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Neutron optical tests of nonlinear wave mechanics

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We analyze the free-space propagation of matter waves with a view to placing an upper limit on the strength of possible nonlinear terms in the Schrödinger equation. Such additional terms of the form $\psi F(|\psi|^2)$ were introduced by Bialynicki-Birula and Mycielski in order to counteract the spreading of wave packets, thereby allowing solutions which behave macroscopically like classical particles. For the particularly interesting case of a logarithmic nonlinearity of the form $F = -\underline{b}\ln|\psi|^2$, we find that the free-space propagation of slow neutrons places a very stringent upper limit on the magnitude of $\underline{b}$. Precise measurements of Fresnel diffraction with slow neutrons do not give any evidence for nonlinear effects and allow us to deduce an upper limit for $\underline{b} < 3.3\times 10^{-15}$ eV about 3 orders of magnitude smaller than the lower bound proposed by the above authors.

I. INTRODUCTION

The epistemological complexity of quantum mechanics has repeatedly led to proposals of nonlinear variants of the Schrödinger equation[1,2] with the aim that such equations may admit solutions with properties closer to classical mechanics. In particular, more localized solutions are desired, resembling classical particles, instead of the wave packets of standard quantum mechanics which, in principle, spread out without limit.

A general class of nonlinear variants of the Schrödinger equation is obtained by adding to the Hamiltonian an extra term which is a function of the probability density

$$\left(-\frac{\hbar^2}{2m}\nabla^2 + V(\vec{r},t) + F(|\psi(\vec{r},t)|^2)\right)\psi(\vec{r},t) = i\hbar\frac{\partial}{\partial t}\psi(\vec{r},t)\ . \quad (1)$$

The properties of this equation and of its solutions have been studied extensively by Bialynicki-Birula and Mycielski (BBM) in Ref. 2, in which a careful analysis shows that solutions of Eq. (1) retain many of the known features of the solutions of the standard Schrödinger equation. A particularly interesting form of the nonlinear term is the so-called logarithmic nonlinearity,[2] where one chooses

$$F(|\psi|^2 = -\underline{b}\ln(\underline{a}^n|\psi|^2)\ . \quad (2)$$

Here, $\underline{b}$ is an universal constant, $\underline{a}$ is an arbitrary, real, positive constant without immediate physical significance, and n in the dimensionality of the configuration space. It was shown by BBM that only a term of this form satisfies the condition of the separability of noninteracting subsystems. In all other forms, the very existence of even an isolated subsystem would influence the physics of all other subsystems of the universe.

The universal constant $\underline{b}$, which has the dimensions of energy, determines the strength of the nonlinear term and, from general physical arguments,[2] has to be non-negative. It is immediately clear that one of the effects of the nonlinear term would be to give rise to small changes in the energy eigenvalues of stationary solutions of the Schrödinger equation. However, the impressive agreement of calculated and observed energy eigenvalues in the case of the Lamb shift obliges BBM to set an upper limit to the strengh of the nonlinear term: $\underline{b} < 4\times 10^{-10}$ eV.

Since, as we mentioned earlier, the desired effect of the nonlinear term is to counteract the spreading of the wave packet, it is physically reasonable to also expect changes in the wave vector during the free-space propagation of matter waves. It turns out that, as far as experimental tests are concerned, such changes in the wave vector place far more stringent limits on the magnitude of any possible nonlinearity. Thus, in a recent paper Shimony[3] proposed the investigation of changes in the *longitudinal* component of the wave vector of a freely propagating particle, using the double-crystal neutron interferometer developed by Zeilinger *et al.*[4] In that experiment a phase shift of the neutron wave (due to the change in the wave vector) would be the detectable effect of the nonlinearity, and it is expected by Shimony that a null result would push down the upper limit of $\underline{b}$ to about 10^{-12} eV.[11]

In our investigation we concentrate on the *lateral* spreading of wave packets: Extra phase shifts (due to changes in the lateral components of the wave vector which follow from the nonlinear term) can, in appropriate circumstances, give rise to measureable lateral deflections. We find that such effects can lead to significantly lower experimental limits to the strength of any nonlinearity. Specifically, we show that experiments on the diffraction of slow neutrons from macroscopic obstacles (i.e., free-space propagation of neutrons after interaction with absorbers in simple geometrical configurations) are inconsistent with any nonlinear term of magnitude larger than that corresponding to $\underline{b} \approx 3.3 \times 10^{-15}$ eV.

In Sec. II we analyze the consequences of nonlinearity in the free-space progation of matter waves and in Sec. III we present the evidence based on Fresnel diffraction experiments. Section IV concludes with some general comments.

II. MODIFICATIONS OF WAVE PROPAGATION

In this section we discuss the effects of a nonlinear term in the Schrödinger equation upon the propagation of matter waves in free space. As will be shown, these effects arise because the extra phase shifts created by the nonlinear term are functions of the local probability density $\rho = |\psi(\vec{r}, t)|^2$.

We begin our analysis of free-space propagation with the time-independent form of Eq. (1):

$$\left(-\frac{\hbar^2}{2m}\nabla^2 + V(\vec{r}) + F(|\psi(\vec{r})|^2)\right)\psi(\vec{r}) = E\psi(\vec{r}) , \quad (3)$$

and analyze its stationary solutions within the eikonal approximation, i.e., in a semiclassical approach equivalent to geometrical optics. We therefore make the ansatz

$$\psi(\vec{r}) = \psi_0(\vec{r}) \exp\left(\frac{i}{\hbar} S(\vec{r})\right) , \quad (4)$$

which leads to

$$(\nabla S)^2 - 2m(E - V - F) = \hbar^2 \frac{\nabla^2 \psi_0(\vec{r})}{\psi_0(\vec{r})} . \quad (5)$$

We introduce the approximation whereby we neglect the right-hand side of Eq. (5); this is equivalent to assuming that the variations of the probability density of the matter waves are very small over distances comparable with a wavelength, i.e.,

$$\lambda\!\!\!^{-2} \frac{\nabla^2 \psi_0}{\psi_0} \ll 1 . \quad (6)$$

We restrict our considerations to experimental situations in which this condition is satisfied.[5] Furthermore, we note that the direction of propagation of the particles is then parallel to ∇S. This property holds also for the BBM-type nonlinear wave mechanics because, as in conventional quantum mechanics, the current is again defined as

$$\vec{j}(\vec{r}, t) = \rho(\vec{r}, t) v(\vec{r}, t) = \frac{\hbar}{2mi}(\psi^* \nabla \psi - \nabla \psi^* \psi) . \quad (7)$$

It is, in fact, the proportionality of the current to ∇S that allows the application of the procedures of standard geometrical optics: the particle propagation direction is always normal to surfaces of constant phase S.

In ordinary quantum mechanics ($F = 0$), the surfaces of constant phase in free space ($V = 0$) follow from (5), subject to the approximation (6),

$$(\nabla S_0)^2 = 2mE . \quad (8)$$

In the case of the modified Schrödinger equation, ($F \neq 0$), if we denote the surfaces of constant phase by S_1 we may write (again for ($V = 0$),

$$(\nabla S_1)^2 = 2m(E - F) = (\nabla S_0)^2 (1 - F/E) \quad (9)$$

and, since we anticipate that $F \ll E$, we may further approximate

$$|\nabla S_1| = |\nabla S_0| (1 - F/2E) . \quad (10)$$

For a wavefront propagating from z to $z + dz$ there will arise an extra phase shift given by

$$d\phi = (|\nabla S_1| - |\nabla S_0|) dz = - |\nabla S_0| F/2E . \quad (11)$$

(This equation is the one exploited in a neutron interferometer experiment of the type proposed by Shimony[3].)

To continue our analysis of free-space propagation, we remark that F is a function of the local density ρ and hence, if the wave function is not constant in space, F will also be a function of position. In particular, we focus our attention on the case where a gradient of the density (and hence of F) exists in the y direction normal to the direction of propagation z. This variation of F gives rise to a variation of phase ϕ which, in turn, is equivalent to a bending of the wavefront (i.e., a change in the direction of propagation). In traveling from z to $z + dz$ the change in direction of propagation is given by

$$d\left(\frac{dy}{dz}\right) = \frac{1}{|\nabla S_0|} \frac{d}{dy} (d\phi) = \frac{1}{2E} \frac{dF}{dy} dz , \quad (12)$$

or

$$\frac{d^2y}{dz^2}=\frac{1}{2E}\frac{dF}{dy} . \tag{13}$$

For a particle traveling from $z=0$ to $z=Z$ (with initial conditions $y=0$, $dy/dz=0$) the accumulated deflection Y is obtained by solving the differential equation (13). An explicit solution[6] is given by

$$Y=\int_0^Z (Z-z)\frac{1}{2E}\frac{dF}{dy}dz . \tag{14}$$

The dependence of this deflection of the density gradient may be studied by writing

$$\frac{dF}{dy}=\frac{dF}{d\rho}\frac{d\rho}{dy} . \tag{15}$$

Thus, the integrand is proportional to the product of the density gradient with the derivative of F with respect to density. It is this latter quantity which vanishes in standard quantum mechanics where we have

$$\frac{dF}{d\rho}\equiv 0 , \tag{16}$$

i.e., in that case F is an arbitrary constant, not dependent on the density at all. In general, however, the experimentally obtainable knowledge of $d\rho/dy$ would, in the case of finite deflection Y, allow us to determine $dF/d\rho$ and hence $F(\rho)$ from Eq. (14). In order to study further the deflection arising from a nontrivial, truly nonlinear term, we will have to make some assumptions on the functional form of F.

Henceforth we shall restrict our attention to the logarithmic nonlinearity proposed by BBM, viz Eq. (2). This, together with Eqs. (4) and (14) leads to

$$Y=\frac{b}{E}\int_0^Z \frac{1}{|\psi_0|}\frac{d|\psi_0|}{dy}(Z-z)dz . \tag{17}$$

Before confronting this predicted deflection with actual experimental tests, the following remarks may be in order.

(a) Since the effect upon the wave function of any nonlinearity is already known to be very small, we have used ψ_0, the unperturbed wave function, in Eq. (17). In the absence of any evidence for a finite size for the hypothetical constant b, we set out primarily to obtain an upper limit for the nonlinearity, to be inferred from any departure of Y from the zero value predicted by ordinary quantum mechanics. (b) We note that Eq. (17) implies that any effect would be directly proportional to the ratio of the strength of the nonlinear term, as measured by b, to the kinetic energy of the particle, E. Hence the use of very slow neutrons has decided advantages in experimental tests. (c) It is immediately apparent from Eqs. (14), (15), and (17) that an experiment would be more sensitive the steeper the density gradient used. It was for this reason that we decided to investigate the Fresnel diffraction of slow neutrons by strongly absorbing obstacles.

III. FRESNEL DIFFRACTION OF NEUTRONS

Figure 1 shows the classical set-up for observing Fresnel diffraction by an absorbing straight-edge, together with the typical diffraction pattern $|\psi_0|^2$ in the plane of observation $z=Z$. From Eq. (17) we learn that any deviation caused by nonlinearities is expected to be proportional to the gradient of the probability density. Hence we conclude that the maxima and minima of the Fresnel diffraction pattern are expected to stay in the same positions, unaffected by nonlinearities. On the other hand, the slopes would be moved by the action of these nonlinearities, towards the positions of the maxima. This feature might already be expected conceptually on the basis of the property of the BBM term that tends to counteract the dispersion of wave packets.

In our experimental analysis we choose to investigate the position of point P (Fig. 1) of the Fresnel diffraction pattern with respect of the position of the first maximum. The point P corresponds to the edge of the geometrical shadow of the diffracting straight edge, i.e., its projection upon the plane of observation. The deflection of this point by an amount Y, due to the hypothetical nonlinearities, may be derived by introducing into Eq. (17) the results of standard Fresnel diffraction theory, which follow from the linear, time-independent Schrödinger equation. We have[7,8]

$$|\psi_0(y,z)|=|\psi_0(u)|=\frac{1}{\sqrt{2}}\{[\tfrac{1}{2}+\mathcal{C}(u)]^2+[\tfrac{1}{2}+\mathcal{S}(u)]^2\}^{1/2} , \tag{18}$$

where

$$u=y\left(\frac{\zeta}{\zeta+z}\right)\left(\frac{2(\zeta+z)}{\lambda\zeta z}\right)^{1/2} =y\left(\frac{2\zeta}{(\zeta+z)\lambda z}\right)^{1/2} \tag{19}$$

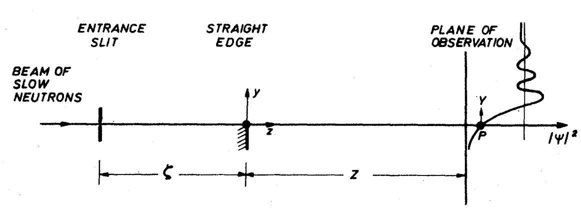

FIG. 1. Schematic of Fresnel diffraction setup.

and the Fresnel integral functions $\mathcal{C}(u)$ and $\mathcal{S}(u)$ are defined by

$$\mathcal{C}(u)+i\ (u)=\int_0^u \exp(i\tfrac{1}{2}\pi w^2)dw\ . \tag{20}$$

We also have [differentiating (20) with respect to u];

$$\frac{d\psi_0}{du}=\frac{1}{\sqrt{2}}[\exp(i\tfrac{1}{2}\pi u^2)]\ , \tag{21}$$

and [by differentiating (19) with respect to y]

$$\frac{du}{dy}=\left(\frac{2\zeta}{(\zeta+z)\lambda z}\right)^{1/2}\ . \tag{22}$$

From (21) and (22) we obtain

$$\frac{d\psi_0}{dy}=\frac{d\psi_0}{du}\frac{du}{dy}=\left(\frac{\zeta}{(\zeta+z)\lambda z}\right)^{1/2}\exp(i\tfrac{1}{2}\pi u^2)\ . \tag{23}$$

For the point P, at $y=0$ (i.e., $u=0$) we evaluate the following quantities:

$$|\psi_0(0)|=\frac{1}{\sqrt{2}}\,|(\tfrac{1}{2}+\tfrac{1}{2}i)|=\tfrac{1}{2}\ , \tag{18$'$}$$

and

$$\left|\frac{d\psi_0}{dy}\right|_{y=0}=\left(\frac{\zeta}{(\zeta+z)\lambda z}\right)^{1/2}\ . \tag{23a}$$

By explicitly substituting these quantities into Eq. (17) we deduce that the point P will be deflected by an amount given by

$$Y=\frac{2b}{E}\int_{z=0}^{Z}(Z-z)\left(\frac{\zeta}{(\zeta+z)\lambda z}\right)^{1/2}dz\ . \tag{24}$$

Evaluation of the definite integral in closed form, gives

$$Y=\frac{2b}{E\sqrt{\lambda}}\,c_1 Z^{3/2}\ , \tag{25}$$

where c_1 is a geometrical constant (of order unity) which depends on the ratio $R=\zeta/Z$.

The deflection Y is most conveniently expressed as a fraction of the width Y_0 of a Fresnel zone in the plane of observation, in the form

$$Y=\epsilon Y_0\ . \tag{26}$$

The value of Y_0 follows from the definition of the scaled variable u in Eq. (19), i.e.,

$$Y_0=\left(\frac{y}{u}\right)_{z=Z}=\left[\frac{\lambda Z}{2}\left(1+\frac{Z}{\zeta}\right)\right]^{1/2}\ , \tag{27}$$

while ϵ is the parameter to be determined experimentally.

In the absence of positive evidence for a finite deflection, as was the case in our experiments, setting ϵ equal to the experimental resolution available leads to an upper limit for $\underline{b}$. Thus, combining the last three equations and making the appropriate substitutions for kinematical quantities we obtain

$$b<c_2\epsilon\hbar\tau^{-1}, \tag{28}$$

where c_2 is another geometrical constant (of order unity) which depends on ζ/Z, and τ is the transit time of the particle between the straight edge and the plane of observation.

We note the striking resemblance of this result to the one obtained by Shimony[3] for his proposed interferometer experiment. Our constants $c_2\epsilon$ correspond to the observable fraction of a fringe shift, Δ, in Shimony's experiment, consistent with the observation that (not surprisingly) our measurement of path deflection corresponds to just another form of phase-shift measurement. It is obvious, however, that by using a longer flight path (hence longer transit time, τ) we can expect a smaller limit on $\underline{b}$ from Fresnel diffraction experiments as compared with neutron interferometer experiments using neutrons of comparable wavelength. It is interesting to point out, in fact, that existing results on Fresnel diffraction of neutrons, based on the experiments of Klein *et al*,[9] already suffice to place an upper limit on any nonlinearity which is lower than that expected from Shimony's proposed interferometer experiment, and therefore almost 3 orders of magnitude lower than the one derived by BBM from Lamb-shift measurements. (Specifically, if we consider that the diffraction pattern of a straight edge measured by Klein *et al*.[9] with 4.3 Å neutrons over a flight-path of 1.8 m agrees with the predictions of standard quantum mechanics to within 0.5 of a Fresnel Zone, i.e., $\epsilon=0.5$, we are led to the conservative conclusion that $\underline{b}<7\times10^{-13}$ eV).

In order to reduce even further the upper limit of the strength of any nonlinearity, $\underline{b}$, we performed more precise Fresnel diffraction experiments using much slower neutrons over a much longer flight path. The apparatus that we used was a 10-meter-long neutron optical bench assembly designed by Gähler *et al*.[10] for an experiment aimed at lowering the experimental limit of the neutron charge. Our setup (Fig. 2) was as follows: A beam of very slow neutrons, originating in the cold source of the High Flux Reactor at the Institut Laue-Langevin, was incident upon the entrance slits S_1 and S_2 of a quartz prism monochromator. The 15 μm-wide exit slit S_3 selected a beam of wavelength 20 Å with a spread of ±0.5 Å and formed the entrance slit, or effective source, of the Fresnel diffraction section. The diffracting obstacle, a highly absorbing straight edge, was placed 5 meters downstream, and a

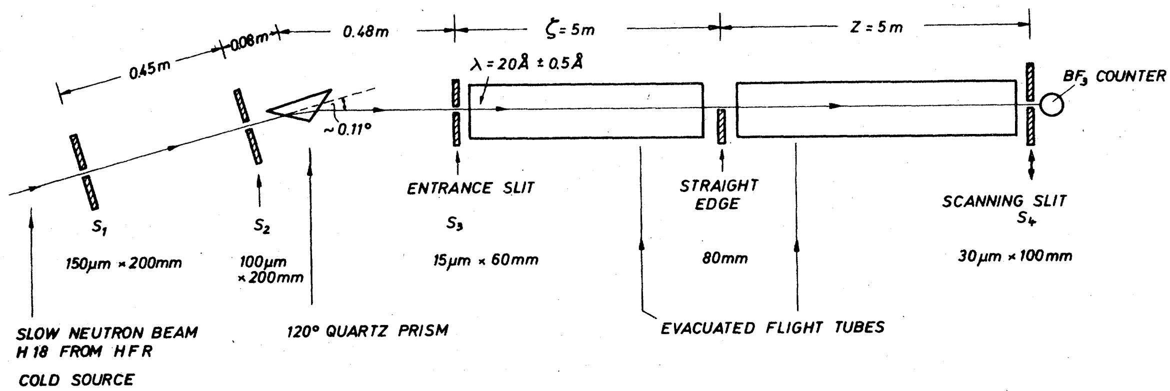

FIG. 2. Layout of the experiment (not to scale).

further 5 meters away the diffraction pattern was scanned with a BF_3 proportional counter placed behind a 30 μm-wide scanning slit S_4. In this configuration, the scale of the diffraction pattern, Y_0, is 100 μm [from Eq. (27)] while the long-term stability of the apparatus is known to be better than 10 μm. The results of the measurements are shown in Fig. 3, together with the theoretical curve calculated on the basis of Standard Fresnel diffraction theory. The calculations take into account the finite resolution of the apparatus, the finite wavelength spread as well as the amplitude distribution and divergence of the illuminating radiation originating in the entrance slit. The vertical scale is normalized to the experimental results, and the origin is arbitrarily located to be in line with the theoretically expected position of the point P, i.e., at $I = 0.25 I_0$.

As explained earlier, the test for the presence of any nonlinear effects consists of measuring

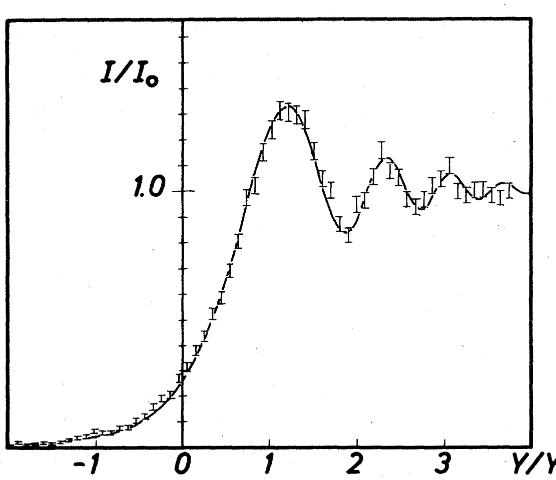

FIG. 3. Measured diffraction pattern of an absorbing straight-edge compared with the curve obtained from standard linear theory.

the distance between the abscissa of point P and that of the first maximum of the diffraction pattern. We find this distance to be 118 ± 5 μm, i.e., $(1.18 \pm 0.05) Y_0$. The theoretically expected distance, on the basis of linear theory, is equal to $1.22 Y_0$. We may therefore conclude, conservatively, that the deflection Y of point P is less than $\approx 0.1\ Y_0$, i.e; that $\epsilon < 0.1$. From the dimensions of the apparatus, the geometrical constants which appear in Eq. (25) and (28) work out to be: $c_1 = 1.23$ and $c_2 = 1.28$. Substituting these values into Eq. (28) yields the following new upper limit to the strength of the BBM-type nonlinearity:

$$\underline{b} < 3.3 \times 10^{-15} \text{ eV}.$$

Another consequence of a nonlinear term in the Schrödinger equation is that Babinet's principle, which follows from linear superposition, would not be valid in wave mechanics. Thus, the diffraction pattern of a single slit would not be complementary to that of an absorbing strip: The two pieces into which an absorbing strip cuts a wave packet would, so to speak, shrink away from each other under the action of the nonlinear term. We have demonstrated experimentally that this is not the case, at least to the same order of accuracy as our previous result. Fig. 4 shows the Fresnel diffraction pattern of a 100-μm Boron wire, measured with the same setup as described above. Once again, the curve is the theoretically predicted result, (based on ordinary, linear quantum mechanics).

IV. CONCLUDING COMMENTS

It is important to note that our experimental curves were compared with the appropriate solutions of the standard linear Schrödinger equation. These solutions have been generated by numerical calculations taking into account all features of the experimental arrangement including the finite width of the entrance slit and hence the lateral

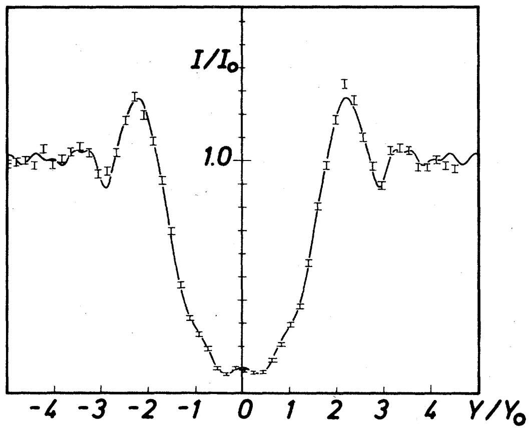

FIG. 4. Measured diffraction pattern of a 100 μm Boron wire compared with the curve obtained from standard linear theory.

spreading of the wave packets after passage through that opening. The agreement of these solutions with the experimental data demonstrates the validity of the standard linear Schrödinger equation for describing our experiment. Nevertheless we are not reluctant to note, that this agreement could, in principle, also be due to a fortuitous coincidence, if, for our specific experimental parameters, a hypothetical nonlinear term were to just cancel the terms neglected in our analysis of the nonlinear Schrödinger equation.[12] This point, which is related to the fact that we use the time-independent Eq. (5), calls for a more detailed theoretical investigation of nonlinear wave mechanics which is beyond the scope of the present paper.

We find it interesting, that the upper limit of $b < 3.3\times10^{-15}$ eV for the strength of the nonlinearities found by our experiment turns out to be already significantly lower than a certain estimated lower limit proposed by BBM ($b > 2.5\times10^{-12}$ eV). This estimate was based by BBM on the assumption that, while the physics of elementary particles ("microphysics") is still governed by quantum mechanics, and "macrophysics" by classical mechanics, there exists an intermediate region where the nonlinear term would be significant.

In order to highlight the implications of our result for the BBM approach to wave mechanics we point out, once again, that one of the main objectives of introducing a nonlinear term into the Schrödinger equation was to obtain nonspreading free-space solutions with more localized properties. But if we were to apply our result to calculate the limiting size to which a free-space electron would spread [the so-called electron "gausson" defined in Ref. 2 as $l = h/(2mb)^{1/2}$], we would get something of the order of $l > 3$ mm, i.e., a macroscopic distance. This, in our view, leaves no room for an intermediate behavior of the electron between macro and microphysics.

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