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Analysis of weak neutral currents in hydrogenic ions

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We have calculated the decay of a metastable hydrogenic ion through the process $2S \to 1S + \gamma$, as a possible source of experiments on weak interactions in atoms. Our results are fairly exhaustive for ions with atomic number up to $Z = 20$ and nuclear spin $I = 0$. We include the effects of applied electric and magnetic fields, and of both $2P_{1/2}$ and $2P_{3/2}$ virtual states. Within this framework, only *one* of seven possible P-odd asymmetries is a likely candidate for experiment, the decay anisotropy of a polarized beam in vacuum. No viable experiments were found using the quench radiation in an electric field.

I. INTRODUCTION

The subject of weak neutral currents in atoms must be considered to have entered a new phase, changing from a period of discovery to one of precision measurement. The earlier search for weak neutral currents in neutrino reactions has already gone through this same change. The first experiments, establishing the existence of neutral currents,[1] have progressed to a model independent determination of the form of the interaction.[2] The results are now well established to agree with the prediction of the "standard model" of Weinberg and Salam,[3] with $\sin^2\theta \simeq 0.25$.

Recent experimental results provide evidence for the existence of weak neutral currents in the electron-nucleon system. The first of these is a series of measurements of optical activity in bismuth vapor.[4] Although the results are not entirely consistent, they indicate the presence of enhanced parity-nonconserving effects of the size predicted for heavy elements.[5] An experiment on deep inelastic scattering of polarized electrons from deuterium provides more conclusive evidence of parity nonconservation and confirms the standard model.[6] Circular dichroism measurements in cesium vapor have not yet been conclusive[7] but a similar experiment in thallium is consistent with the standard model.[8] Thus, there is strong evidence for the existence of neutral currents in atoms. The next challenge is to provide independent precision measurements of the four empirical coupling constants C_{1p}, C_{1n}, C_{2p}, C_{2n}.

Parity-nonconserving effects in heavy atoms are probably not suited to this task for two separate reasons. One reason, which underlies the early success of these experiments, is that heavy atoms enhance one particular combination of coupling constants $Q = 2(C_{1p}Z + C_{1n}N)$. High-Z atoms are sensitive primarily to this one parameter, giving a one-parameter fit to the theory. The other reason is that the atomic theory of these forbidden transitions is very sensitive to details of the electronic wave functions, leading to significant theoretical uncertainties in analyzing the data. This source of uncertainty probably explains part of the disagreement between the standard model and the bismuth results. It seems unlikely to us that experiments with heavy atoms will ever lead to a determination of all of the coupling constants, and to complete determination of the model for weak neutral currents in atoms.

In this next phase of precision measurements, the role of the hydrogen atom will probably be important.[9] Our understanding of the electronic wave functions is sufficient to reduce the theoretical uncertainty well below experimental errors, thus eliminating this source of error. Experiments with these atoms will not have the advantage of a coherent enhancement arising from a large number of nucleons, but this disadvantage will be offset by a different type of enhancement, coming from the close spacing of $2S$-$2P$ levels in the Coulomb potential. It has already been shown that parity mixing among $2S$, $2P$ states in the vicinity of the level crossings in hydrogen and deuterium is equally sensitive to all four coupling constants.[10] Metastable beam experiments on microwave transitions at the first level crossing ($B \simeq 572$ G) are already underway at Michigan, Washington, and Yale. These experiments are sensitive only to C_{2p} and C_{2n}, and should eventually provide precision measurements of these constants. Parity mixing at the second level crossing ($B \simeq 1194$ G) provides an opportunity of measuring C_{1p} and C_{1n}, but there are no experiments now in progress at this level crossing.

We intend to analyze in this paper the prospects for precision experiments sensitive to Q using metastable beams of hydrogenic ions of moderate

Z. Our aim is to exploit *both* the advantages of the Coulomb degeneracy as well as the coherent enhancement due to larger numbers of nucleons. We will single out the one-phonon decay $2S_{1/2} \to 1S_{1/2} + \gamma$ as a candidate for a parity experiment. The analysis will be based on a spinless nucleus ($I=0$) in order to remove the complications of hyperfine structure and the contributions of C_{2p}, C_{2n}. Our results provide a systematic computation of the various decay asymmetries, together with a discussion of some practical realities for observing certain terms.

Various parts of this analysis have already been published. The $2S \to 1S$ decay of metastable hydrogen ($Z=1$) was analyzed in some detail by Moskalev *et al.*[11] They considered the parity-conserving terms for a beam perturbed by both electric and magnetic fields and the parity-nonconserving terms in a pure magnetic field. Feinberg and Chen[12] have discussed the vacuum decay asymmetry for hydrogenic ions of arbitrary Z. More recently, Mohr[13] and Drake[14] have calculated the parity-conserving terms for a polarized hydrogenic ion of arbitrary Z, perturbed by both electric and magnetic fields. Our work is an extension of these previous publications, giving the decay asymmetries of a polarized beam perturbed by electric fields, magnetic fields, and weak interactions. The main motivation for experiments with hydrogenic ions is still the measurement of the Lamb shift ($\mathcal{S}$) but we will direct attention to the possibility of measuring weak interactions. As in Lamb-shift measurements, there is a variety of different parity experiments possible, using both intrinsic or induced dipole moments, and both $2P_{1/2}$ and $2P_{3/2}$ virtual states. We will include all of these possibilities, and assess their role in weak interaction experiments.

II. PRELIMINARY DISCUSSION

Before presenting the details of our calculation, it is necessary to discuss why we retain certain variables and not others. Since any experiment aimed at detecting weak interactions will have to cope with a very small effect, it is vital to consider only processes with the largest possible asymmetries and event rates, and to retain only those variables which can be accurately controlled. This requires some preliminary discussion of the practical realities governing heavy-ion experiments.

Beams of hydrogenic ions are available with atomic numbers from $Z=1$ to $Z=20$, with about 1% of the beam in the metastable $2S$ state.[15] In vacuum, this state decays with a rate dominated by two decay modes[16]

$$R(2S \to 1S) \cong (8.2Z^6 + 2.5\times 10^{-6} Z^{10}), \qquad (1)$$

with units of $\sec^{-1}$. The first term, which dominates at low Z, gives the rate of two-photon ($E1$-$E1$) decays. The resulting continuum is very insensitive to weak interactions, which mix in some $E1$-$M1$ amplitude. The weak parity mixing is suppressed by the small size of magnetic-dipole moments relative to electric-dipole moments $M1/E1 \approx \mu_0/ea_0 = \frac{1}{274}$.

The second term gives the rate of one-photon ($M1$) decays, which form a line spectrum with energy

$$E_\gamma \cong 10.2Z^2, \qquad (2)$$

in units of eV, at the upper end of the two-photon continuum. The one-photon decays are more sensitive to weak interactions, which are enhanced by the inverse of the same ratio of dipole moments. The remainder of our discussion is concerned with these one-photon decays. At low Z there is need for good energy resolution to separate the line from the stronger continuum; at high Z there is a need for good spatial resolution to deal with the short metastable lifetime. We want to explore the various asymmetries as a function of Z to assess the experimental opportunities.

Assuming the line spectrum can be resolved, it is reasonable to determine the angular distribution of the photons and so we retain the wave vector $\hat{k}$ in all calculations. But since polarimetry in the soft x-ray region is difficult, we will always average over the photon polarization vector $\hat{e}$. On the other hand, polarization of metastable beams can be produced by the tilted-foil method[17] and perhaps other means, so we retain the incident beam polarization $\vec{P}$ in our analysis. For $2S$ atoms with $I=0$, no other orientation tensors are possible. We note that it is also feasible to analyze the polarization of the transmitted metastable beam[14] which would be an alternative to dealing with a polarized incident beam.

One must not limit the discussion to spontaneous decays in free space. These events are usually a small fraction of the decays, and give a distributed line source which is not easy to detect. A more intense localized source can be made by applying an electric field to the beam. This mixes the $2P$ and $2S$ states, allowing decay of the beam via the $E1$ amplitude of the $2P$ states. This Stark-induced amplitude depends on the $2S$-$2P$ energy separation, and therefore on the magnetic-field strength. The beam enters the E and B fields adiabatically without generating quantum beats; we are only concerned with the mean decay rate of the beam. We will limit our treatment to fields which are sufficiently small that the amplitudes are linear in $\vec{E}$ and $\vec{B}$. This can be guaranteed by keeping only the leading terms of order

$$\frac{eEa_0}{Z\mathcal{S}}, \quad \frac{\mu_0 B}{\mathcal{S}} \ll 1 \tag{3}$$

in an expansion about the zero-field limit. Since $\mathcal{S}$ increases strongly with Z while the dipole moments either decrease or remain constant, these field strengths can be quite strong without exceeding the linearity condition

$$E \ll 830\ Z^5\ (\text{V/cm}), \quad B \ll 760\ Z^4\ (\text{G}). \tag{4}$$

Indeed, the difficulty of perturbing these ions with static fields makes it impossible to satisfy the condition for a level crossing of $2S$-$2P$:

$$B_0 = 570\ Z^4\ \text{G} \tag{5}$$

except for very light ions. Our task will therefore be to discuss the asymmetries in the weak-field limit. This forces attention to the design of parity experiments away from the level crossings, which is of interest for hydrogen as well as for hydrogenic ions.[18]

III. ONE-PHOTON-DECAY AMPLITUDES

We will give in this section a compilation of the decay amplitudes for the transition $2S_{1/2} \rightarrow 1S_{1/2} + \gamma$ for arbitrary Z, in combined electric and magnetic fields. Despite our comments about the precision of atomic theory for these ions, our aim here is to explore various asymmetries rather than to give accurate numerical results. For this reason, we keep only the leading contribution to each amplitude. This is not a systematic expansion in powers of αZ; some amplitudes (especially the $M1$) require inclusion of $(\alpha Z)^2$ corrections to the nonrelativistic theory, while others do not. These omissions can be remedied, as was done in hydrogen by Brodsky and Parsons,[19] using numerical methods including the entire $n=2$ shell. By limiting the treatment to the leading terms, we are able to rely on simple matrix methods.

The basic formula for the rate is well known,[20]

$$\frac{dR}{d\Omega} = \frac{e^2}{2\pi\hbar}\frac{\omega}{c^3}\sum \left| \left\langle 1Sm' \left| \hat{\epsilon}\cdot\frac{\vec{p}}{m}\, e^{-i\vec{k}\cdot\vec{r}} \right| 2S_{1/2}m \right\rangle \right|^2, \tag{6}$$

where $\omega, \vec{k}$ are the photon frequency wave vector and where $\sum$ stands for the average over initial states and the sum over final states. Since there are only two projections of the initial and final spins, we can express the amplitudes in terms of 2×2 matrices M,

$$\left(1Sm' \left| \hat{\epsilon}\cdot\frac{\vec{p}}{m}\, e^{-i\vec{k}\cdot\vec{r}} \right| 2Sm \right) \equiv (m'|M|m), \tag{7}$$

where M contains all the dependence on $\hat{e}$, $\vec{k}$ and $\vec{E}, \vec{B}$. In relativistic units ($\hbar, m, c \Rightarrow 1$) the frequency is

$$\omega = \tfrac{3}{8}\alpha^2 Z^2. \tag{8}$$

We obtain from (6)

$$\frac{dR}{d\Omega} = \frac{3\alpha^3 Z^2}{16\pi}\,\frac{1}{2}\sum{}' \operatorname{tr}[M(1+\vec{\sigma}\cdot\vec{P})M^\dagger]. \tag{9}$$

Here $\sum'$ is the sum over photon polarization, and the factor $\frac{1}{2}(1+\vec{\sigma}\cdot\vec{P})$ is the density matrix of the partially polarized incident beam. We will list the various contributions to M, ordered roughly by their relative magnitudes.

A. Magnetic dipole

Our basic formula does not strictly pertain to this amplitude, which vanishes in the nonrelativistic limit. A derivation which includes relativistic and finite wavelength corrections gives the result[21]

$$M_1 = -\frac{(\alpha Z)^4}{27\sqrt{2}}\,\vec{\sigma}\cdot\hat{e}\vec{\sigma}\cdot\hat{k}. \tag{10}$$

This amplitude is linear in $\hat{k}$ and grows rapidly with αZ. It is independent of external $\vec{E}, \vec{B}$ fields except for terms arising from a shift in the Lyman-α frequency in Eq. (8). We will not include these corrections, which are of order $(ea_0E/Z\ \text{Ry})$ or $(\mu_0 B/\text{Ry})$, and are much smaller than $(ea_0E/Z\mathcal{S})$ or $(\mu_0 B/\mathcal{S})$.

B. Stark-induced electric dipole ($2P_{1/2}$ contribution)

An electric field mixes the nearby $2S_{1/2}$ and $2P_{1/2}$ states, giving the perturbed $2S_{1/2}$ state an electric-dipole amplitude for decay to the $1S_{1/2}$ ground state. To first order on $\vec{E}$ the amplitude is given by

$$(m'|M_2|m) = -i\omega\sum_{m''}\frac{(1S_{1/2}m'|\hat{\epsilon}\cdot\vec{r}|2P_{1/2}m'')(2P_{1/2}m''|e\vec{E}\cdot\vec{r}|2S_{1/2}m)}{\mathcal{S}+i(\Gamma/2)}, \tag{11}$$

where Γ is the radiative width of the $2P$ states (we neglect the width of the $2S$ states). A short derivation gives the result

$$M_2 = +i\,\frac{32e}{81\sqrt{2}}\,\eta\vec{\sigma}\cdot\hat{e}\vec{\sigma}\cdot\vec{E}, \tag{12}$$

where $\eta \equiv [\mathcal{S}+i(\Gamma/2)]^{-1}$. This amplitude is independent of $\hat{k}$ and decreases like Z^{-4} due to the growth of the Lamb shift $\mathcal{S}$.

C. Zeeman corrections to M_2

A magnetic field alters Eq. (11), to first order in B, by shifting the energy eigenvalues in the denominator, without changing the eigenfunctions. Expanding the denominator leads to the following additional amplitude:

$$M_3 = -i\,\frac{32\alpha}{243\sqrt{2}}\,\eta^2\vec{\sigma}\cdot\hat{e}(\vec{E}\cdot\vec{B}+2i\vec{\sigma}\cdot\vec{E}\times\vec{B})\,. \quad (13)$$

This introduces a dependence of the amplitude on $\vec{B}$, of order $(eB/\mathcal{S})$ relative to M_2.

D. Stark-induced electric dipole ($2P_{3/2}$ contribution)

We can obtain the relatively small contribution of the $2P_{3/2}$ states by extending the sum in Eq. (11) to include them. We find for this term

$$M_4 = i\,\frac{32}{81\sqrt{2}}\,\zeta(2\hat{\epsilon}\cdot\vec{E}-i\vec{\sigma}\cdot\hat{e}\times\vec{E})\,, \quad (14)$$

where $\zeta\equiv[\mathcal{S}-\Delta E+i(\Gamma/2)]^{-1}$. Here $\Delta E=E(2P_{3/2})-E(2P_{1/2})$ is the fine-structure separation. This amplitude is smaller than M_2 by about $\mathcal{S}/\Delta E\cong 10\%$. It has a Zeeman correction of order $eB/\Delta E$ smaller than M_4, which we will neglect.

E. Weak induced electric dipole ($2P_{1/2}$)

Finally, we come to the weak neutral-current interaction, which also mixes $2S_{1/2}$ and $2P_{1/2}$, giving a decay amplitude

$$(m'|M_5|m) = -i\omega\sum_{m''}\frac{(1S_{1/2}m'|\hat{\epsilon}\cdot\vec{r}|2P_{1/2}m'')(2P_{1/2}m''|H_{wk}|2S_{1/2}m)}{\mathcal{S}+i(\Gamma/2)}\,. \quad (15)$$

The weak matrix elements have already been given [Ref. 10, Eq. (25)] and the sum easily leads to the amplitude

$$M_5 = -\frac{QG(\alpha Z)^5}{324\pi}\,\eta\vec{\sigma}\cdot\hat{e}\,, \quad (16)$$

where $Q=2(ZC_{1p}+NC_{1n})$ is the coupling constant we want to measure. We note that for $I=0$ the weak interaction preserves $\vec{J}$ and does not lead to any mixing of $2P_{3/2}$ states.

F. Zeeman correction to M_5

Expanding the energy denominators as before, leads to a Zeeman correction of the weak amplitude M_6,

$$M_6 = +\frac{QG(\alpha Z)^5 e}{972\pi}\,\eta^2\vec{\sigma}\cdot\hat{e}\vec{\sigma}\cdot\vec{B}\,. \quad (17)$$

This completes our list of amplitudes; one need only insert their sum in Eq. (9) and grind. The number of terms is large and we find it appropriate to present only some of them for discussion.

IV. DECAY DISTRIBUTION

The one-photon-decay distribution given by Eq. (9) can now be examined term by term. In relativistic units, each term is in units of $mc^2/\hbar = 7.76\times10^{20}\ \mathrm{sec}^{-1}$. We begin with the decay of an unpolarized beam, neglecting the contributions of the $2P_{3/2}$ states (M_4) and of weak interactions (M_5 and M_6),

$$\frac{dR}{d\Omega} = \frac{64\alpha^3Z^2}{2187\pi}\left(\frac{9(\alpha Z)^8}{1024}+\frac{\alpha}{(\mathcal{S}^2+\Gamma^2/4)}E^2 - \frac{3(\alpha Z)^4 e\Gamma}{32(\mathcal{S}^2+\Gamma^2/4)}(\vec{E}\cdot\hat{k}) + \frac{\alpha(\alpha Z)^4(\mathcal{S}^2-\Gamma^2/4)}{8(\mathcal{S}^2+\Gamma^2/4)^2}(\vec{B}\times\vec{E}\cdot\hat{k})\right). \quad (18)$$

These terms have all been published earlier and are given again only for reference. The first term is the intrinsic magnetic-dipole distribution, and the second is the electric-dipole distribution arising from the mixing of $2S_{1/2}$ and $2P_{1/2}$ states in an electric field. Intereference of these $M1$ and $E1$ amplitudes gives two additional anisotropic terms proportional to $\vec{E}\cdot\hat{k}$ and to $\vec{B}\times\vec{E}\cdot\hat{k}$, previously analyzed by Moskalev,[11] Mohr,[13] and Drake.[14] Each of these anisotropies provides a possible method of measuring the Lamb shift in high-Z ions.

The strength of these intereference terms is dependent on the relative phases of the amplitudes M_1, M_2, and M_3, which are governed by T invariance. In a T-invariant theory with negligible damping, the $\vec{B}\times\vec{E}\cdot\hat{k}$ term occurs but not the $\vec{E}\cdot\hat{k}$ term. The reason is that $\vec{B}$ and $\hat{k}$ are odd, and $\vec{E}$ is even in T; in lowest order the transition rate can only depend on even combinations. If damping is included in the T-invariant theory, then the phases of the amplitudes are changed by small angles of order $\Gamma(2P)/\mathcal{S}$, and odd terms like $\vec{E}\cdot\hat{k}$ occur. This is consistent with the reciprocity of transition rates and with the conservation of probability.[22] Since $\Gamma(2P)/\mathcal{S}$ is of order 10%, these odd terms are rather small.

If the incident beam is polarized, then we must add three further terms to the large parentheses in Eq. (18),

$$\left(-\frac{3(\alpha Z)^4 e\mathcal{S}}{16(\mathcal{S}^2+\Gamma^2/4)}\vec{P}\cdot\vec{E}\times\hat{k}-\frac{2e\alpha\mathcal{S}}{3(\mathcal{S}^2+\Gamma^2/4)^2}(2E^2\vec{P}\cdot\vec{B}-\vec{E}\cdot\vec{B}\vec{E}\cdot\vec{P})\right.$$
$$\left.+\frac{(\alpha Z)^4\alpha\mathcal{S}\Gamma}{16(\mathcal{S}^2+\Gamma^2/4)^2}(\vec{E}\cdot\vec{B}\vec{P}\cdot\hat{k}-2\vec{E}\cdot\vec{P}\vec{B}\cdot\hat{k}+2\vec{P}\cdot\vec{B}\vec{E}\cdot\hat{k})\right). \quad (19)$$

The first of these was emphasized by Drake[14] as a means of determining the beam polarization; the others are new results. A note of caution should be added about the use of Eq. (19). It gives the instantaneous decay distribution in terms of the instantaneous polarization $\vec{P}(t)$ for fields $\vec{E},\vec{B}$ in the proper frame of the beam. In some circumstances, the polarization does not depend on time, and no problems arise. An example would be a longitudinally polarized beam moving parallel to the laboratory magnetic field; the polarization stays parallel to the field. But generally the polarization will change through precession about the proper magnetic fields, and this precession will tend to reduce the observed anisotropy. A separate calculation of this spin precession is required. This precession will set an upper limit on the magnitudes of the fields which can be applied, often a more restrictive limit than that in Eq. (4).

If the $2P_{3/2}$ states are included in the calculation, then there are small changes in the anisotropic terms in Eqs. (18) and (19), as well as some new terms.[23] Rather than write out the general result, we will simply list the five new terms which appear:

$$E^2-3(\vec{E}\cdot\hat{k})^2,\quad (\vec{E}\cdot\hat{k})(\vec{E}\times\vec{P}\cdot\hat{k}),$$
$$[E^2-3(\vec{E}\cdot\hat{k})^2](\vec{P}\cdot\vec{B}),$$
$$(\vec{E}\cdot\hat{k})(\vec{E}\times\vec{B}\cdot\hat{k}),$$
$$(\vec{E}\cdot\vec{P})(\vec{E}\cdot\vec{B})-(\vec{E}\cdot\hat{k})(\vec{P}\cdot\hat{k})(\vec{E}\cdot\vec{B})-2(\vec{E}\cdot\hat{k})(\vec{P}\cdot\vec{E})(\vec{B}\cdot\hat{k}). \quad (20)$$

These terms are all suppressed by $\mathcal{S}/\Delta E\simeq 10\%$, and are too small to be of much interest in Lamb-shift experiments. Their presence is simply a complication in precision experiments, or in the search for weak interactions, since they may contribute to systematic errors, proportional to stray fields instead of applied fields.

Finally we will give the parity-nonconserving terms arising from the interference of the weak amplitudes (M_5 and M_6) and the dominant amplitudes (M_1, M_2, and M_3). We should add the following four terms to Eq. (19):

$$\left[\left(\frac{GQ(\alpha Z)^5}{1024\pi\sqrt{2}}\right)\left(\frac{3(\alpha Z)^4\mathcal{S}}{(\mathcal{S}^2+\Gamma^2/4)}\vec{P}\cdot\hat{k}-\frac{e(\alpha Z)^4(\mathcal{S}^2-\Gamma^2/4)}{(\mathcal{S}^2+\Gamma^2/4)^2}\vec{B}\cdot\hat{k}+\frac{e(\alpha Z)^4\mathcal{S}\Gamma}{(\mathcal{S}^2+\Gamma^2/4)^2}\vec{P}\cdot\vec{B}\times\hat{k}+\frac{32\alpha\mathcal{S}}{3(\mathcal{S}^2+\Gamma^2/4)^2}\vec{P}\times\vec{B}\cdot\vec{E}\right)\right]. \quad (21)$$

The first term gives an anisotropy in the vacuum decay of a polarized beam; its coefficient is the same as the circular polarization of the emission line from an unpolarized beam, first calculated by Feinberg and Chen.[24] It comes from the interference of the magnetic dipole and the weak amplitude (M_1 and M_5). The second and third terms give the anisotropy for a polarized beam in a magnetic field, first derived by Moskalev.[11] They come from the interference of the magnetic dipole and the Zeeman correction to the weak amplitude (M_1 and M_6). The last term is a new one which gives a dependence of the quench rate in crossed fields. It comes from the interference of the induced electric dipole and the weak amplitude (M_2 and M_6 or M_3 and M_5).

It should be noted that the decay distribution does *not* depend on the pseudoscalars $\vec{E}\cdot\vec{B}$ or $\vec{E}\cdot\vec{P}$. The absence of these terms results from the combined effects of T invariance and unitarity. Either of these terms would survive the

TABLE I. Parity-nonconserving asymmetries in the one-photon decay distribution. Asymmetries A_1–A_3 are evaluated at $E=0$ and A_4–A_7 at $E=E_0(Z)$ to optimize them.

Asymmetry	Pseudoscalar	Maximum size
A_1	$\vec{P}\cdot\hat{k}$	$\frac{\sqrt{2}G}{\alpha^4}\frac{Q}{Z^3F}P$
A_2	$\vec{B}\cdot\hat{k}$	$-\frac{2\pi\sqrt{2}G}{e\alpha^8}\frac{Q}{Z^7F^2}B$
A_3	$\vec{P}\times\vec{B}\cdot\hat{k}$	$\frac{2^{10}\pi^2\sqrt{2}G}{3^7e\alpha^8}\frac{Q}{Z^7F^3}PB$
A_4	$\vec{P}\cdot\vec{B}\times\vec{E}$	$\frac{\sqrt{2}\pi G}{e\alpha^8}\frac{Q}{Z^7F^2}PB$
A_5	$\vec{E}\cdot\vec{B}-3\vec{E}\cdot\hat{k}\vec{B}\cdot\hat{k}$	$-\frac{2^{12}\pi G}{3^7\sqrt{2}e\alpha^7}\frac{Q}{Z^7F^2}B$
A_6	$\vec{E}\cdot\vec{P}-3\vec{E}\cdot\hat{k}\vec{P}\cdot\hat{k}$	$\frac{2^{10}G}{3^7\sqrt{2}\alpha^3}\frac{Q}{Z^3F}P$
A_7	$\vec{E}\cdot\vec{P}\times\vec{B}-3\vec{E}\cdot\hat{k}\vec{P}\times\vec{B}\cdot\hat{k}$	$\frac{2^4G}{3\sqrt{2}e\alpha^7}\frac{Q}{Z^7F}PB$

summation over photon directions, and therefore be related by unitarity to the total disappearance rate of metastable states in the beam. But T invariance implies the absence of these T-odd terms in the disappearance rate; they would invalidate the reversibility of the beam-quenching reactions.[22]

The inclusion of $2P_{3/2}$ contributions adds three more parity-nonconserving terms to Eq. (18), with coefficients which are suppressed by a factor $\mathcal{S}/\Delta E \simeq 10\%$ relative to Eq. (21). We find for the interference of M_4 and M_5, M_6,

$$\left[\left(\frac{QG(\alpha Z)^5}{1024\pi\sqrt{2}}\right)\left(-\frac{8e\Gamma}{\Delta E(\mathcal{S}^2+\Gamma^2/4)}(\vec{E}\cdot\vec{P}-3\vec{E}\cdot\hat{k}\vec{P}\cdot\hat{k}) +\frac{16\alpha\mathcal{S}\Gamma}{3\Delta E(\mathcal{S}^2+\Gamma^2/4)^2}(\vec{E}\cdot\vec{B}-3\vec{E}\cdot\hat{k}\vec{B}\cdot\hat{k}) +\frac{16\alpha(\mathcal{S}^2-\Gamma^2/4)}{3\Delta E(\mathcal{S}^2+\Gamma^2/4)^2} \times(\vec{E}\cdot\vec{P}\times\vec{B}-3\vec{E}\cdot\hat{k}\vec{P}\times\vec{B}\cdot\hat{k})\right)\right]. \quad (22)$$

Each of these terms produce tensor terms in the angular distribution of the photon, odd under spatial inversions of all the variables but even in $\hat{k}$.

Finally let us record the result for the total decay rate, integrated over all photon directions,

$$R=\frac{256\alpha^3Z^2}{2187}\left(\frac{9(\alpha Z)^8}{1024}+\frac{\alpha}{(\mathcal{S}^2+\Gamma^2/4)}E^2 +\frac{QG\alpha(\alpha Z)^5\mathcal{S}}{96\pi\sqrt{2}\,(\mathcal{S}^2+\Gamma^2/4)^2}\vec{P}\cdot\vec{B}\times\vec{E}\right). \quad (23)$$

This still contains a parity-nonconserving term.

V. DISCUSSION OF ASYMMETRIES

The general form of our analysis shows that there is a wide variety of parity-nonconserving asymmetries in this decay mode. As a first step in assessing the significance of the various terms in the decay distribution, we consider the magnitude of the asymmetries in Eqs. (21) and (22). For convenience we label the seven possible asymmetries $A_1, \ldots, A_7$. To simplify the discussion of their Z dependence, we represent the Lamb shift by Erickson's formula[25]

$$\mathcal{S}(Z)=\alpha(\alpha Z)^4F(Z)/6\pi, \quad (24)$$

where $F(Z)$ is a slowly varying function decreasing roughly as $Z^{-1/2}$. Similarly, the fine-structure interval and the $2P$ level width can be written as

$$\Delta E(Z)\cong(\alpha Z)^4/32, \quad \Gamma(Z)=(\tfrac{2}{3})^8\alpha(\alpha Z)^4. \quad (25)$$

Using these expressions, we have listed in Table I the pseudoscalar asymmetries and their Z dependence after optimizing the applied fields.

The asymmetry A_1 decreases with Z roughly as $Z^{-3/2}$. A_2 and A_3 decrease with Z roughly as Z^{-5} and $Z^{-9/2}$, although some of this decrease can be offset by an increase in the value of B, according to Eq. (4). All three of these asymmetries depend linearly on $\hat{k}$, and require observation of an anisotropy in the directional distribution. Since they originate from the intrinsic magnetic-dipole amplitude, they are all optimized by choosing zero electric field.

The other four terms come from the induced-electric-dipole amplitude, and are optimized at a field strength which equalizes the electric quench rate and the spontaneous decay rate

$$E_0\equiv\frac{e(\alpha Z)^8F(Z)}{64\pi}\cong 3.9\times10^{-6}Z^8F(Z) \quad (26)$$

with E_0 given in units of V/cm. This field strength is easily achieved either with applied or motional fields. For field strengths larger than E_0, these asymmetries decrease like $A_{\max}(2E_0/E)$. These four asymmetries also fall off with increasing Z.

This approach, which concentrates on the *asymmetries*, seems to imply that one should work with the lightest ions, since they all decrease with Z. This conclusion overlooks the fact that the *intensity* of the one-photon line is very small there, relative to the two-photon decays and to background. In fact, the one-photon transition has not been observed for low Z, and so the asymmetry in its distribution is somewhat irrelevant. To balance the discussion, we will also include the role of the intensity by giving an estimate of the integration time T for detecting the asymmetry. This complicates the presentation but makes it much more realistic.

The time required for the signal to equal the shot noise is

$$T=1/fJ\nu A^2, \quad (27)$$

where f is the fraction of the incident metastable atoms which decay by emitting one photon within the fiducial volume of the detectors, J is the metastable beam flux, and ν is the product of the detector solid angle and quantum efficiency. It is important to recognize that the variation of these other variables, we well as the asymmetry A, must be considered when we optimize Z, E, B, and P.

Several different cases must be examined. First consider the vacuum decay of a low-Z beam. The mean decay length is too long to be viewed by the detector, and so the fraction f is determined by the time of flight (τ) through the fiducial volume viewed by the detector

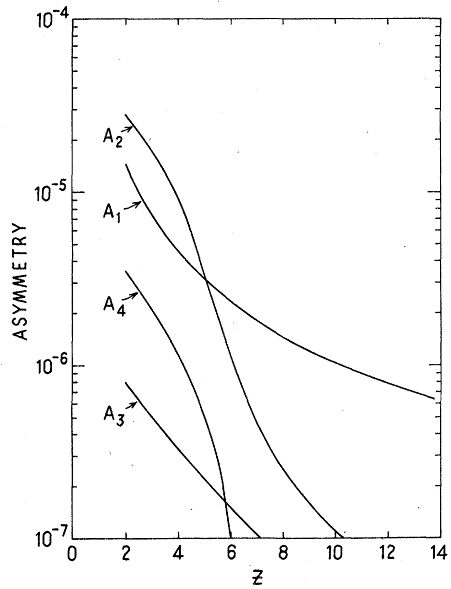

FIG. 1. Asymmetry for the most interesting terms versus nuclear charge Z.

$$f \cong 2.5\times 10^{-12} Z^{10}\tau \tag{28}$$

given in μsec. This result is strongly dependent on Z, and describes the difficulty in observing $M1$ decays in light elements. As Z increases, the mean decay length becomes shorter and is easily viewed by a single detector. The fraction f then is limited by the branching ratio for one-photon decays

$$f = 3.1\times 10^{-7} Z^{4}. \tag{29}$$

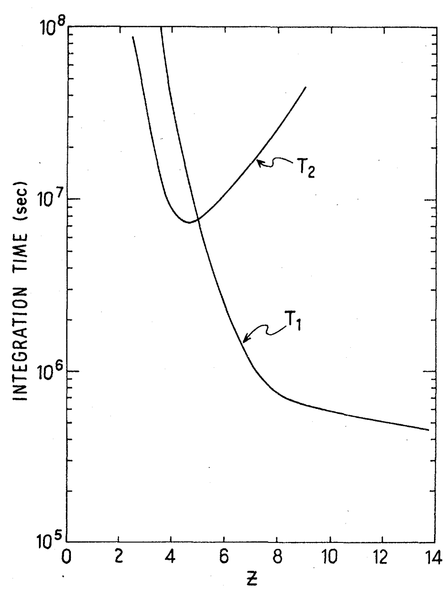

FIG. 2. Integration time T_i required to measure asymmetries A_1 and A_2 versus nuclear charge Z.

These formulas assume that f is much less than unity.

The application of an electric field can give a large increase in f, since the induced decay rate is quadratic in E. Both Eqs. (28) and (29) should be changed by a factor $(1+E^2/E_0^2)$. For fields stronger than the critical strength E_0, the integration time is independent of E, due to a compensation between the increase in the event rate and the decrease in the asymmetry. Nevertheless, it is an important advantage to have the events occurring in a smaller fiducial volume, and with a rate controlled by the field. We assume that a measurement of $A_4, \ldots, A_7$ would be done with $E \gtrsim E_0$.

The results of this section are gathered together in numerical form in Figs. 1 and 2. Figure 1 shows the asymmetries A_1, A_2, A_3, and A_4 as a function of nuclear charge while Fig. 2 shows the integration time for measurement of A_1 and A_2 versus nuclear charge. The figures are based on the following set of parameters[26]:

$$Q=Z,\quad P=0.25\,,$$

$$J\nu = 5\times 10^{8}\ \text{sec}^{-1},\quad \tau = 1\ \mu\text{sec}\,,$$

$$E=0\quad (i=1\text{--}3),\quad E=E_0(Z),\quad (i=4\text{--}7)$$

$$B=0.8B_0(Z)\quad (Z\leqslant 3),\quad B=100\ \text{kG}\quad (Z>3)$$

where E_0, B_0 have been defined earlier. In the next section we will use these results for an assessment of the prospects for parity experiments with hydrogenic ions.

VI. CONCLUSIONS

There is no question that weak-interaction experiments with heavy-ion beams are much more difficult than Lamb-shift measurements; the experiments we are discussing are all quite difficult. There are questions, however, as to the relative difficulties of detecting weak-interaction effects with light ions versus heavy ions, or with polarized beams versus unpolarized beams, or with spontaneous magnetic-dipole amplitudes versus induced electric-dipole moments. These are the kinds of questions we are now prepared to address.

It is evident from Figs. 1 and 2 that $A_1 \propto \vec{P}\cdot\hat{k}$ is a promising choice for higher-Z ions. Despite the slow decrease of A_1 with Z, the rapid increase in the one-photon rate gives an integration time which slowly decreases with Z. Another important consideration is that the one-photon line must be resolved from the two-photon continuum in order to suppress background. It is impossible to resolve the line unless its integrated intensity is at least a few percent of the total; this requires

$Z \geq 13$. All considered, it appears to us that an experiment of this type is reasonably attractive for moderately heavy ions. It would require development of beams with greater intensity and higher polarization than those available at the present time.

Substitution of a magnetic field instead of beam polarization, for the measurement of $A_2 \propto \vec{B} \cdot \hat{k}$, is a less attractive alternative. Under our assumptions the signal to noise is optimized at $Z \simeq 5$. The growth in the level separation at higher Z makes it increasingly difficult to perturb the states effectively ruling out the extension of this measurement to higher Z. There would be a serious problem in resolving the one-photon transitions from the two-photon decays for $Z \simeq 5$.

The measurement of $A_3 \propto \vec{P} \times \vec{B} \cdot \hat{k}$ has no advantages over A_1 and A_2. The asymmetry is smaller and falls off rapidly with Z. The resolution problem is the same, and the spin precession about $\vec{B}$ adds further complications.

If we turn next to quenching experiments, in which a strong electric field is applied to the beam, then some things appear to improve substantially. There is a strong attenuation of the beam in the quench region, giving a more localized source and a more intense one-photon line. There is the added possibility of asymmetries which survive the average over photon directions, such as $\vec{P} \cdot \vec{E}$, $\vec{B} \cdot \vec{E}$, and $\vec{P} \times \vec{B} \cdot \vec{E}$. These asymmetries could be detected by monitoring the metastable beam intensity downstream of the quench region where the fields are applied, instead of the one-photon distribution in this region. Unfortunately, for reasons already described, the simplest of these asymmetries do not occur in the transitions we are discussing. Only the term $A_4 \propto \vec{P} \times \vec{B} \cdot \vec{E}$ will be present, and its size decreases rapidly with Z. This experiment would also suffer from precession of the spin in the magnetic field, which would limit the strength of the applied magnetic field. It appears that quenching experiments are not a viable source of parity experiments under our assumptions.

The final three asymmetries A_5, A_6, A_7 arising from the virtual $2P_{3/2}$ states, are too small and too rapidly decreasing with Z to be of interest. Although these states have made possible some alternative experiments on the Lamb shift, the same is apparently not true for the weak interactions.

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