm, the Bohr radius). These computations were extended up to 8

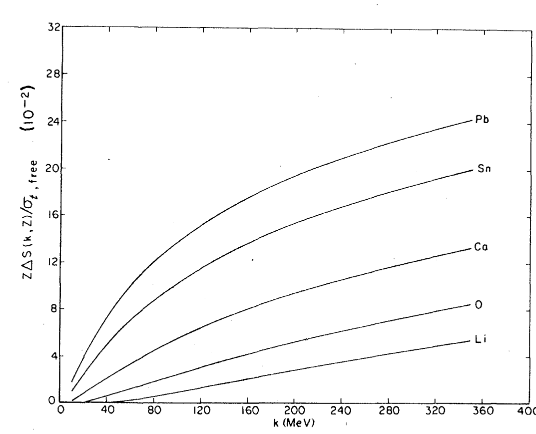

FIG. 7. The screening correction, $Z\Delta S(k,Z)$ as a percentage of the total triplet cross section for free electrons, $\sigma_{t,\,\mathrm{free}} = Zf_{\mathrm{rad}}[\sigma_{\mathrm{Bors}}(k) + \sigma_{\mathrm{Haug}}(k)]$.

TABLE II. The screening correction $Z\Delta S(k,Z)$ for pair production in the field of the atomic electrons, in mb, as a function of the photon energy k, in MeV, for various elements. The numbers in parentheses indicate powers of ten, i.e., $a(b)=a\times 10^{b}$.

k (MeV)	$Z=3$	$Z=4$	$Z=6$	$Z=8$	$Z=13$
9.0	3.4529(−07)	3.2742(−06)	4.9712(−05)	3.0837(−04)	4.4940(−03)
10.0	1.1263(−06)	8.5698(−06)	1.0282(−04)	6.0852(−04)	7.7397(−03)
15.0	3.1683(−05)	1.7089(−04)	1.3090(−03)	5.8116(−03)	4.3734(−02)
20.0	2.0006(−04)	9.5657(−04)	5.8725(−03)	2.0812(−02)	1.1303(−01)
25.0	7.1837(−04)	3.0710(−03)	1.5821(−02)	4.7625(−02)	2.0974(−01)
50.0	1.7502(−02)	5.0016(−02)	1.5352(−01)	3.1582(−01)	9.4251(−01)
75.0	6.8319(−02)	1.5753(−01)	3.8345(−01)	6.9504(−01)	1.8928(+00)
100.0	1.4896(−01)	3.0195(−01)	6.5193(−01)	1.1190(+00)	2.9237(+00)
125.0	2.4646(−01)	4.6068(−01)	9.3081(−01)	1.5546(+00)	3.9523(+00)
150.0	3.5934(−01)	6.3121(−01)	1.2252(+00)	2.0118(+00)	4.9832(+00)
175.0	4.7872(−01)	8.0420(−01)	1.5229(+00)	2.4721(+00)	5.9832(+00)
200.0	6.0108(−01)	9.7700(−01)	1.8209(+00)	2.9308(+00)	6.9471(+00)
250.0	8.4695(−01)	1.3171(+00)	2.4099(+00)	3.8313(+00)	8.7641(+00)
300.0	1.0877(+00)	1.6464(+00)	2.9829(+00)	4.6994(+00)	1.0442(+01)
350.0	1.3201(+00)	1.9637(+00)	3.5359(+00)	5.5301(+00)	1.1997(+01)
k (MeV)	$Z=20$	$Z=29$	$Z=50$	$Z=73$	$Z=82$
9.0	2.9262(−02)	1.0108(−01)	4.3913(−01)	1.0707(+00)	1.3979(+00)
10.0	4.4761(−02)	1.4352(−01)	5.9052(−01)	1.4111(+00)	1.8314(+00)
15.0	1.7307(−01)	4.5036(−01)	1.6013(+00)	3.5591(+00)	4.5367(+00)
20.0	3.7118(−01)	8.8119(−01)	2.9111(+00)	6.1903(+00)	7.8156(+00)
25.0	6.2193(−01)	1.3993(+00)	4.3981(+00)	9.0872(+00)	1.1390(+01)
50.0	2.3038(+00)	4.6201(+00)	1.2677(+01)	2.4513(+01)	2.9980(+01)
75.0	4.2264(+00)	8.1617(+00)	2.0876(+01)	3.9090(+01)	4.7197(+01)
100.0	6.1532(+00)	1.1662(+01)	2.8507(+01)	5.2248(+01)	6.2605(+01)
125.0	7.9927(+00)	1.4971(+01)	3.5461(+01)	6.4005(+01)	7.6306(+01)
150.0	9.7843(+00)	1.8156(+01)	4.1955(+01)	7.4809(+01)	8.8844(+01)
175.0	1.1493(+01)	2.1165(+01)	4.7966(+01)	8.4697(+01)	1.0029(+02)
200.0	1.3122(+01)	2.4010(+01)	5.3558(+01)	9.3812(+01)	1.1081(+02)
250.0	1.6158(+01)	2.9256(+01)	6.3688(+01)	1.1015(+02)	1.2962(+02)
300.0	1.8932(+01)	3.3995(+01)	7.2679(+01)	1.2449(+02)	1.4609(+02)
350.0	2.1484(+01)	3.8312(+01)	8.0767(+01)	1.3729(+02)	1.6076(+02)

GeV for comparison with previous calculations[20] and experiments.[21,22] As noted in Ref. 20, the contribution of molecular binding to the screening correction is smaller than the experimental errors.

It is often convenient to write the total cross section for pairs, $\sigma_n+\sigma_t$, in the form

$$\sigma_n+\sigma_t=Z(Z+\eta)(\sigma_n/Z^2)\,. \tag{19}$$

Here

$$\eta=Z\sigma_t/\sigma_n \tag{20}$$

and is close to unity for all elements at high energies. In particular we have seen that if we neglect screening as well as higher-order corrections (radiative corrections and Coulomb corrections) then $\eta\to 1$ as $k\to\infty$. In Fig. 8 we show η as a function of photon energy for Li, Sn, and Pb. The values of η have been calculated from (20), in which screening and radiative corrections have been included in both σ_t and σ_n, but Coulomb cor-

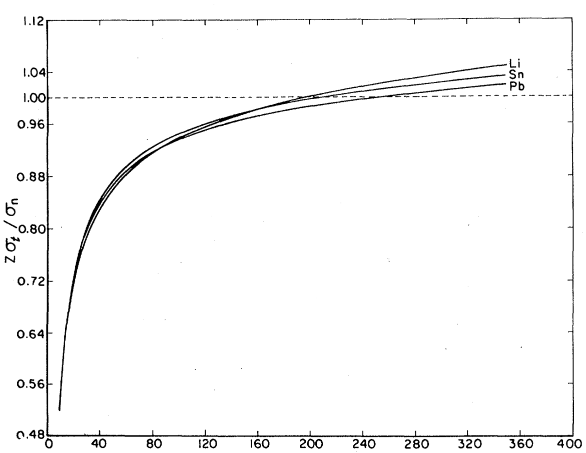

FIG. 8. The ratio of triplet to nuclear pair production, $\eta=Z\sigma_t/\sigma_n$, including screening and radiative corrections (to both σ_t and σ_n), as a function of photon energy k for Li, Sn, and Pb. Coulomb corrections have, however, been omitted from both σ_t and σ_n.

TABLE III. Screening correction for hydrogen $\Delta S(k, Z=1)$ in mb as a function of the photon energy k, in MeV, calculated using the exact incoherent scattering function $S(q)=1-F^2(q)$, where $F(q)$ is the hydrogen atomic form factor.

k (MeV)	$\Delta S(k, Z=1)$ (mb)
125	0.00433
150	0.00970
175	0.0175
200	0.0277
225	0.0401
250	0.0543
275	0.0701
300	0.0873
325	0.106
350	0.125
400	0.166
500	0.252
1000	0.686
2000	1.37
3000	1.86
4000	2.24
5000	2.56
6000	2.83
7000	3.06
8000	3.27

rections have not been included in either.

Previous calculations of the triplet screening cross section[23] were obtained by using the recoil distribution $d\sigma_{\text{Coul}}/dq$ in the high energy approximation of Ref. 4 instead of the recoil distribution $d\sigma_{\text{Bors}}/dq$. A comparison with our present results shows rather large differences, especially in the low energy part of the energy region we deal with. Figure 5 shows this very clearly for Pb.

In Ref. 2 the screening correction was also calculated with the recoil distribution $d\sigma_{\text{Coul}}/dq$ but with a correction factor taking into account the difference between $d\sigma_{\text{Coul}}/dq$ and $d\sigma_{\text{Bors}}/dq$. A comparison with these results leads to smaller deviations than with those of Ref. 23. At 10 and 15 MeV the screening corrections $Z\Delta S(k, Z)$ differ from our present evaluation of them by 15% and 10%, respectively, decreasing with higher energies. This behavior reflects the fact that the difference between the recoil distributions used is largest at low energies (Fig. 6).

The high energy screening corrections for hydrogen (Table III) can be compared with the results of Ref. 20, which were obtained by using the recoil distribution $d\sigma_{\text{Coul}}/dq$ of Ref. 12 instead of $d\sigma_{\text{Bors}}/dq$ together with a term correcting for the use of the incorrect recoil distribution. The agreement of these results with the present calculation is excellent and can again be understood as a consequence of the decreasing difference between $d\sigma_{\text{Coul}}/dq$ and $d\sigma_{\text{Bors}}/dq$ (Fig. 6) at these very high energies.

A comparison of the triplet cross section with experimental data is rather difficult since there exist no specific precision measurements of the triplet cross section σ_t. The absorption experiments[1] which motivated our investigation of the triplet cross section measure the total photoabsorption cross section σ_{tot}, of which the triplet cross section is only a fraction. From Eq. (19) we have $\sigma_t/\sigma_{\text{tot}} \approx 1/(Z+1)$. Thus total absorption measurements on high Z elements will not be particularly useful for accurate determinations of the triplet cross section. In view of the ratio $1/(Z+1)$, hydrogen is most suited for a comparison of theory with experiment. Figure 9 displays the experimental data of Ref. 21 and Ref. 22 along with the calculated sum of the triplet cross section σ_t and the cross section for pair production in the nuclear Coulomb field σ_n (computed from Eq. (5), with the radiative correction). The cross section for pair production in the nuclear Coulomb field has been checked on high Z elements[2] and is known to an accuracy of at least 0.5% for high Z elements and even better for low Z. If we now turn to hydrogen, we can thus assume that the nuclear pair cross section is known, and thus use the hydrogen data to check the triplet cross section. Figure 9 shows good agreement with the experiment.

It should be noted, however, that for low Z elements, for which the triplet cross section is an appreciable fraction of σ_{tot}, the various calculations of σ_t are distinguished only in the different values given for the screening correction $Z\Delta S(k, Z)$ and the even smaller term, $Z\Delta\sigma_{\text{Haug}}(k)$ [Eq. (18)]. However, for moderate energies the screening correction is very small for low Z. For example,

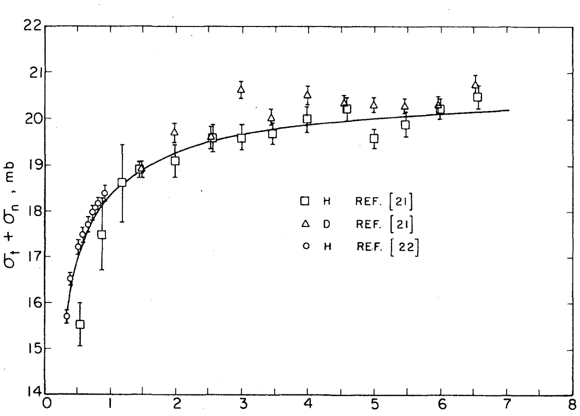

FIG. 9. The total cross section for pairs, $\sigma_t + \sigma_n$, together with experimental cross sections from H and D, as a function of photon energy k.

for Li at 350 and 100 MeV, $Z\Delta S(k,Z)$ is 1.3% and 0.1% of σ_{tot}, respectively. (See Table II.) On the other hand, at very high energies where the screening correction is large, the difference between $d\sigma_{\text{Coul}}/dq$ and $d\sigma_{\text{Bors}}/dq$ becomes small so that the various calculations again agree, as we have noted. Thus, the difference between the various calculations of the triplet screening correction has, generally, a negligible impact on the analysis of the total absorption cross section. Nonetheless, the present work gives for the first time a consistent treatment of the triplet screening correction which avoids the various *ad hoc* correction factors used in previous calculations.[2,20,23] Furthermore, we can now specify the errors which were made by using the recoil distribution $d\sigma_{\text{Coul}}/dq$. Previously, these errors could only be estimated.

APPENDIX A: BORSELLINO RECOIL DISTRIBUTION AND TOTAL CROSS SECTION FOR TRIPLETS

In this appendix we give the recoil distribution for pair production in the field of a free electron, $d\sigma_{\text{Bors}}/dq$, obtained when one retains only the two diagrams I and II in Fig. 3, as well as the total cross section $\sigma_{\text{Bors}}(k)$, obtained by integrating $d\sigma_{\text{Bors}}/dq$ over all kinematically allowed momenta [Eq. (4′)]:

$$\sigma_{\text{Bors}}(k)=\int_{q_m}^{q_M}\frac{d\sigma_{\text{Bors}}}{dq}\,dq\,. \tag{A1}$$

The recoil distribution was first derived by Borsellino[5] for arbitrary target mass M. For the case $M=m$ (m, the electron mass) this is given below in expressions (A1)–(A10). We show that $d\sigma_{\text{Bors}}/dq$ can in fact be written in terms of the same functions that appear in the expression for the recoil distribution for pair production in a Coulomb field, $d\sigma_{\text{Coul}}/dq$, viz., logarithms and dilogarithms (see Ref. 12). The total cross section for the case $M=m$ was derived by Ghizzetti;[6] his expansion of $\sigma_{\text{Bors}}(k)$ for large k is given below in (A23).

We start with the expression for the total cross section as given on page 33 Eq. (31a) of Ref. 5, which becomes (A2), for $M=m$ and upon taking all momenta in units of mc and all energies in units of mc^2, as on page 38 of Ref. 6. (We have made minor changes in notation. In place of $\Gamma(k)$, q_2 and q_1 in Ref. 5, we have written $\sigma_{\text{Bors}}(k)=\alpha r_0^2\Gamma(k)$, $q_M=q_2$ and $q_m=q_1$, respectively.)

$$\sigma_{\text{Bors}}(k)=\alpha r_0^2\frac{1}{k^2}\int_{q_m}^{q_M}G(q)\,\frac{q\,dq}{W(W-1)^2}\,, \tag{A2}$$

where

$$W=W(q)=(q^2+1)^{1/2}\,, \tag{A3}$$

$$\left.\begin{matrix}q_M\\ q_m\end{matrix}\right\}=\frac{k(k-1)\pm(k+1)[k(k-4)]^{1/2}}{2k+1} \tag{A4}$$

and

$$G(q)=A(q,k)+B(q,k)\int_{q_m}^{q}C(q',k)\,dq'\,, \tag{A5}$$

with

$$A(q,k)=S_1\sqrt{R}+S_2L+S_3N\,, \tag{A6a}$$

$$S_1=-\frac{4}{3k}\,\frac{(W-1)(4W-5)}{D^3}+\frac{2}{k}\,\frac{(W-1)(k+4W-5)}{D^2}+\frac{2}{3k}\,\frac{[k^2-9k(W-1)-3(W-2)(2W-3)]}{D}-2(W-2)\,, \tag{A6b}$$

$$\begin{aligned}S_2=&\frac{8}{3k}\,\frac{(W-1)[1+3(W-1)-(W-1)^2]}{D^3}+\frac{4}{k}\,\frac{(W-1)[(k+W-1)(W-2)-(2W-1)]}{D^2}\\&+\frac{4}{k}\left(\frac{(W-2)[1-(W-1)(W-2)]-2k(W-1)(W-2)-k^2(W-1)}{D}\right)\\&+2k(W-2)D+2(W-2)(3W-5)+4k(W-1)\,,\end{aligned} \tag{A6c}$$

$$S_3=2(W-2)q-\frac{2k^2}{3}\,\frac{(2W-3)}{q}\,, \tag{A6d}$$

$$D=q-W+1\,, \tag{A7}$$

$$R=R(q)=(kD-W)^2-1\,, \tag{A8a}$$

$$L=L(q)=\ln\left(\frac{(kD-W+1)^{1/2}+(kD-W-1)^{1/2}}{\sqrt{2}}\right), \tag{A8b}$$

$$N=N(q)=\ln\left(\frac{kDW-q^2-q\sqrt{R}}{kD}\right), \tag{A8c}$$

$$B(q,k)=-4[(W-2)^2+k(W-1)]\,, \tag{A9}$$

$$C(q,k)=\frac{N}{2W}+\frac{(W-q)L}{WD}\,. \tag{A10}$$

In the following part of this appendix we show that the integral in (A5) may be expressed in terms of dilogarithms:

$$\int_{q_m}^{q} C(q',k)dq' = \tfrac{1}{2}\ln(t_2)\ln\left(\frac{2kD}{W+q}\right) + \tfrac{1}{2}L_2\left(-\frac{t_2}{W+q}\right) - \tfrac{1}{2}L_2\left(-\frac{1}{t_2(W+q)}\right), \tag{A11}$$

where t_2 is given by (A19) and (A13).

The expressions (A1)–(A10) give the recoil distribution and total cross section in the form given on page 38 of Ref. 6. The details are given on pages 31–33 of Ref. 5, Eqs. (27a)–(31a). It may be noted that the limits on the integral in (A5) are not given in the corresponding expression, Eq. (30a) in Ref. 5. They may be obtained, however, from the analysis presented on pages 31 and 32 of Ref. 5, and this will also enable us to write this integral in terms of dilogarithms.

As is shown in Eqs. (26a) of Ref. 5 and may be verified directly by differentiation,

$$C(q,k) \equiv \frac{N(q)}{2W} + \frac{(W-q)L(q)}{WD} = \frac{d}{dq}\int_{x_1}^{x_2} \frac{\lambda}{x}\,dx\,, \tag{A12}$$

where [Eq. (20a), Ref. 5]

$$x_1 = x_1(q) = W+1\,,\qquad x_2 = x_2(q) = kD\,, \tag{A13}$$

and $\lambda = \lambda(x,q)$ is given [Eq. (16a) Ref. 5] by

$$\lambda = \ln\left(\frac{(x-W+1)^{1/2} + (x-W-1)^{1/2}}{\sqrt{2}}\right). \tag{A14}$$

It should be noted in (A14) that W is a function of q, as defined in (A3). Moreover, the limits of the integral in (A2), q_m and q_M as given in (A4), are determined from the solution of the equation $x_1(q) = x_2(q)$ [note Eqs. (20a) and (21a) in Ref. 5], from which it follows that

$$\int_{x_1}^{x_2} \frac{\lambda}{x}\,dx = 0 \quad \text{for } q = q_m \text{ or } q = q_M\,. \tag{A15}$$

We then have, on integrating (A12),

$$\int_{x_1}^{x_2} \frac{\lambda}{x}\,dx = \int_{q_m}^{q} C(q',k)dq'\,. \tag{A16}$$

We now transform the left-hand side of (A16), writing it in terms of the dilogarithm function (also called the Spence function). With (A14) in view, we first make the change of variable

$$x - W = \frac{t^2+1}{2t}\,, \quad t \geq 1 \tag{A17}$$

(in which W does not depend on x), and obtain

$$\int_{x_1}^{x_2} \frac{\lambda}{x}\,dx = \tfrac{1}{2}\int_{1}^{t_2} \ln(t)\left(\frac{1}{t+W-q} + \frac{1}{t+W+q} - \frac{1}{t}\right)dt\,, \tag{A18}$$

where

$$t_2 = x_2 - W + [(x_2 - W)^2 - 1]^{1/2}. \tag{A19}$$

In the first term in the integrand in (A18) we write $t = 1/t'$, and integrate by parts. This gives

$$\int_{1}^{t_2} \frac{\ln t}{t+W-q}\,dt = \tfrac{1}{2}\ln^2(t_2) + \int_{1/t_2}^{1} \frac{\ln t'}{t'+W+q}\,dt'\,,$$

which, upon substitution in (A18) results in

$$\int_{x_1}^{x_2} \frac{\lambda}{x}\,dx = \tfrac{1}{2}\int_{1/t_2}^{t_2} \frac{\ln t}{t+W+q}\,dt\,. \tag{A20}$$

Substituting $t = (W+q)s$ in (A20) and integrating by parts we then obtain this integral in terms of the dilogarithm function

$$\int_{x_1}^{x_2} \frac{\lambda}{x}\,dx = \tfrac{1}{2}\ln(t_2)\ln\left(\frac{2kD}{W+q}\right) + \tfrac{1}{2}L_2\left(-\frac{t_2}{W+q}\right) - \tfrac{1}{2}L_2\left(-\frac{1}{t_2(W+q)}\right), \tag{A21}$$

where $L_2(x)$ is the dilogarithm function, defined by

$$L_2(x) = -\int_0^x \frac{\ln(1-t)}{t}\,dt\,. \tag{A22}$$

Comparing (A16) and (A21) we then obtain (A11).

For the purpose of numerical evaluation of the dilogarithm function we note that the series expansion

$$L_2(x) = \sum_{n=1}^{\infty} \frac{x^n}{n^2}$$

is useful in the range $0 \leq x \leq \frac{1}{2}$. For $x = 0.5$ the first 10 terms give a precision of $< 2\times 10^{-5}$, the first 20 terms of $< 5\times 10^{-9}$, and the first 30 terms of $< 2\times 10^{-12}$. For all other argument values the dilogarithm can be expressed in terms of $L_2(0 \leq x \leq \frac{1}{2})$ with the following transformation[24]:

$$L_2(x) = L_2\left(\frac{1}{1-x}\right) - \frac{\pi^2}{6} - \tfrac{1}{2}\ln(1-x)\ln\frac{x^2}{1-x}\,, \quad x < -1$$

$$L_2(x) = -L_2\left(\frac{-x}{1-x}\right) - \tfrac{1}{2}\ln^2(1-x), \quad -1 \leq x < 0$$

$$L_2(x) = -L_2(1-x) + \frac{\pi^2}{6} - \ln x\ln(1-x), \quad \tfrac{1}{2} < x < 1$$

$$L_2(x) = \frac{\pi^2}{6}\,, \quad x = 1$$

$$L_2(x) = L_2\left(\frac{x-1}{x}\right) + \frac{\pi^2}{6} - \tfrac{1}{2}\ln x\ln\left(\frac{(x-1)^2}{x}\right), \quad 1 < x \leq 2$$

$$L_2(x) = -L_2\left(\frac{1}{x}\right) + \frac{\pi^2}{3} - \tfrac{1}{2}\ln^2 x . \quad 2 < x$$

{Note that $L_2(x)$ has a singularity at $x=1$, and is usually defined by a cut along the real axis, from $x=+1$ to $x=+\infty$. The expressions given above for $x>1$ refer to the principal value of $L_2(x)$, defined by $L_2(x) \equiv \frac{1}{2}\lim_{\epsilon\to 0}[L_2(x+i\epsilon) + L_2(x-i\epsilon)]$.}

Finally, for completeness, we give the full expansion of $\sigma_{\text{Bors}}(k)$ as given by Ghizzetti.[6]

$$\begin{aligned}\sigma_{\text{Bors}}(k) = \alpha r_0^2 \Bigg[&\frac{28}{9}\ln 2k - \frac{218}{27} + \frac{1}{k}\left(-\frac{4}{3}\ln^3 2k + 3\ln^2 2k - \frac{(60+16a)}{3}\ln 2k + \frac{(123+12a+16b)}{3}\right) \\ &+ \frac{1}{k^2}\left(\frac{8}{3}\ln^3 2k - 4\ln^2 2k + \frac{(51+32a)}{3}\ln 2k - \frac{(123+32a+64b)}{6}\right) \\ &+ \frac{1}{k^3}\left(\ln^2 2k - \frac{53}{9}\ln 2k - \frac{(2915-288a)}{216}\right) + \frac{1}{k^4}\left(-\frac{49}{18}\ln 2k - \frac{115}{432}\right) + \frac{1}{k^5}\left(-\frac{77}{36}\ln 2k + \frac{10\,831}{8640}\right) \\ &+ \frac{1}{k^6}\left(-\frac{641}{300}\ln 2k + \frac{64\,573}{36\,000}\right) + \frac{1}{k^7}\left(-\frac{4423}{1800}\ln 2k + \frac{394\,979}{216\,000}\right) + \cdots \Bigg], \end{aligned} \tag{A23}$$

with $a = -2.467\,40$ and $b = -1.803\,10$.

APPENDIX B: CRITICAL COMMENTS ON REFERENCE 16

The radiative corrections to high energy bremsstrahlung and pair production in the field of a nucleus were calculated by Mork and Olsen.[16] They give the radiative corrections to the total pair cross section on p. B 1670 of Ref. 16, in Table IV for incomplete screening, in Eq. (X.3) for complete screening, and in Eq. (X.4) for no screening. These final results are correct as given, within the framework of the Weiszäcker-Williams approximation. More recently, these radiative corrections have been recalculated, by Kuraev, Lipatov, Merenkov, and Fadin[18] for bremsstrahlung, and by Vinokurov, Kuraev, and Merenkov,[17] for pair production, also using the Weiszäcker-Williams approximation. For pair production with complete screening, the latter authors have performed all of the integrations analytically. Their result, given in Eq. (11) on p. 945 of the first citation in Ref. 17, verifies the result given previously by Mork and Olsen in Eq. (X.3) of Ref. 16.

However, in spite of the correctness of the results in Table IV and Eqs. (X.3) and (X.4) of Ref. 16, there are a number of misprints in other equations and tables of Ref. 16. Some of these are mentioned in Ref. 25 and the second citation in Ref. 16. Others are mentioned in Ref. 17, and still others in Ref. 18. Since Ref. 16 remains an extremely useful reference, we give here what we believe to a complete list of the misprints in Ref. 16, with comments where necessary:

(1) On page B1666 the expressions for a_8, a_9 and a_{11} are incorrect as given in Eq. (VI.7). The correct expressions are

$$a_8 = (3\gamma^2 + 5\gamma)/4 ,$$

$$a_9 = \frac{11\,333}{105^2}\gamma^2 + \frac{9443}{2\times 105^2}\gamma + \frac{6608}{105^2}$$

$$a_{11} = -\tfrac{8}{105}\gamma^3 - \tfrac{1}{10}\gamma + \tfrac{119}{105} .$$

(2) On page B1668 the expressions for c_2 and c_4 are incorrect as given in Eq. (VII15). The correct expressions are

$$c_2 = \pi^2\left(\tfrac{8}{105}\gamma_p^3 - \tfrac{32}{35}\gamma_p^2 + \tfrac{174}{105}\gamma_p - \tfrac{92}{105}\right),$$

$$c_4 = -\tfrac{8}{105}\gamma_p^3 + \tfrac{109}{60}\gamma_p^2 - \tfrac{31}{12}\gamma_p .$$

(3) On page B1666 the values given for $F_1 \times 10^2$ in Table I are incorrect. The correct values are given in the second citation in Ref. 16 and in Ref. 25.

(4) On page B1667 in Fig. 3 the curves are incorrect due to numerical errors in the values for F_1.

It should be noted, however, that the values of G_1 are correct within 2% as given in Table II, page B1669, as are also the curves in Fig. 4, page B1668 and Fig. 5, page B1669, both of which depend on G_1. One may thus assume that G_1 was in fact calculated with the correct expressions for c_2, c_4 and a_9.

As a check we have recalculated all of the values given in Tables I and II of Ref. 16, using the corrected expressions for a_8, a_9, a_{11}, c_2 and c_4 given here. For F_1 we find agreement within 1% with the values given in the errata, the second citation in Ref. 16. For F_2, F_{vac}, G_1, G_2 and G_{vac} we find agreement within 1% with the values given in Tables I and II of the first citation in Ref. 16, apart from the following, all of which constitute rather minor corrections.

Corrected values for Table I of Ref. 16

ω_1/ϵ_1	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$F_2 \times 10^2$	0.0147	0.0488	0.0986	0.164	0.247	0.353	0.494	0.695	1.038
$F_{vac} \times 10^2$			0.196	0.194	0.192				

Corrected values for Table II of Ref. 16

ϵ_+/ω_1	0.2	0.4	0.6	0.8
$G_1 \times 10^2$		2.27	2.27	
$G_2 \times 10^2$	0.777			0.777

[1]J. Ahrens, H. Borchert, K. H. Czock, H. B. Eppler, H. Gimm, H. Gundrum, M. Kröning, P. Riehm, G. Sita Ram, A. Zieger, and B. Ziegler, Nucl. Phys. A251, 479 (1975).

[2]H. A. Gimm and J. H. Hubbell, *Total Photon Cross Section Measurements, Theoretical Analysis and Evaluation for Energies above 10 MeV* (Nat. Bur. Stand., Washington, D.C., 1978), Tech. Note 968.

[3]L. C. Maximon and H. A. Gimm, Comments on the Analysis of Total Photoabsorption Measurements in the Energy Range 10–150 MeV, National Bureau of Standards Internal Report No. NBSIR-78-1456, June, 1978 (unpublished).

[4]J. A. Wheeler and W. E. Lamb, Jr., Phys. Rev. 55, 858 (1939); 101, 1836 (1956).

[5]A. Borsellino, Rev. Univ. Nac. Tucumán A6, 7 (1947). This article is much more detailed than two slightly earlier articles by the same author, in Helv. Phys. Acta 20, 136 (1947), and Nuovo Cimento 4, 112 (1947). It also corrects a number of serious misprints in the article in Nuovo Cimento 4, 112 (1947).

[6]A. G. Ghizzetti, Rev. Univ. Nac. Tucumán A6, 37 (1947).

[7]K. S. Suh and H. A. Bethe, Phys. Rev. 115, 672 (1959).

[8]K. J. Mork, Phys. Rev. 160, 1065 (1967).

[9]E. Haug, Z. Naturforsch. A 30a, 1099 (1975).

[10]H. A. Bethe and W. Heitler, Proc. R. Soc. London A146, 83 (1934); W. Heitler, *The Quantum Theory of Radiation* (Oxford University Press, London, 1954), 3rd ed.

[11]H. A. Bethe, Cambridge Phil. Soc. 30, 524 (1934).

[12]R. Jost. J. M. Luttinger, and M. Slotnick, Phys. Rev. 80, 189 (1950). For corrections to misprints in this paper, see pgs. 6 and 7 in Ref. 2 above.

[13]L. C. Maximon, Journal of Research, NBS 72B, 79 (1968).

[14]H. A. Bethe and J. Ashkin, in Experimental Nuclear Physics, edited by E. Segrè (Wiley and Sons, New York, 1953) Vol. I, Part II. Note p. 259–266 for bremsstrahlung and p. 325–342 for pair production, and, in particular, Eqs. (115), (119), and (124). It should be noted that in our Eq. (16), the number which appears in the argument of the logarithm is 1271, rather than 1440 as in p. 263 Eq. (59) and p. 332 Eq. (119) of this reference. The number 1440 is derived from Fig. 1 on p. 862 of the first article by Wheeler and Lamb, listed here in Ref. 4. The scale in that figure was incorrect, as the authors later noted and corrected in the erratum given in the second citation in Ref. 4. This correction of the scale requires that the argument of the logarithm given by Bethe and Ashkin be multiplied by $e^{-0.125}$, from which we obtain $1440 \times e^{-0.125} = 1271$. It is worthy of note that although the article of Bethe and Ashkin predates much of the work listed in the references we have given, their discussion of the exchange contribution is still valid. As is pointed out on page 675 of Ref. 7, the later review article of Joseph and Rohrlich, Rev. Mod. Phys. 30, 354 (1958), used a recoil distribution due to Votruba which is incorrect for $q \sim 1$. The additional term that this introduces was interpreted by them to imply that an exchange correction should be applied to the high energy cross section of Wheeler and Lamb.[4] This error has unfortunately been propagated in many subsequent reviews.

[15]Ingjald Øverbø, thesis, Arkiv for Det Fysiske Seminar i Trondheim, No. 9, 1970 (unpublished).

[16]Kjell Mork and Haakon Olsen, Phys. Rev. 140, B1661 (1965); 166, 1862 (1968).

[17]E. A. Vinokurov, E. A. Kuraev, and N. P. Merenkov, Zh. Eksp. Teor. Fiz. 66, 1916 (1974) [Sov. Phys.—JETP 39, 942 (1974)].

[18]E. A. Kuraev, L. N. Lipatov, N. P. Merenkov, and V. S. Fadin, Zh. Eksp. Teor. Fiz. 65, 2155 (1973) [Sov. Phys.—JETP 38, 1076 (1974)].

[19]J. H. Hubbell, Wm. J. Veigele, E. A. Briggs, R. T. Brown, D. T. Cromer, and R. J. Howerton, J. Phys. Chem. Ref. Data 4, 471 (1975).

[20]T. M. Knasel, The Total Pair Production Cross Section in Hydrogen and Helium, DESY Report Nos. 70/2 and 70/3, 1970 (unpublished).

[21]H. Meyer, B. Naroska, J. H. Weber, M. Wong, V. Heynen, E. Mandelkow, and D. Notz, Phys. Lett. 33B, 189 (1970). The cross sections at 0.55, 0.87 and 1.18 GeV are given in Table 5 of D. Notz, Diplomarbeit Universität Hamburg, Measurement of the Pair Production Cross Section for Hydrogen and Deuterium between 1 and 7 GeV (unpublished), as well as in DESY Report No. 70/3 listed in Ref. 20 above.

[22]H. Fujii, S. Homma, H. Okuno, N. Yamashita, I. Arai, H. Ikeda, A. Itano, E. Ohshima, Y. Hoshi, T. Ishii, K. Maruyama, and A. Sasaki, Nucl. Phys. B114, 477

(1976).
[23]J. H. Hubbell, *Photon Cross Sections, Attenuation Coefficients, and Energy Absorption Coefficients from 10* keV *to 100* GeV, Nat. Stand. Ref. Data Ser., Nat. Bur. Stand. (Washington, D.C., 1969), Report No. NSRDS-NBS 29.
[24]K. Mitchell, Philos. Mag. 40, 351 (1949). Also useful is W. Gröbner and N. Hofreiter, *Integraltafel*, Part II (Springer, Vienna, 1961), 3rd ed., pp. 71–73.
[25]H. D. Schulz and G. Lutz, Phys. Rev. 167, 1280 (1968).