

$1s2s2p\ ^2P$ resonance of He^-

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By using a saddle-point technique, the $1s2s2p\ ^2P$ resonance of He^- is found to be a Feshbach resonance. It is formed by the strong coupling of $[(1s2s)\ ^1S, 2p]$ and $[(1s2p)\ ^1P, 3d]$ configurations. By using a seven-partial-wave 54-term wave function and neglecting the open channels, the energy is $-2.149\ 045$ a.u. This is higher than the $-2.175\ 23$ a.u. $(1s2s)\ ^3S$ threshold, but significantly below the $-2.145\ 97$ a.u. $(1s2s)\ ^1S$ threshold. A quasi-projection-operator method is also carried out. Using the same wave function, the energy is $-2.150\ 54$ a.u. These results generally agree with that of experiment.

I. INTRODUCTION

In the electron-helium scattering experiment, the excitation cross section for the $(1s2s)\ ^3S$ state gives rise to a maximum at about 0.5 eV above the triplet threshold. This excitation enhancement has been observed in many experiments,¹⁻⁸ in total cross section as well as differential cross sections. Ehrhardt and Willmann⁶ also find that the angular distribution of the scattered electrons associated with this excitation exhibits a p -wave character. Hence, this resonance is designated to be a 2P state.

Theoretically this resonance has been investigated extensively by Burke and collaborators using close-coupling calculations and R -matrix theory,⁹ Nesbet and collaborators using a matrix-variation method,¹⁰ Wichmann and Heiss,¹¹ and Wakid *et al.*¹² The results generally agree with those of the experiment.

Until very recently, this resonance has been designated to be a shape resonance. The precise reason for this designation is not very clear. In most theoretical investigations (except Ref. 12), the calculation is done with both open and closed channels. Therefore, when a resonance arises, it is difficult to tell whether it is associated with only the open channels or the closed channels. Perhaps the fact that this 2P is particularly broad and that it lies above the 3S threshold has led to the conclusion that it is a shape resonance.

However, the origin of a shape resonance is very different from that of a Feshbach resonance (closed-channel resonance). It is associated with an open channel and formed by the centrifugal potential barrier of the scattering electron. The feshbach resonance, on the other hand, involves the excitation of the target electrons into a closed channel. The energy of the incoming electron becomes negative and the scattering system becomes quasi-bounded. It decays through the open channel due to the coupling of coulomb interaction between

the electrons. These closed-channel resonances are usually narrow (≈ 0.05 eV) if they lie in the elastic scattering region. They may become broader above the inelastic threshold.

In the study of electron-atom scattering, the electron-hydrogen system offers the simplest model. However, many features of this system are unique and cannot be generalized to other scattering systems. For example, the exact degeneracy of the $2s$ and $2p$ target state enable the system to sustain an infinite number of resonances.^{13, 14} Hence, a detailed study of the electron-helium scattering system becomes very important as a tool for understanding more general aspects of electron-atom scattering systems.

In this work, the nature of the 2P resonance will be studied by using closed-channel wave functions alone. This is to illustrate that this resonance may actually be a closed-channel resonance, contrary to the general assumption in the literature. Two methods will be used: the first is the saddle-point technique by Chung¹⁵ and the second is the quasi-projection-operator method by Temkin¹⁶ and collaborators. Both methods are suitable for studying closed-channel resonances. In Sec. II, the general approach of the saddle-point technique will be outlined. Section III gives the computational aspects and the calculation results. Section IV gives the results of the quasi-projection-operator method. Section V is a brief discussion.

II. THE SADDLE-POINT TECHNIQUE

To study this 2P resonance, the helium target states of interest are $(1s^2)\ ^1S$, $(1s2s)\ ^3S$, $(1s2s)\ ^1S$, $(1s2p)\ ^3P$, and $(1s2p)\ ^1P$. These are the usual target states used in previous theoretical investigations. The ground-states energy of helium is $-2.903\ 724$ a.u.¹⁷ Compared with this, the $(1s2s)\ ^3S$ ($E = -2.175\ 229$ a.u.)¹⁷ is at 19.824 eV and the $(1s2s)\ ^1S$ ($E = -2.145\ 97$ a.u.)¹⁷ is at 20.620

eV. The maximum of the 3S excitation cross section is observed to be at about 20.3 to 10.45 eV. Therefore, if the resonance is associated with the $(1s2s)^3S$ channel only, it must be a shape resonance. On the other hand, if the resonance is caused by virtual excitation to the other closed channels, it would be a Feshbach resonance.

Should the resonance in question be a closed-channel resonance, the most likely configuration will be the $[(1s2s)^1S, 2p]$. This wave function can be constructed by building in a $1s$ vacancy in the 2P wave function with the saddle-point technique.

For the electron-helium system, the nonrelativistic Hamiltonian is given by

$$H = - \sum_{i=1}^3 \left(\frac{\nabla_i^2}{2} + \frac{2}{r_i} \right) + \sum_{i < j} \frac{1}{r_{ij}}, \quad (1)$$

where atomic units are used. A quasi-bound-state wave function can be expanded by a set of basis functions of the form

$$\psi_{mnk}^{i_1 i_2 i_3 L M}(\vec{r}_1, \vec{r}_2, \vec{r}_3) = \phi_{mnk}(r_1, r_2, r_3) Y_{i_1 i_2 i_3}^{L M}(\hat{r}_1, \hat{r}_2, \hat{r}_3) \times \chi(1, 2, 3), \quad (2)$$

where the spatial part is given by

$$\phi_{mnk}(r_1, r_2, r_3) = r_1^m r_2^n r_3^k e^{-(\alpha_j r_1 + \beta_j r_2 + \gamma_j r_3)} \quad (2a)$$

and

$$Y_{i_1 i_2 i_3}^{L M}(\hat{r}_1, \hat{r}_2, \hat{r}_3) = \sum_{m_1, m_2, m_3, \mu} Y_{i_1}^{m_1}(\hat{r}_1) Y_{i_2}^{m_2}(\hat{r}_2) Y_{i_3}^{m_3}(\hat{r}_3) \times \langle l_1 l_2 m_1 m_2 | l_{12} \mu \rangle \times \langle l_{12} l_3 \mu m_3 | L M \rangle \quad (2b)$$

and the spin part is given by

$$\chi(1, 2, 3) = [\alpha(1)\beta(2) \mp \alpha(2)\beta(1)] \alpha(3) - (1 \mp 1) \alpha(1) \alpha(2) \beta(3). \quad (2c)$$

The \mp is chosen such that the total wave function satisfies the Pauli antisymmetry principle. The exponents $\alpha_j, \beta_j, \gamma_j$ are the nonlinear parameters for the j th partial wave (with l_1, l_2, l_{12}, l_3). These are determined by optimizing the energy.

Since the LS coupling scheme is used, L, M, S , and S_z are all good quantum numbers; the energy of the state is independent of M and S_z . With the understanding that it is a 2P state, the notation of the angular and spin part of the wave function will be simplified as

$$Y_{i_1 i_2 i_3}^{L M} \chi = [(l_1 l_2)^k l_{12}; l_3], \quad (3)$$

where k is the multiplicity of the (l_1, l_2) core;

$k=1$ if the minus sign is adopted in Eq. (2c), $k=3$ if the plus sign is adopted.

Since a $1s$ vacancy is present in the resonance of interest, we assume the vacancy orbital takes the form

$$\phi_{1s}(\vec{r}) = 2q^{3/2} e^{-qr} Y_0^0(\theta, \varphi), \quad (4)$$

where q is a parameter to be optimized. The total wave function takes the form

$$\Psi = A \Sigma C_{mnk}^{i_1 i_2 i_3 L M} [1 - P(i)] \psi_{mnk}^{i_1 i_2 i_3 L M}(\vec{r}_1, \vec{r}_2, \vec{r}_3), \quad (5)$$

where A is the antisymmetrization operator and

$$P(i) = |\phi_{1s}(\vec{r}_i)\rangle \langle \phi_{1s}(\vec{r}_i)| \quad (5a)$$

is a projection operator which projects electron i out of the vacancy orbital

For example, for a $1s(1)2s(2)2p(3)$ wave function, $i=2$. On the other hand, a $1s(1)2p(2)3d(3)$ wave function would not be affected by the presence of this projection operator if $i=2$ or 3 .

To utilize the saddle-point technique, we first minimize the expression

$$E = \frac{\langle \Psi | H | \Psi \rangle}{\langle \Psi | \Psi \rangle} \quad (6)$$

with respect to the linear parameters C to obtain a secular equation. The lowest eigenvalue of the secular equation is then minimized with respect to the nonlinear parameters $\alpha_j, \beta_j, \gamma_j$ and maximized with respect to q .

III. COMPUTATIONAL ASPECT

A three-electron $^2P^o$ wave function can be constructed through various angular terms. For example $[(s, s)^3, ^1S, p]$, $[(s, p)^3, ^1P, d]$, $[(p, p)^3, ^1S, p]$, $[(p, p)^3, ^1P, p]$, etc. If all of these partial waves were to be used in the computation, the size of the wave function would become extremely large. Fortunately, many of these will not contribute significantly to the energy for this negative-ion system. For example, the terms with triplet intermediate coupling contribute little to the lowering of the 2P total energy. This is because in the absence of a net attractive long-range potential, the exchange effect of the third electron with the triplet core tends to raise the total energy of the system. Hence, only partial waves with singlet intermediate coupling are used. The angular terms used in this calculation are $[(s, s)^1S, p]$, $[(s, p)^1P, d]$, $[(p, p)^1S, p]$, $[(s, d)^1D, p]$, $[(d, d)^1S, p]$, $[(p, d)^1P, d]$, and $[(s, d)^1D, f]$. Notice, the terms we use here emphasize the improvement of two-electron core states. For the

²P, the relevant core states are (1s2s)¹S, (1s2p)¹P, and (1s3d)¹D. Other partial waves such as [(p,p)¹P,p], [(p,p)¹D,p] are tested but their contribution is found to be too small.

In order to see how the ²P resonance is formed, we perform a test calculation with only three partial waves: a 19-term [(s,s)¹S,p], a 10-term [(p,p)¹S,p], and a 10-term [(s,p)¹P,d]. The *q* for the vacancy orbital is chosen at the optimized value of 1.5. The nonlinear parameters α_j and β_j in the (s,s)¹S, and (p,p)¹S, and (s,p)¹P are chosen such that a low minimum energy is obtained as $\gamma \rightarrow 0$. With α_j and β_j fixed, the energy of He⁻ is calculated as a function of γ from 0 to 1. When the terms in the wave function are fixed, this is equivalent to bringing the third electron in from infinity towards the two-electron targets. The result of this calculation is shown in Fig. 1.

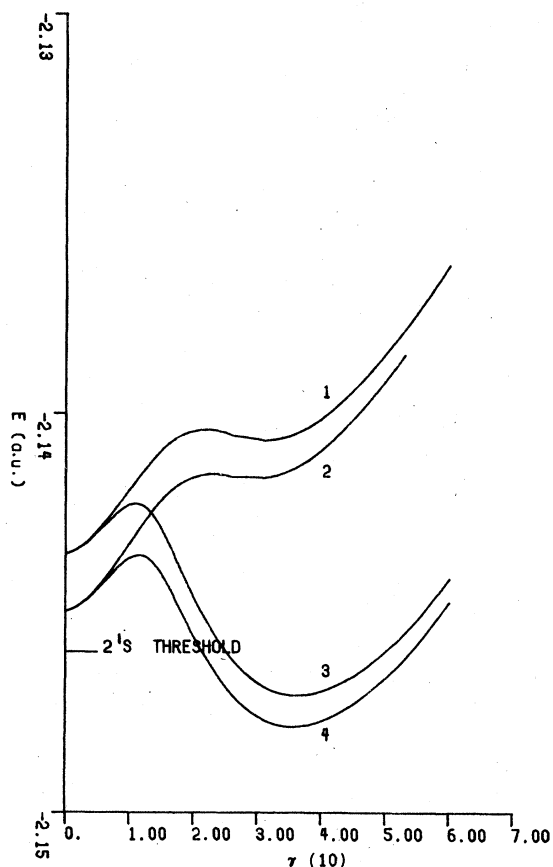


FIG. 1. Energy calculation for ²P resonance of He⁻ as a function of the nonlinear parameter of the third electron. Curve 1 is a 19-term [(s,s)¹S,p] calculation, $\alpha_1 = 2.05$, $\beta_1 = 0.415$. Curve 2 is a two-partial-wave calculation by superposing a 10-term [(p,p)¹S,p] to curve 1, $\alpha_2 = 1.90$, $\beta_2 = 1.80$. Curve 3 is a two-partial-wave calculation by superposing a 10-term [(s,p)¹P,d] to curve 1, $\alpha_2' = 2.0$, $\beta_2' = 0.49$. Curve 4 is a 39-term calculation obtained by combining all three partial waves.

In this figure, four curves are plotted. Curve 1 is the result of a 19-term [(s,s)¹S,p] calculation. As $\gamma \rightarrow 0$ the energy is about -2.1435 a.u. This energy rises as γ increases, showing the effect of the centrifugal potential as the electron approaches the target. There is a slight dip in energy when $\gamma \approx 0.3$. However, the energy is well above the 2¹S threshold, indicating that the polarization and exchange effects of this configuration are not sufficient to produce a resonance. This situation remains even if the 2¹S target wave function is improved. This can be seen from curve 2 in which a 10-term [(p,p)¹S,p] is added to curve 1. At $\gamma \rightarrow 0$, the energy becomes -2.1450 a.u., an improvement of over 60% towards the threshold energy -2.14597 a.u. This partial wave gives an overall lowering of the total energy but it does not lead to a resonance.

The situation is drastically changed if a 10-term [(s,p)¹P,d] partial wave is used instead of [(p,p)¹S,p]. This is illustrated in curve 3. As expected, this term does not contribute to the energy as $\gamma \rightarrow 0$. But, as γ increases, it leads to a dramatic lowering of the total energy. The minimum at $\gamma = 0.37$ is about -2.1471 a.u., significantly below the 2¹S threshold. It is this curve 3 that leads me to believe that the ²P resonance is a Feshbach resonance, arising from the strong dipole coupling of [(1s2s)¹S,2p] and [(1s2p)¹P,3d] configurations. It is worthwhile to mention that this second partial wave is not affected by the projection operator $P(i)$ in Eq. (5); hence, this result can not depend critically on the saddle-point technique used. On the other hand, the energy near $\gamma \rightarrow 0$ is well above the threshold, indicating that this resonance cannot be attributed to the improper optimization of *q*.

Curve 4 is the net result of all three partial waves. Compared with curve 3, the energy is generally lowered, but the results are qualitatively unchanged.

Although Fig. 1 clearly suggests the possibility of a ²P resonance associated with the closed channels (1s2s)¹S and (1s2p)¹P, the energy is not optimized in the sense that γ is restricted to be the same for all three partial waves. In an attempt to obtain a better result for the energy, γ is allowed to be optimized for each partial wave. Furthermore, each term in the partial wave is examined in terms of its contribution to the normalized wave function as well as its contribution to the energy. Terms with a small contribution are dropped. The nonlinear parameters for an angular partial wave are optimized individually but in the presence of the other partial waves.

In the first calculation, we used 97 terms and

TABLE I. Energy calculation of $1s2s2p\ ^2P$ resonance of He^- (in a.u.).

Angular partial waves		35-term wave function			54-term wave function			
j	$[(l_1, l_2)^{k_1} l_{12}, l_3]^2 P$	Nonlinear parameter ($\gamma_j, \beta_j, \gamma_j$)	N	$-E$	$^a \psi_j ^2$	N	$-E$	$ \psi_j ^2$
1	$[(s, s)^1 S, p]$	2.05, 0.415, 0.37	19	2.140 429	0.931 64	38 ^b	2.141 133	0.932 92
2	$[(s, p)^1 P, d]$	2.00, 0.515, 0.42	4	2.147 053	0.084 60	4	2.147 441	0.082 96
3	$[(p, p)^1 S, p]$	2.00, 1.60, 0.24	5	2.147 900	0.000 25	5	2.148 301	0.000 25
4	$[(s, d)^1 D, p]$	2.00, 0.44, 0.77	3	2.148 522	0.006 66	3	2.148 912	0.006 53
5	$[(d, d)^1 S, p]$	2.90, 3.00, 0.25	2	2.148 588	7×10^{-6}	2	2.148 978	7×10^{-6}
6	$[(p, d)^1 P, d]$	1.60, 1.84, 0.37	1	2.148 623	14×10^{-6}	1	2.149 013	14×10^{-6}
7	$[(s, d)^1 D, f]$	2.00, 0.69, 0.87	1	2.148 655	0.000 10	1	2.149 045	0.000 10

^a $|\psi_j|^2$ is the contribution of the j th partial wave to the normalized wave function. Since the 2nd and 4th, and the 5th and 6th partial waves are not orthogonal after permutation, their net contributions to the normalization are 0.067 99 and 21×10^{-6} , respectively, for a 35-term wave function; 0.066 70 and 21×10^{-6} , respectively, for the 54-term wave function.

^b The optimized nonlinear parameters for this 38-term partial wave are (2.00, 0.54, 0.345). The rest of the partial waves have essentially the same nonlinear parameters as the 35-term calculation.

seven-partial waves. Out of these, a 35-term wave function is selected. The results are shown in Table I. In this table, E is the energy obtained after the corresponding partial wave is adopted, we see that with a two-partial-wave, 23-term wave function, the optimized energy is $-2.147\ 05$ a.u. It is lower than the 2^1S threshold. The seven-partial-wave result, $-2.148\ 655$ a.u. is at 20.547 eV compared with the helium ground state.

One interesting observation we have made is that except for the $[(s, s)^1 S, p]$ partial wave, the radial part of other partial waves converges quickly if a set of suitable nonlinear parameters is used. This is true even for the $[(s, p)^1 P, d]$ which contributes greatly to the resonance. For this partial wave, a 14-term wave function is taken in an initial calculation, but we found that most of this energy can be accounted for by a careful selection of four terms. In the case of $[(s, d)^1 D, f]$, only one term is selected to approximate the contribution of 18 terms. In Table I, the optimized nonlinear parameter and the number of terms are given for each partial wave. Also given in this table is the contribution of each partial wave to the normalized wave function when all seven partial waves are included. This gives some idea about the composition of this 2P resonance.

To improve the 35-term result, the $[(s, s)^1 S, p]$ partial wave is expanded to 57 terms and a 38-term function is selected from it. For reasons discussed above, the terms from the six other partial waves are kept unchanged. The final result, $-2.149\ 045$ a.u. is at 20.536 eV relative to the ground-state target. These results are also included in Table I.

The results obtained thus far all have $q=1.5$. This optimized value is quite stable and it does not change significantly when the total wave func-

tion changes. To illustrate this point, the energy dependence as a function of q is given for two rather different wave functions. This is given in Table II. In this table, A is the seven-partial-wave 35-term wave function of Table I and B is the three-partial-wave 39-term wave function of curve 4 in Fig. 1 with $\gamma_1=0.37$, $\gamma_2=0.42$, and $\gamma'_2=0.27$. Although the energies are very different, the optimized q values all appear to be near 1.50. This seems to suggest that the $1s$ vacancy orbital is half screened by the presence of the $1s$ electron. Whereas from Table I the α value of the $1s$ electron is about 2.0, implying that the $1s$ electron is not screened by the vacancy. This, of course, is reasonable and should be expected.

IV. QUASI-PROJECTION-OPERATOR METHOD

Another method which is very effective in locating Feshbach resonances was developed by Temkin and collaborators.¹² The basic idea of this method is to obtain a closed-channel wave function by projecting out the open channels from the

TABLE II. Saddle-point search for $1s2s2p\ ^2P$ resonance of He^- (in a.u.). A is the seven-partial-wave, 35-term wave function as in Table I. B is the three-partial-wave, 39-term wave function as in curve 4 of Fig. 1 with γ_j optimized.

q	A	$-E$	B
1.47	2.148 696		2.147 888
1.48	2.148 670		2.147 859
1.49	2.148 657		2.147 847 3
1.50	2.148 655		2.147 846 8
1.51	2.148 665		2.147 857
1.52	2.148 684		2.147 878
1.53	2.148 714		2.147 908
1.54	2.148 753		2.147 948

total wave function. It is very similar to the Feshbach projection-operator formalism¹⁸ for two-electron systems except that the idempotent condition of the projection operators is relaxed. This is necessary in order to apply the method to systems with more than three electrons. In some cases, this method may give rise to spurious solutions, but these can be predicted and discarded. For ²P^o He⁻, no spurious solution arises.

To project out the open-channel component, one constructs the closed-channel operator

$$\hat{Q} = 1 - \sum_{i=1}^3 P_i, \quad (7)$$

where

$$P_i = |\phi_o(\vec{r}_j, \vec{r}_k)\rangle \langle \phi_o(\vec{r}_j, \vec{r}_k)|, \quad i \neq j \neq k. \quad (7a)$$

ϕ_o is the ground-state wave function of the two-electron target. Since the resonance of interest is above the 2³S target, in general this target state should also be projected out. However, the two wave functions we used in Table I do not contain 2³S components. If the same wave function is to be used, there is no need to have the 2³S target state included in the projection operator.

The ground-state wave function is chosen to be

$$\phi_o(\vec{r}_j, \vec{r}_k) = (\zeta^3/\pi) e^{-\zeta(r_j+r_k)}, \quad (8)$$

where

$$\zeta = \frac{27}{16} = 1.6875, \quad (8a)$$

is the optimized effective nuclear charge. This wave function is quite approximate as compared with the exact ground-state wave function. However, using an accurate wave function would make the computation prohibitively complicated. On the other hand, use of this projection operator has been successful in the literature.¹⁹

The energy of the ²P resonance is then solved by minimizing

$$\delta \frac{\langle \hat{Q}\Psi | H | \hat{Q}\Psi \rangle}{\langle \hat{Q}\Psi | \hat{Q}\Psi \rangle} = 0, \quad (9)$$

with respect to the parameters in Ψ . This method is carried out using the same two functions as in Table I. The new results are in good agreement with those of the saddle-point technique. These results are given in Table III. Again, the formation of the resonance is mainly caused by the coupling of the first two partial waves. The energy is at 20.504 eV for the 35-term wave function and at 20.495 eV for the 54-term wave function. These are slightly lower than the results of Table I. This is reasonable in view of the approximation, Eq. (8). The energy would be raised if ϕ_o could be improved.

V. DISCUSSION

The results of both theoretical methods show the possible existence of a ²P resonance lying above the 2³S threshold but below the 2¹S threshold. Since the resonant wave function is orthogonal to the 2³S target, it should be considered as a closed-channel resonance. This Feshbach resonance differs from those in the elastic scattering region by having a much larger width. This can be seen as follows. Using Feshbach formalism, the width of a resonance can be defined approximately, as¹⁸

$$\Gamma = 2\pi |\langle Q\Psi | H | P\Psi \rangle|^2, \quad (10)$$

where $Q\Psi$ is the resonant closed-channel wave function and $P\Psi$ includes the open channel as well as the nonresonant closed-channel part. For elastic scattering, the configurations in $Q\Psi$ and $P\Psi$ differ by at least two electronic configurations. However, for ²P of He⁻, the main configuration in $Q\Psi$ is [(1s2s)¹S, 2p]. It differs with the [(1s2s)³S, kp] in $P\Psi$ by only one configuration. The width, therefore, is larger. This also leads to a large increase in the excitation cross section for the (1s2s)³S state.²⁰

Experimentally, the observed position of this resonance lies from 20.3 to 20.45 eV. The standard deviations range from ± 0.3 to ± 0.05 eV.¹ The result of the present work is 20.536 eV from the saddle-point technique and 20.496 eV from the quasi-projection-operator method. In making this comparison, one should keep in mind that the coupling with the open-channel has not been fully accounted for, especially in the quasi-projection-operator method. There will be a shift in the resonance position due to this coupling. For the same reason discussed for the width, this shift could be larger than those in the elastic scattering region.

TABLE III. Quasi-projection-operator calculation of 1s2s2p²P resonance of He⁻ (in a.u.). The nonlinear parameter in the quasi-projection-operator ground-state orbital is $q = 27/16$. The nonlinear parameters and the number of terms in each partial wave is the same as those in Table I.

Angular partial wave	-E	
	35 term	54 term
[(s, s) ¹ S, p]	2.142 18	2.142 89
[(s, p) ¹ P, d]	2.148 80	2.149 12
[(p, p) ¹ S, p]	2.149 47	2.149 82
[(s, d) ¹ D, p]	2.150 10	2.150 43
[(d, d) ¹ S, p]	2.150 14	2.150 48
[(p, d) ¹ P, d]	2.150 18	2.150 51
[(s, d) ¹ D, f]	2.150 21	2.150 54

TABLE IV. $1s2s2p\ ^2P$ resonance energy for He^- .

	Energy (eV)	Method
Theory	20.17	Matrix-variation method, Ref. 10
	20.19	Close-coupling and R -matrix method, Ref. 9
	20.536 ^a	Present work saddle-point technique
	20.495 ^a	Present work quasi-projection-operator technique
Experiment	20.3 ± 0.3	Ref. 8
	20.35 ± 0.3	Ref. 3
	20.45 ± 0.05	Ref. 5

^a Relative to the "exact" nonrelativistic ground-state energy of He at $-2.903\ 724$ a.u. The conversion factor used is $1\ \text{a.u.} = 27.211\ 652\ \text{eV}$.

Most of the previous theoretical calculations were carried out by using target state expansions. The results are about 20.2 eV, lying on the low side of the experiment. It is not clear whether the inaccuracy in the target-state wave functions, especially the large error in the ground-state target (~ 1 eV),¹⁰ may have lowered the calculated resonance position. A comparison of the theoretical and experimental result is given in Table IV.

An interesting analogy to the $\text{He}^- 1s2s2p\ ^2P$ state is the $1s2s2p\ ^4P$. The latter is the lowest bound state of this negative ion in the nonrelativistic approximation. It is formed by the dipole coupling of $[(1s2s)^3S, 2p]$ and $[(1s2p)^3P, 3d]$ with an electron affinity of 78 meV.²¹ In this work the results

of the two 2P calculations are lower than the $(1s2s)^1S$ state by 83.8 meV and 124 meV, respectively. This is perhaps caused by the larger polarizability of the singlet targets.²²

The result of the quasi-projection-operator method calculated here is different from that of Wakid *et al.*¹² The main reason is probably the different choice of angular terms and the optimization on nonlinear parameters.²³

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²³Upon the completion of this manuscript, I was informed by Dr. A. Temkin and Dr. A. K. Bhatia that a new calculation had been done on the $(1s2s2p)^2P$ of He^- . Their result also showed a Feshbach resonance below the $(1s2s)^1S$ target energy, thus confirming the present finding.