Quantum approach to $1/f$ noise

Peter H. Handel
University of Missouri-St. Louis, St. Louis, Missouri 63121
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On the basis of the known experimental properties of $1/f$ noise, some previous models are analyzed. The presence of $1/f$ noise in the simplest systems such as beams of charged particles in vacuum, the existence of $1/f$ noise in currents limited by the surface recombination rate, bulk recombination rate, or by the finite mobility determined by interaction with the phonons in solids, suggests a fundamental fluctuation of the corresponding elementary cross sections. This leads to fluctuations of the kinetic transport coefficients such as mobility μ or recombination speed, observable both in equilibrium and nonequilibrium. In the first case the available Johnson noise power kT, determined by the Nyquist theorem, is free of this type of $1/f$ fluctuation. An elementary calculation is presented which shows that any cross section, or process rate, involving charged particles, exhibits $1/f$ noise as an infrared phenomenon. For single-particle processes, the experimental value of Hooge's constant is obtained as an upper limit, corresponding to very large velocity changes of the current carriers, close to the speed of light. The obtained $\sin^2(\theta/2)$ dependence on the mean scattering angle predicts much lower $1/f$ noise for (low-angle) impurity scattering, showing a strong ($\sim \mu^2/\mu_{\rm latt}^2$) noise increase with temperature at the transition to lattice scattering. This is in qualitative agreement with measurements on thin films and on heavily doped semiconductors, or on manganin. The theory is based on the infrared quasidivergence present in all cross sections (and in some autocorrelation functions) due to interaction of the current carriers with massless infraquanta: photons, electron-hole pair excitations at metallic Fermi surfaces, generalized spin waves, transverse phonons, hydrodynamic excitations of other quanta, very low-energy excitations of quasidegenerate states observed, e.g., in disordered materials, at surfaces, or at lattice imperfections, etc. The observed $1/f$ noise is the sum of these contributions, and can be used to detect and study new infraquanta.

I. THE PHENOMENON OF $1/f$ NOISE

The problem of $1/f$ noise has captivated the attention of workers in the field of electrophysics for half a century, ever since problems of amplification took command of this field.[1,2] The more research was done, the more elusive its causes proved to be, and the more enigmatic its nature turned out to be.

If a constant voltage is applied to a semiconductor sample or device, to a carbon resistor or vacuum tube, the current will exhibit fluctuations. The frequency spectrum, obtained e.g., by bandpass filtering the fluctuations and recording the mean-squared output per unity passband as a function of the tuning frequency, is in general constant at high frequencies (white, thermal, or Nyquist noise), has one or more types of shot-noise components proportional to $[1+(2\pi f\tau)^2]^{-1}$ at intermediate frequencies f, and is proportional to f^{-1} and to the square of the averaged current at low frequencies, i.e., below some limit of 10^3–10^6 Hz. No lower limit of the $1/f$ spectrum has ever been found, although measurements down to 5×10^{-7} Hz have been performed.[3] The absence of a lower frequency limit was the main difficulty on the way to a theory of $1/f$ noise.

Another difficulty was the universality of $1/f$ noise. Electric $1/f$ noise is present whenever a current is carried by a small number of carriers, or when a bottleneck exists in an electric circuit. Indeed, $1/f$ noise is found also in electrolytic cells, electron beams in vacuum, thin metallic films and bad contacts, being also known as flicker noise, excess noise, or contact noise. There are excellent review articles[4-6] and books[7,8] in which the subject of $1/f$ noise is treated, and references are given.

The universality of $1/f$ noise transcends the limits of electrophysics. Indeed, $1/f$ noise is present in the phase and frequency of all known frequency standards[9,10] (clocks), determining in general the minimal error bars of frequency and time measurements. Furthermore, $1/f$ noise has been found in nerve cells as fluctuations of the voltage across the membrane of the node of Ranvier, below 10^{-8} Hz in the angular velocity of the earth's rotation, below 10^{-4} Hz in the relativistic neutron flux in the terrestrial magnetosphere, and in seasonal temperature fluctuations. If has also been found in the flow rate of sand in an hourglass, the frequency of sunspots, the light output of quasars, the flow rate of the Nile over the last 2000 years, the central Pacific ocean current velocities at a depth of 3100 meters, the flux of cars on an expressway, and in the loudness and pitch fluctuations of classical music. This enumeration is far from being complete.[11]

We define the universal phenomenon of $1/f$ noise here in general on the basis of three properties:

(1) The exponent of f in the spectral density differs by less than 20% from -1, over more than five units of the decimal logarithmic frequency scale with no observed indication of a low-frequency limit.

(2) The process is Gaussian; this means not only a Gaussian amplitude distribution, but also the relations between higher-order moments characteristic for a Gaussian process.

(3) The spectral density is proportional to the squared average value of the fluctuating quantity in linear systems.

As a consequence of the first defining property, once a measurement has become $1/f$-noise limited, no further reduction of the error is possible by extension of the measurement time. The limiting variance of the result obtained in a measurement which averages the quantity of interest over a time T and subtracts the average over the immediately preceding interval T is $4C \ln 2$, where C is the coefficient of $1/f$ in the spectral density. For shorter measurement times, i.e., as long as white noise was the limiting factor, the variance of the result decreased proportional to $T^{-1/2}$. Strictly speaking, this interesting property is applicable only to f^{-1} spectra. However, the slow increase ($\sim T^{0.2}$) of the variance for $Cf^{-1.2}$, or the slow decrease ($\sim T^{-0.2}$) for $Cf^{-0.8}$ do not change the nature of the process significantly from a practical point of view, although the theoretical and gnoseological implications are important. It is difficult to perform an accurate subtraction of superposed phenomena, such as various shot-noise components (proper or of generation-recombination type) which perturb the $1/f$ spectrum; cleaner $1/f$ spectra seem to be correlated with measurements over many frequency decades, while perturbed $1/f$ spectra are better represented among those measured over 2–3 decades only, in this author's opinion. This author also believes that after the subtraction of perturbing components, a subtraction of linear drift should be done, particularly if the spectrum becomes steeper than f^{-1} at low frequencies.

The spectral density of $1/f$ noise also determines all statistical properties of the process, according to the second defining property. This has been verified experimentally, by Stoisiek and Wolf,[12] up to the level of fourth-order, two-time moments, for band-limited $1/f$ spectra of various spectral widths. We shall keep the second and third property in the definition of $1/f$ noise until there is any experimental evidence against them.

The aim of this paper is to show that from basic quantum mechanics one can derive a certain level of electric $1/f$ noise without making any hypothesis. Furthermore, the properties of $1/f$ noise can be derived and new results can be predicted.

In the first two papers we restrict ourselves to *electric $1/f$ noise* at $T=0$. A third paper in this series could be viewed in principle as independent of the problem of $1/f$ noise and will generalize the known theory of infrared radiative corrections to finite temperatures, i.e., to the case in which a thermal equilibrium radiation background is present (e.g. the cosmic 3K background). A fourth paper is dedicated to the theory of $1/f$ noise in the presence of a thermal background radiation field. For most of the derivations, the idealized case of a beam of electrons scattered in vacuum will be considered, although a generalization to macroscopic samples and a summary of the "second-order" derivation are included in the fifth paper.

In order to gain clarity in the presentation of the fundamental ideas, after a brief analysis of the experimental facts and models of $1/f$ noise, the general scheme of the theory will be discussed first in Sec. III. An elementary derivation of the $1/f$ spectrum will be presented in Secs. IV and V. The essence of the quantum theory of $1/f$ noise presented in this series of papers is the interpretation of $1/f$ noise as an infrared phenomenon. Specifically, a new self-interference effect is found, which creates a bridge between the infrared quasidivergence of the cross sections as a function of the energy loss parameter (generalized bremsstrahlung) on one hand, and the observed $1/f$ fluctuation of the cross section, mobility, resistance, recombination speed (and often implicitly carrier concentration also) in the time domain. For the first time, this bridge translates the $1/\epsilon$ spectrum of generalized bremstrahlung energy losses into a $1/f$ frequency spectrum and into an infrared-divergent response in the time domain. A slightly different bridge developed by the author in the framework of the same theory for closed electric circuits is known as second-order $1/f$ noise (see Ref. 43 in Sec. VI). There may be other bridges in solids, e.g., due to the dependence of the scattering cross section of the current carriers on the relative separation and configuration of two or more scatterers with the time autocorrelation function of spatial correlations corresponding to an infrared-divergent ($1/f$) spectrum in the generalized spin-lattice description. While only one bridge is presented and only the case of photons is analyzed in this paper, the case of other infraquanta is similar. Finally, Sec. VI brings a discussion of the results, a summary of results to be derived in other papers of this series, and a comparison with experimental properties of $1/f$ noise.

The theory presented in these papers does not attempt to explain all spectra of electric current noise which show a $1/f$-like dependence over a limited frequency interval. Not accidental $1/f$ spectra, but the fundamental $1/f$ problem is our subject. On the other hand, to this author's knowledge, there is no other theory in which a $1/f$ spectrum down to $f=0$ is derived, i.e., in which no lower frequency limit is present.

II. EXPERIMENTAL PICTURE AND MODELS OF $1/f$ NOISE

Out of the large amount of experimental data published on electric $1/f$ noise, the following most important facts can be crystalized in addition to the properties already included in the general definition.

(1) In general, in homogeneous samples, $1/f$ noise is due to resistance fluctuations. Indeed, a constant current I through a homogeneous sample causes $1/f$ voltage fluctuations with a spectral density $\sim I^2$, while a constant voltage V across the sample causes $1/f$ current fluctuations $\sim V^2$ with the same coefficient of proportionality for relative (fractional) fluctuations. This is also true in electrolytes and liquid metals. We can write the spectral density of relative resistance-fluctuations

$$\frac{\langle(\Delta R)^2\rangle_f}{\langle R\rangle^2}=\frac{C}{f}, \tag{2.1}$$

where C is a constant.[13]

In junction diodes and bipolar transistors, Eq. (2.1) is not applicable and the current spectral density is observed to be proportional to I^γ in general,[14] with $1<\gamma<2$. This is true, e.g., for the $1/f$ noise in the basis current of bipolar transistors, which, according to van der Ziel[15] and Fonger, is caused by fluctuations of the recombination velocity. The latter may be located either on the surface, or in the bulk, the $1/f$ noise is the same.[16]

The question arises: Why do such different quantities as the resistance of a homogeneous sample and the recombination speed on the surface, or in the volume, all fluctuate with the same $1/f$ spectrum? Why should they fluctuate at all? As we shall see later, the correct answer to this question is: They all fluctuate with a $1/f$ spectrum because they are process rates, in essence cross sections, of current carriers with infrared-divergent coupling to photons and other infraquanta present in condensed matter (R is a reciprocal process rate, in essence a cross section). All such process rates and cross sections present this fundamental fluctuation,[17,18] as we shall see, but only the $1/f$ noise in the cross sections which in fact determine the current in the given circuit will determine the current noise.

The fundamental $1/f$ fluctuation of electromagnetic cross sections is neither a nonequilibrium process nor an equilibrium process, but an elementary process such as, e.g., scattering, as the author has pointed out[19-21] from the beginning of the quantum theory of $1/f$ noise. Therefore, it should be present both in equilibrium and in nonequilibrium. In thermal equilibrium, the expectation level of Nyquist noise in a bandwidth Δf is $N=4kTR\Delta f$ in voltage, $N'=4kTG\Delta f$ in current, and $N''=kT\Delta f$ in available power, where $G\equiv R^{-1}$ is the conductance of the sample. At sufficiently low frequencies, the fundamental $1/f$ fluctuation of the scattering cross sections, and of R and G, should therefore be observable in thermal equilibrium as a slow modulation of the expectation level of Nyquist noise, with a $1/f$ spectrum in voltage, or in current, but not in available power:

$$\frac{\langle(\Delta N)^2\rangle_f}{\langle N\rangle^2}=\frac{\langle(\Delta N')^2\rangle_f}{\langle N'\rangle^2}=\frac{C}{f}, \qquad \frac{\langle(\Delta N'')^2\rangle}{\langle N''\rangle^2}=0, \tag{2.2}$$

where C is the constant defined by Eq. (2.1). At higher frequencies above $C\Delta f$, this effect is covered by the white noise resulting from the squaring of thermal noise in Eq. (2.2). Furthermore, if the coefficient C is small, everything can be masked by bolometer effect,[8] which represents resistance fluctuations generated by the well-known energy fluctuations (formally equivalent to temperature fluctuations by division with the small heat capacity of the sample C_v) present in thermal equilibrium: $\langle(\Delta T)^2\rangle=kT^2/C_v$. These temperature fluctuations have a spectrum determined by the equation of heat diffusion. For three-dimensional diffusion, the spectrum has four regions, proportional to ω^0, ln ω^{-1}, $\omega^{-1/2}$, and $\omega^{-3/2}$. For two dimensions there are three regions, proportional to $\ln\omega^{-1}$, $\omega^{-1/2}$, and $\omega^{-3/2}$. Finally, for one-dimensional diffusion, there are regions proportional to $\omega^{-1/2}$ and $\omega^{-3/2}$. At sufficiently low frequencies, below the lowest relaxation time determined by the heat diffusion equation, the spectrum will always be white, as in the three-dimensional case, no matter what the geometry of the sample is. In the intermediary region, the superposition of contributions from various time constants[8] will often give a power-law-like spectrum, e.g., $f^{-1.3}$ or $f^{-1.4}$ over an extended frequency region, e.g., three or four frequency decades, which is easily confused with $1/f$ noise. The confusion with the bolometer effect must be avoided in the study of $1/f$ noise, although temperature fluctuations are also important in practical applications, and although true $1/f$ temperature fluctuations are likely to be present in nature and in the laboratory, as we shall see in the study of nonelectromagnetic $1/f$ noise, and may, in principle also induce electric nonelectromagnetic $1/f$ noise, i.e., electric $1/f$ noise of nonelectromagnetic origin.

We have already mentioned $1/f$ seasonal temperature fluctuations, and we have to agree that, beyond the predictions of the heat diffusion equation, the possibility of a low-frequency physical bolometer effect with a true $1/f$ spectrum, i.e., with no lower cutoff, has to be treated in the same context of nonelectromagnetic $1/f$ noise in a different paper; a simple reduction to temperature fluctuations does not solve the $1/f$ problem in any case.

Low-frequency fluctuations (with a $f^{-1.4}$ of $f^{-3/2}$ spectrum from 10^{-2} to 10^{-4} Hz) in the level of Nyquist noise have been first reported[22,23] in small (10^6 atoms) films of InSb and Nb, and may have to be associated with the bolometer effect. While the initial thick strip of InSb exhibited $1/f$ noise, the actual sample used for the measurement was cut with a diamond knife until a narrow bridge of 10^6 atoms only remained; in this state the strip seems to have been dominated by some diffusion mechanism, as the spectral density also indicates. The following year Hooge[24] reported measurements by Beck and Spruit,[25] who have verified Eq. (2.2) with the same C as in dc $1/f$ noise, on a carbon paper resistor showing exact $1/f$ noise over six frequency decades. Note that the C value of dc $1/f$ noise would have to be multiplied by a factor of $(1+\beta T)^2/\beta^2 T^2$ to obtain the C value applicable to Nyquist noise level fluctuations, if $1/f$ noise would be caused by temperature fluctuations. Indeed, temperature fluctuations would affect not only R, but also the available power level, or the factor T in N and N'. Therefore, at least in this instance, $1/f$ noise is not due to temperature fluctuations. While Voss and Clarke have verified that the bolometer effect,[8] i.e., resistance modulation noise from temperature fluctuations, has a spectral density proportional to $\beta^2 \equiv (R^{-1}dR/dT)^2$, Vandamme[26] has verified on manganin contacts that the spectral density of $1/f$ noise is independent of β. Furthermore, analyzing temperature fluctuations in metals and semiconductors, Kleinpenning[27] showed that $1/f$ noise has to be clearly distinguished from the bolometer effect which is negligible in semiconductors but often dominant in metals at frequencies close to the reciprocal thermal relaxation time. Finally, Eberhard and Horn[28,29] have overcome the bolometer effect limitation in metals, showing that the temperature dependence of $1/f$ noise cannot be explained by the temperature-fluctuation model.[23] They also prove that the bolometer effect, which they call type A noise, has to be distinguished from $1/f$ noise (type B in their papers).

We conclude that, as the experimental evidence suggests, $1/f$ noise is caused by fundamental fluctuations in the resistance, or recombination speed, or in general the kinetic coefficient which limits the current in the circuit, and that any identification of the bolometer effect with $1/f$ noise is misleading and should be avoided. Although $1/f$ temperature fluctuations may be present in nature (e.g., seasonal) and will cause some nonelectromagnetic $1/f$ noise in thermometric systems (large β), this is not the cause of the phenomenon of $1/f$ noise.

(2) For homogeneous samples the constant C in Eq. (2.1) is roughly proportional to the reciprocal number N of free charge carriers in the sample and to the squared ratio of mobility μ and mobility μ_{1att} determined by lattice vibrations only:

$$C = (\alpha_0/N)(\mu^2/\mu_{1att}^2), \tag{2.3}$$

where $\alpha_0 \simeq 2\times 10^{-3}$ is a universal dimensionless constant. This empirical relation established by Hooge[30,4] has been tested for many p-type and n-type semiconductors and for metals, including mercury. For the case of point contacts[30,4] the relation gave excellent results when applied independently to thin shells between equipotential surfaces around the contact, both for metals and high-ohmic semiconductors. By taking into account the presence of the oxide film on the surface, the agreement was extended also to low-ohmic semiconductors. Hooge's treatment of point contacts is based on an approximation of the electric field in the vicinity of the contact, which is excellent at low f. Sometimes Eq. (2.3), or the method of addition of independent noise contribution from various volume elements, fails if the sample is inhomogeneous, e.g., in presence of a carrier-concentration gradient perpendicular to the direction of the current. In Eq. (2.3) the factor μ^2/μ_{1att}^2 shows that no $1/f$ noise is observed in impurity scattering, as expected in the present theory; see the discussion after Eq. (5.6).

In electrolytes the constant C is inversely proportional to the volume and independent of the concentration of ions, i.e., α_0 in Eq. (3) is proportional to the concentration, with $\alpha_1 = 1$ for decimolar solutions.[31,32]

(3) The $1/f$ fluctuations present in the resistance (or conductance) arise mainly from mobility fluctuations. This has been first demonstrated for electrolytes[31,4] by comparing diffusion voltage fluctuations and resistance fluctuations in concentration cells. Later, this property was demonstrated for the case of semiconductors[33,34,4] by comparing thermoelectric emf fluctuations with resistance fluctuations in a thermocell. The experimental results are often ambiguous; concentration fluctuations cannot be ruled out completely, but mobility fluctuations seem to prevail.

This fundamental property, combined with the conclusion reached above under number (1) in this section, on the limiting kinetic coefficient as a source of the $1/f$ fluctuations, leads us to the cross sections which determine these coefficients, as cause of the $1/f$ fluctuations in general: the scattering cross sections σ_s (by impurities, phonons, and lattice defects), the recombination cross sections σ_r (on the surface and in the volume) in junctions and transistors, the hopping cross sections in some materials, tunneling cross sections in others, etc. Indeed, the expression $\mu = e\tau/m$ leads us to the collision time—and to the collision frequency which is proportional to σ_s.

(4) In general, $1/f$ noise has insignificant temperature dependence. Often, e.g., in semiconductors, there is a certain dependence due to absorption or desorption of gases or water vapor on the surface, or due to changes in the concentration of carriers. Very low T dependence is also found in thermally stable carbon resistors. Until recently the same was considered true for metals.[35] On 800-Å and 1400-Å Ag, Cu, Au, and Ni films, however, a strong, exponential activation type temperature variation over two orders of magnitude has been observed[28] between 100 and 500 K. A strong temperature dependence of this type was also observed for surface conductance in the presence of an inversion layer created by field effect in semiconductors.[36] I think both[28,36] arise [See VI (e), (f)] from a T-dependent scattering angle, due to changes in scattering mechanism.

(5) In semiconductors and semiconductor devices there is a strong dependence of $1/f$ noise on the state of the surface. Adsorbed gases or water may increase the noise from etched semiconductor surfaces by more than an order of magnitude. A thick oxide layer reduces the level of $1/f$ noise in general (passivation). In the case of MOST's, i.e., metal-oxide-silicon transistors, $1/f$ noise has been observed to be proportional to the concentration of surface states at the Fermi level, over a wide range of concentrations down to 10^9 cm^{-2}. This fact is usually interpreted in terms of McWhorter's model.[37] According to this model, the low-frequency fluctuations arise from transitions of electrons to and from "slow states" present in the oxide layer on the surface. Assuming a uniform distribution of these states throughout the oxide layer, and considering tunneling from the bulk, one obtains a distribution of lifetimes τ, proportional to $1/\tau$. Assuming all carriers in interaction with only one slow state at a time, a $1/f$ spectral density is obtained as superposition of independent exponential relaxation spectra between certain limits:

$$\int_{\tau_1}^{\tau_0} \frac{d\tau}{\tau} \frac{4\tau}{1+\omega^2\tau^2} = \frac{4}{\omega}(\arctan \omega\tau_0 - \arctan \omega\tau_1)$$

$$\approx \frac{2\pi}{\omega} = 1/f \quad (\omega\tau_1 \ll 1 \ll \omega\tau_0) . \quad (2.4)$$

However, there is $1/f$ noise even in the absence of the oxide layer. Therefore, there is fundamental $1/f$ noise on which accidental $1/f$-noise spectra may be superposed. Note, however, that a certain dependence on the concentration of surface states should also be expected on the basis of the fundamental fluctuation of surface-recombination and -scattering cross sections predicted by the quantum theory of $1/f$ noise for any electrically charged carriers.

III. ELEMENTARY SCHEME OF THE QUANTUM THEORY OF 1/f NOISE

The presence of electric $1/f$ noise in a large number of systems which contain a small number of carriers, including the most simple systems carrying a current, suggests a fundamental, universal and simple mechanism. This does not exclude the presence of accidental $1/f$ spectra superposed on the universal phenomenon.

The simplest system which carries a current I is a beam of free electrons, or other charged particles, moving uniformly in vacuum, in stationary conditions. Any beam originates in some process which can always be described as a form of scattering. Consider, for example, a beam of electrons which emerges from scattering on a fixed charge Ze, as in Rutherford scattering (Fig. 1). The beam is defined by the diaphragm D. The electric current carried by this beam can be measured, e.g., by catching the beam with an electrode leading to an amplifier, or directly to a galvanometer. We are tempted to say that in stationary conditions the current should be given by a constant plus small shot noise. The purpose of the subsequent analysis is to prove that, according to basic electrodynamics and quantum mechanics, the current carried by the beam should present $1/f$ noise in addition to the shot noise mentioned above.

It is well known that the beam of electrons will emit bremsstrahlung in the scattering process. The power spectrum $W(f)$ of the emitted radiation is independent of frequency (W = const) at low frequencies (low compared to the reciprocal duration of the scattering process of one particle) and decreases to zero at an upper frequency limit f_m which is approximated by E/h, where E is the kinetic energy of the electrons. This decrease is

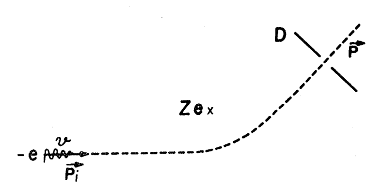

FIG. 1. Scattering on a fixed point charge Ze.

schematized in Fig. 2 by a step function. Consequently, the rate of photon emission per unit frequency interval is $N(f)=W/hf$, i.e., proportional to $1/f$. Here h is Planck's constant and $hf=\epsilon$ is the energy loss of some electron in the—otherwise elastic—scattering process. We conclude that the fraction of electrons scattered with energy loss ϵ is proportional to $1/\epsilon$, i.e., the relative squared matrix element for scattering with energy loss ϵ is $|b_T(\epsilon)|^2 \sim 1/\epsilon$.

If the incoming beam of electrons is described by a wave function $\exp[(i/\hbar)(\vec{p}_i\cdot\vec{r}-Et)]$, the scattered beam will contain a large nonbremsstrahlung part of amplitude a, and an (incoherent) mixture of waves of amplitude $ab_T(\epsilon)$ with bremsstrahlung energy loss ϵ ranging from some resolution threshold ϵ_0 to an upper limit $\Lambda \lesssim E$, of the order of the kinetic energy E of the electrons

$$\Psi_T=\exp[(i/\hbar)(\vec{p}\cdot\vec{r}-Et)]a\left(1+\int_{\epsilon_0}^{\Lambda} b_T(\epsilon)e^{i\epsilon t/\hbar}d\epsilon\right), \quad |t|<\tfrac{1}{2}T. \tag{3.1}$$

Here $b_T(\epsilon)\equiv|b_T(\epsilon)|e^{i\gamma_\epsilon}$ has a random phase γ_ϵ which implies incoherence of all bremsstrahlung parts. This incoherence may be related to the undetermined character of the time of the photon emission. The threshold ϵ_0 is given by the lowest frequency f_0 measured ($\epsilon_0=hf_0$). The subscript T indicates that Eq. (3.1) represents only a sample of duration $T>f_0^{-1}$ of the Schrödinger field of the scattered wave. Since we are dealing with a stationary process, the Fourier transform $b_T(\epsilon)$ can be defined

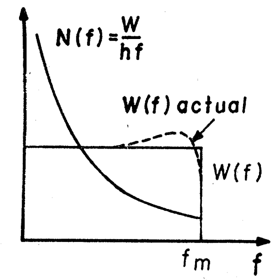

FIG. 2. Spectral density of bremsstrahlung in power W and photon emission rate N.

only for a finite duration sample, and $|b_T(\epsilon)|^2\sim T$ for large values of T. We shall give a more detailed formulation in a second paper, in terms of Fourier series rather than integrals.

In Eq. (3.1) the frequency-shifted components present in the integral interfere with the elastic term, yielding beats of frequency ϵ/h, i.e., with the same frequency as the emitted photons which have caused the slight bremsstrahlung energy loss ϵ initially. The amplitude of the electric current oscillations given by these beats at the frequency $f=\epsilon/h$ is linear both in the nonbremsstrahlung amplitude a and in $ab_T(\epsilon)$, and is therefore proportional to $\epsilon^{-1/2}$. This means that the power spectrum of the current fluctuations will be proportional to $|a^2 b_T(\epsilon)|^2$, i.e., to $1/\epsilon$, or $1/f$. We conclude that the electric current carried by the scattered beam of electrons exhibits fluctuations in time at any point, and that the power spectral density of these fluctuations is proportional to $1/f$. This derivation will now be formulated quantitatively.

IV. ELEMENTARY DERIVATION OF THE $1/f$ SPECTRUM

The particle density in the scattered Schrödinger field given by Eq. (3.1) is

$$|\Psi_T|^2=|a|^2\left(1+2\int_{\epsilon_0}^{\Lambda}|b_T(\epsilon)|\cos(\epsilon t/\hbar+\gamma_\epsilon)d\epsilon+\int_{\epsilon_0}^{\Lambda}\int_{\epsilon_0}^{\Lambda}b_T^*(\epsilon)b_T(\epsilon')e^{i(\epsilon'-\epsilon)t/\hbar}d\epsilon d\epsilon'\right). \tag{4.1}$$

The second term in large parentheses describes the particle density beats.

Before we continue the derivation of the fundamental $1/f$ noise spectrum, we average Eq. (4.1) over time and let $T\to\infty$:

$$\langle|\Psi|^2\rangle_t\equiv\lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}|\Psi|^2dt=|a|^2\left(1+\int_{\epsilon_0}^{\Lambda}\int_{\epsilon_0}^{\Lambda}\lim_{T\to\infty}\frac{b_T^*(\epsilon)b_T(\epsilon')}{T}\int_{-T/2}^{T/2}e^{i(\epsilon'-\epsilon)t/\hbar}\,dt\,d\epsilon\,d\epsilon'\right). \tag{4.2}$$

The time integral yields $2\pi\hbar\delta(\epsilon-\epsilon')$ for $T\to\infty$, and with the relative scattering rate density $|b(\epsilon)|^2$ with energy loss ϵ defined by

$$\lim_{T\to\infty}\frac{2\pi\hbar}{T}|b_T(\epsilon)|^2\equiv|b(\epsilon)|^2\sim 1/\epsilon, \tag{4.3}$$

where "relative" refers to the nonbremsstrahlung scattering rate, we obtain

$$\langle|\Psi|^2\rangle_t=|a|^2\left(1+\int_{\epsilon_0}^{\Lambda}|b(\epsilon)|^2d\epsilon\right). \tag{4.4}$$

Note that the random phases γ_ϵ drop out in the calculation of the time average. The latter coincides with the expectation value calculated for the statistical ensemble, i.e., obtained by averaging over the phases γ_ϵ present in $b_T(\epsilon)\equiv |b_T(\epsilon)|e^{i\gamma_\epsilon}$,

$$\langle|\Psi|^2\rangle = |a|^2\left(1+\int_{\epsilon_0}^{\Lambda}\int_{\epsilon_0}^{\Lambda}\langle b_T^*(\epsilon)b_T(\epsilon')\rangle e^{i(\epsilon-\epsilon')t/\hbar}d\epsilon\, d\epsilon'\right). \tag{4.5}$$

With

$$b_T(\epsilon)=(T/2\pi\hbar)^{1/2}b(\epsilon),$$
$$\langle e^{i(\gamma_\epsilon-\gamma_{\epsilon'})}\rangle=(2\pi\hbar/T)\delta_T(\epsilon-\epsilon'), \tag{4.6}$$

where δ_T becomes a δ function for $T\to\infty$, we obtain

$$\lim_{T\to\infty}\langle b_T^*(\epsilon)b_T(\epsilon')\rangle = |b(\epsilon)|^2\delta(\epsilon-\epsilon'). \tag{4.7}$$

Substitution into Eq. (4.5) yields again Eq. (4.4), without the subscript. We shall use this ensemble (phase) average from now on, in order to define expectation values. The expectation value of the current density $\vec{j}=(\hbar/2mi)(\Psi^*\nabla\Psi-\Psi\nabla\Psi^*)$, corresponding to Eq. (4.4), is obtained by multiplying Eqs. (4.2), (4.4), and (4.5) by $\vec{p}/m$:

$$\langle\vec{j}\rangle=\left(\frac{\vec{p}}{m}\right)|a|^2\left(1+\int_{\epsilon_0}^{\Lambda}|b(\epsilon)|^2d\epsilon\right). \tag{4.8}$$

Inside the second set of large parentheses, we distinguish the nonbremsstrahlung part (1) and the bremsstrahlung scattering part. The constant $|a|^2$ in front will contain three factors, proportional to the incoming current density, to the total (Rutherford) scattering cross section into the direction considered, and to the inverse of the expression in the second set of large parentheses, respectively. The latter factor is required for normation (unitarity). We also realize that the total scattering cross section will contain nonbremsstrahlung and bremsstrahlung components proportional to the terms in large parentheses in Eq. (4.8).

We continue the derivation of the $1/f$ spectrum now by computing the autocorrelation function for the probability density from Eq. (4.1),

$$\begin{aligned}\langle|\Psi|_t^2|\Psi|_{t+\tau}^2\rangle = |a|^4\Big(1&+\int\int\langle b_T^*(\epsilon)b_T(\epsilon')\exp[i(\epsilon'-\epsilon)t/\hbar+i\epsilon'\tau/\hbar]+b_T(\epsilon)b_T^*(\epsilon')\exp[i(\epsilon-\epsilon')t/\hbar-i\epsilon'\tau/\hbar]\rangle d\epsilon\, d\epsilon'\\ &+\int\int\langle b_T^*(\epsilon)b_T(\epsilon')\exp[i(\epsilon'-\epsilon)(t+\tau)/\hbar]+b_T(\epsilon)b_T^*(\epsilon')\exp[i(\epsilon-\epsilon')t/\hbar]\rangle d\epsilon\, d\epsilon'\\ &+\int\int\int\int\langle b_T(\epsilon)b_T^*(\epsilon')b_T^*(\epsilon'')b_T(\epsilon''')\rangle\exp[i(\epsilon-\epsilon'+\epsilon'''-\epsilon'')t/\hbar+i(\epsilon'''-\epsilon'')\tau/\hbar]d\epsilon\, d\epsilon' d\epsilon'' d\epsilon'''\Big).\end{aligned} \tag{4.9}$$

With the help of Eqs. (4.6) and (4.7) we obtain

$$\langle|\Psi|_t^2|\Psi|_{t+\tau}^2\rangle=|a|^4\left(1+2\int_{\epsilon_0}^{\Lambda}|b(\epsilon)|^2[1+\cos(\epsilon\tau/\hbar)]d\epsilon+\int_{\epsilon_0}^{\Lambda}\int_{\epsilon_0}^{\Lambda}|b(\epsilon)|^2|b(\epsilon')|^2[1+e^{i(\epsilon'-\epsilon)\tau/\hbar}]d\epsilon\, d\epsilon'\right). \tag{4.10}$$

If the particle concentration fluctuation is defined by $\delta|\Psi|^2=|\Psi|^2-\langle|\Psi|^2\rangle$, its autocorrelation function will be, according to Eq. (4.4),

$$\begin{aligned}\langle\delta|\Psi|_t^2\delta|\Psi|_{t+\tau}^2\rangle&=\langle|\Psi|_t^2|\Psi|_{t+\tau}^2\rangle-\langle|\Psi|^2\rangle^2\\ &=2|a|^4\int_{\epsilon_0}^{\Lambda}|b(\epsilon)|^2\cos(\epsilon\tau/\hbar)d\epsilon+|a|^4\int_{\epsilon_0}^{\Lambda}\int_{\epsilon_0}^{\Lambda}|b(\epsilon)|^2|b(\epsilon')|^2e^{i(\epsilon'-\epsilon)\tau/\hbar}d\epsilon\, d\epsilon'.\end{aligned} \tag{4.11}$$

For the autocorrelation function of fluctuations in the particle current density we obtain by multiplication with p^2/m^2

$$\begin{aligned}\langle\delta j(t)\delta j(t+\tau)\rangle&=2\left(\frac{p}{m}\right)^2|a|^4\left(\int_{\epsilon_0}^{\Lambda}|b(\epsilon)|^2\cos(\epsilon\tau/\hbar)d\epsilon+\int_{\epsilon_0}^{\Lambda}\int_{\epsilon_0}^{\Lambda}|b(\epsilon)|^2|b(\epsilon')|^2\cos(\epsilon-\epsilon')\tau/\hbar d\epsilon d\epsilon'\right)\\ &\approx 2\left(\frac{p}{m}\right)^2|a|^4\int_{\epsilon_0}^{\Lambda}|b(\epsilon)|^2\cos(\epsilon\tau/\hbar)d\epsilon,\end{aligned} \tag{4.12}$$

where the term in $b^4(\epsilon)$ has been neglected as a "noise of noise" term which will be shown to be very small. According to the Wiener-Khintchine theorem, the autocorrelation function is the Fourier transform of the spectral density of the fluctuations. Therefore, Eq. (4.12) shows that the spectral density of the probability current density fluctuations is

$$S_j(f) = 2h(p/m)^2 |a|^4 |b(\epsilon)|^2, \quad \epsilon_0 < \epsilon = hf < \Lambda. \qquad (4.13)$$

Since we know that the squared matrix element $|b(\epsilon)|^2$ is inversely proportional to ϵ, the spectrum obtained is a $1/f$ spectrum.

Equation (4.13) also expresses the fact that the fluctuations of the electric current density exhibit a $1/f$ spectrum. The $1/f$ spectrum will be present also in the total particle current carried by the scattered beam. *This means that in fact the scattering cross section fluctuates with the same $1/f$ spectrum.* This $1/f$ fluctuation is neither a thermal equilibrium process nor a nonequilibrium process, but a fundamental quantum-mechanical one-electron process such as, e.g., "scattering," or "recombination." Any cross section involving charged particles will exhibit this fundamental fluctuation, not only scattering cross sections included in the definition of the electrical resistance, but also, e.g., surface (or bulk) recombination cross sections for carriers in solids. Although it is a one-particle effect, the $1/f$ fluctuation of cross sections is statistical in character and is similar to the diffraction of matter waves: It takes many particles to observe the diffraction pattern or the $1/f$ spectrum.

V. BREMSSTRAHLUNG AND $1/f$ NOISE

The relative bremsstrahlung rate $|b(\epsilon)|^2$ can be derived from simple classical considerations. The power P radiated by an accelerated charge e is $P = (2e^2/3c^3)\dot{\vec{v}}^2$. The acceleration $\dot{\vec{v}} = \Delta\vec{v}\delta(t)$ has a Fourier transform $\dot{\vec{v}}_f = \Delta\vec{v}$, where $\Delta\vec{v}$ is the velocity change in the scattering process. Consequently, the one-sided spectral energy density $4e^2|\dot{\vec{v}}_f|^2/3c^3$ can be written $w(f) = 4e^2(\Delta\vec{v})^2/3c^3$, and does not depend on f. The relative scattering rate density with energy loss ϵ, $|b(\epsilon)|^2$, is obtained by dividing $w(f)$ by the energy of a photon $\epsilon = hf$:

$$|b(f)|^2 = \frac{4e^2(\Delta\vec{v})^2}{3c^3 2\pi\hbar f} = \frac{e^2}{\hbar c}\frac{2(\Delta\vec{\beta})^2}{3\pi} = \frac{\alpha A}{f}, \qquad (5.1)$$

$$|b(\epsilon)|^2 = \frac{4e^2(\Delta\vec{v})^2}{3c^3 h\epsilon} = \frac{\alpha A}{\epsilon}, \qquad (5.2)$$

where

$$A \equiv \frac{2(\Delta\vec{\beta})^2}{3\pi}, \quad \Delta\vec{\beta} = \frac{\Delta\vec{v}}{c}. \qquad (5.3)$$

Substitution of Eq. (5.2) or (5.1) into Eq. (4.13) yields

$$S_j(f) = 2(p/m)^2 |a|^4 \alpha A/f, \quad \epsilon_0/h < f < \Lambda/h. \qquad (5.4)$$

The spectral density of the relative fluctuations is, according to Eq. (4.8),

$$S_{|\Psi|^2}(f)\langle|\Psi|^2\rangle^{-2} \equiv S_j(f)\langle\vec{j}\rangle^{-2} \equiv S_\sigma(f)\langle\sigma\rangle^{-2} = 2[1+\alpha A\ \ln(\Lambda/\epsilon_0)]^{-2}\alpha A/f \approx 2\alpha A/f, \qquad (5.5)$$

the same for current density fluctuations, or cross sections σ, as it is for probability density fluctuations. The last form in Eq. (5.5) is a very good approximation, since αA is small: $\alpha A < 8\alpha/3\pi \approx (161)^{-1}$. This is also why the "noise of noise" is negligible in Eq. (4.12). If s were the transversal coherence area and S the cross section of the beam, we would expect the relative $1/f$ noise in the total current to be $N_1 = S/s$ times smaller than the result obtained in Eq. (5.5), since the variances of the independent contributions would be additive. Also, if l is the coherence length and $L = N_2 l$ a length along the beam over which we average the current fluctuations, the total number of carriers in the sample considered is $N = N_1N_2 = LS/ls$, since the incoming particles are Poisson distributed once they were described by a $\exp[i(\vec{p}\cdot\vec{r} - Et)/\hbar]$ incoming wave. Averaging over $L/l = N_2$ independent contributions introduces a factor N_2 into the denominator. We obtain this way

$$S_I(f) = 2\frac{\alpha A}{N}\frac{1}{f}\langle I\rangle^2. \qquad (5.6)$$

Notice that for $|\Delta\vec{\beta}|^2 = |\Delta\vec{v}/c|^2 = \frac{1}{6}$ one obtains $2\alpha A \approx 2\times10^{-3}$, i.e., the value of Hooge's constant.[30] In general, lower values of αA will be obtained. One will have to replace c by the speed of light in the medium considered if the current is not a beam in vacuum. This will lead to larger αA values in metals than in semiconductors. Furthermore, the average velocity change $\Delta\vec{\beta}$ in the expression of A will be much larger for lattice (mainly umklapp) scattering than impurity scattering.[30] $1/f$ noise in condensed matter will be treated in a later paper of this series.

In the derivation of Eqs. (5.1) and (5.2), we have identified the relative energy-loss spectrum of the electrons with the photon number spectrum. By including infrared radiative corrections, we shall see in the next paper that actually the relative energy loss spectrum $|\rho_\epsilon|^2\epsilon^{-1} = (\epsilon/\epsilon_0)^{\alpha A}|b_\epsilon|^2$ is slightly smaller than the photon number spectrum, due to the possibility of multiple photon emission. Although this difference is negligible for practical purposes due to the smallness of αA, it provides for a finite, physically meaningful, integral of the resulting $1/f$ noise spectrum. The reason the difference is small is that the probability of additional photon emission into the interval df is small and given by $\alpha A f^{-1} df$.

In the derivation of the $1/f$ spectrum, we have considered the outgoing beam of electrons as our system, described by the mixture in Eq. (3.1). The electromagnetic field was not included. The system thus defined is open, i.e., energy was lost during the scattering process to the electromagnetic field by bremsstrahlung. If the electromagnetic field is included in the system as part of the final state, the latter becomes again a pure state rather than a mixture, with the only time dependence expressed by the $e^{i(E/\hbar)t}$ factor present in the incoming wave already. However, in the case of any real $1/f$ noise measurement of the beam, the emitted bremsstrahlung has left the system, or has been absorbed by the shielding. Therefore, we have to consider the system open, even if the measurement proceeds during the time in which the beam is scattered.[38] This approach was also used in the previously published form of the theory,[17] in which creation and annihilation operators were used. Both formulations are equivalent. Philosophically, the openness of the system, or the interaction of the system with the rest of the world, can thus be considered the cause of $1/f$ noise. This view has also been expressed by Gagnepain and Uebersfeld.[10] Furthermore, an evaluation[19] of the vector potential term in the expression of the current density $\Psi^*\vec{A}\Psi$, where $\vec{A}$ is the vector potential of the emitted photons, shows that this term is negligible.

The small momentum losses $p=\epsilon/v$ of the electrons required by energy conservation in the emission process have been neglected in the present elementary derivation, but have been considered earlier.[17-19] They can be taken into account by replacing τ with $\theta \equiv \tau - x/v$ in Eqs. (4.11) and (4.12), where x is a displacement in space along the scattered beam. This variation in space is slow at the low frequencies characteristic to $1/f$ noise. At higher frequencies, all the way to $\Lambda \approx E/2$ the variation in space ($1/k$ distribution spectrum) becomes important[19] and is just what we expect from scattering cross-section fluctuations. Equations (4.13) and (5.4)–(5.6) can therefore be viewed also as wave-vector spectra. The presence of the momentum corrections $\Delta p = \epsilon/v$ in our expressions[19] does not mean that energy conservation holds in the system considered, i.e., that the system of the electron beam is not open. Although the system described is open, energy conservation in the global system was used in order to determine the small momentum losses present in the subsystem described. Note also that the photon absorption part present in a previous description[19] has to be dropped, as we are going to prove in a subsequent paper that the presence of a thermal equilibrium radiation background has no influence on the $1/f$ noise.

VI. DISCUSSION AND COMPARISON WITH THE EXPERIMENT

The elementary derivation of $1/f$ noise in an electron beam is in fact applicable to any beam of charged particles. Indeed, the spin, or any property of the electrons other than their charge, is irrelevant. From this point of view $1/f$ noise is a semiclassic effect. Its quantum character is determined by the necessity of a description of the beam in terms of de Broglie waves and by the presence of the fine-structure constant α.

The following three questions are raised by the derivation:

(1) How are the bremsstrahlung spectrum, the corresponding energy-loss spectrum of the current carriers, and the obtained current noise spectrum affected by the presence of the thermal radiation background?

(2) How can the long-range correlations between the particles, obtained in the present derivation, survive when the actual case of a beam of particles, with a finite coherence length, is considered?

(3) How is the current noise in a closed electric circuit containing a noisy element (semiconductor, vacuum tube, etc.) related to the noise present in a beam in vacuum?

The first question is natural if we think of the large number $N = kT/hf \gg 1$ of photons present in a field mode frequency f, and of the corresponding large induced bremsstrahlung and absorption (inverse bremsstrahlung) contributions. Indeed, the average number of photons emitted by an electron per unit frequency interval is now $(\alpha A/f)(1 + kT/\hbar f) \approx \alpha A kT/\hbar f^2 =$ number of photons absorbed. One is led to expect therefore a $1/f^2$ noise proportional to the radiation temperature T, which may eventually be set equal to 3 K at sufficiently low frequencies, independent of laboratory conditions. This T/f^2 guess is wrong, however, since this time the energy-loss distribution of the carriers turns out to be quite different from the photon number spectrum: The energy-loss distribution is constant[39] (uniform) for $-kT < \epsilon < kT$ rather than $\sim T/f^2$. To yield this result, the theory of infrared radiative corrections was extended to the case in which a thermal radiation background is present and will be presented in a subsequent paper. On the basis of the uniform energy-loss distribution at $T \neq 0$ one is tempted to expect white noise rather than $1/f$. However, the exact solution of the infrared problem, in terms of a convolution, proves the statistical independence of spontaneous and stimulated processes, allows to factorize the Schrödinger field of the outgoing particles with respect to their energy losses, and finally, the same $1/f$ noise as for $T = 0$ emerges,[39] as will be shown in a subsequent paper. The observed $1/f$ noise will be the same as the noise one would expect if the induced energy gains and losses would compensate each other exactly for each carrier. The compensation is real and can be viewed as a consequence of the fact that the number of induced photon transitions per scattered particle is large and the relative number fluctuations are small.

The second question is based on the fact that any real beam of particles is described by a mixture of wave packets of finite length in space and time (coherence length and time). If a mixture of wave packets incident with a Poisson distribution is scattered, all plane waves which form the packet will split into a nonbremsstrahlung component and a small bremsstrahlung part. When $|\Psi|^2$ and $\vec{j}$ are calculated, the $1/f$ part factors out,[40] and the arbitrary phases of the bremsstrahlung parts drop out, as we have seen in Sec. IV. The Poisson distribution of the incoming wave packets corresponds to a constant probability density in space and time for the centers of incoming wave packets. The factored $1/f$ part yields $1/f$ noise in the same way as before. The essential fact is the presence of the same type of component plane waves in all wave packets, i.e., with the same relative phase shifts γ_ϵ of the bremsstrahlung part. In the same time, the plane waves used in different wave packets are completely incoherent, even for the same wave vector, but the phases drop out of the noise calculation, and are irrelevant. This calculation is planned to be presented in a subsequent paper, along with a density-matrix formulation of the theory.

The third question is related to the second. Indeed, the current through a bad contact, or noisy sample, can be conceived as a sequence of incoherent wave packets. The new elements are the finite time each wave packet is in stationary translational motion, and the fact that repeated measurements may be performed on the same quantum object. The first new element can be treated by using the concept of coherent waves,[41] describing the average motion of the carriers in the multiple scattering process. This concept is related to the definition of the index of refraction.[42] The second element requires a treatment which takes into account the evolution of the wave function between two consecutive measurements on the same system. This treatment has been given[43] so far only in the (equivalent) formulation in terms of photon annihilation and creation operators, but will be presented in a different form in another paper (second-order $1/f$ noise[43]).

An important experimental characteristic of $1/f$ noise, the Gaussian amplitude distribution of $1/f$ noise, has recently been derived[44] without the use of the creation and annihilation operators present in an earlier derivation.[45] $1/f$ noise is similar to diffraction.[46] The most important experimental verifications other than spectrum and amplitude distribution are as follows:

(a) The presence of $1/f$ noise both in the (surface[16]- or volume[16]-) recombination speed and in the resistance. The theory predicts $1/f$ noise in both the recombination and scattering cross sections, whether located on the surface, or in the volume, independent of the scattering agent (phonons or impurities). It also predicts $1/f$ noise in the ionic scattering cross section in electrolytes. In general, the $1/f$ noise will be in the current-determining cross section, i.e., in the kinetic coefficient limiting the current in the given circuit.

(b) The theory predicts a fundamental fluctuation of scattering cross sections, i.e., of the collision time (frequency) and therefore of the mobility. Experimentally, it is also apparent that the mobility fluctuations cause the $1/f$ resistance fluctuations.

(c) The fundamental scattering cross section, mobility, and resistance $1/f$ fluctuations are neither an equilibrium process nor a nonequilibrium process, but a fundamental one—such as, e.g., scattering. They will be present both if nonequilibrium currents or thermal equilibrium currents are used to test their existence. Therefore, this theory predicts $1/f$ fluctuations of the level of Nyquist noise of the same relative magnitude as observed for dc $1/f$ noise, with no "available power" level fluctuations. This is also the experimental fact[22-25] as was mentioned in Sec. II. This equality of relative Nyquist level fluctuations and dc $1/f$ noise *distinguishes* $1/f$ noise from the bolometer effect always present in small samples, with a steeper spectrum which flattens out at sufficiently low frequency. Indeed, in the bolometer effect the relative Nyquist level fluctuations *exceed* the relative dc current noise due to the available power fluctuations.

(d) On the basis of the quantum theory of $1/f$ noise, by simply associating the fluctuations with the kinetic coefficient determining entropy generation in the oscillant system, we can derive the empirical Q^{-4} law of Gagnepain and Uebersfeld,[10] which relates the flicker of frequency of quartz oscillators to their total quality factor Q. This derivation will be reported in detail elsewhere.[47]

(e) Owing to the proportionality [Eq. (5.3)] with an average value of $\alpha A \equiv (\alpha/3\pi)4\beta^2 \sin^2\frac{1}{2}\theta$, where θ is the scattering angle, the theory predicts a very low $1/f$ noise level for impurity scattering compared to lattice scattering. Indeed, this resistivity arises from many small-angle scattering processes with large impact parameters in the Coulomb field of the scattering impurity ions, while phonons cause large-angle scattering, particularly in umklapp processes. Scattering from neutral impurities or lattice defects should yield intermediate $1/f$ noise, closer to the case of lattice scattering. If, therefore, the fluctuations in the mobility μ_i defined by impurity scattering are neglected in the approximate relation $\mu^{-1} = \mu_i^{-1} + \mu_{1att}^{-1}$, one obtains $\langle(\delta\mu)^2\rangle_f = (\mu/\mu_{1att})^2\langle(\delta\mu_{1att})^2\rangle_f$ in accord with recent experiments.[30] The usual mechanism at room temperature is lattice scattering in metals and semiconductors. Finally, the theory is consistent with large αA, and large $1/f$ noise when hopping is the conduction mechanism, due to the large velocity changes $\Delta\vec{\beta}$.

(f) Although the predicted $1/f$ noise for a given scattering mechanism does not show substantial temperature dependence, a strong dependence is expected in the temperature region in which a transition to a different type of scattering occurs. This qualitatively explains the temperature dependence observed in thin films[28] and in inversion layers[36]; a similar temperature dependence should be present in manganin, and may help reconcile apparently contradictory measurements.

(g) It is interesting to note that the fundamental $1/f$ fluctuation of the elementary cross sections also entails $1/f$ fluctuation in the concentration of particles which have been scattered (or recombined, etc.). This can be seen in Eqs. (4.11) and (5.5). In general, the cross-section fluctuations will carry over in the kinetic coefficients (mobility, surface and bulk recombination speed, etc.), while the concentration fluctuations in the scattered waves of all scattering centers will yield a noticeable resultant concentration fluctuation only if relatively few, noise-coherent, scatterers are present and if the concentration is not subject to fast relaxation (electrostatic in metals, recombinational in semiconductors, etc.). The fundamental presence of concentration fluctuations along with the cross-section fluctuations, as seen in Eqs. (4.11) and (5.5), allows us to understand the ambiguous result of some experiments trying to distinquish mobility fluctuations from number fluctuations.

The quantum approach presented in this paper reduces fundamental $1/f$ noise to an infrared phenomenon with contributions from all massless quanta, or excitations, which couple to the current carriers. If $(\alpha A)_i$ are the coupling constants, $\alpha A = \sum_i (\alpha A)_i$ is the total $1/f$ noise coefficient. Thus, for electron-hole excitations at the Fermi surface of metals,[48-53] $(\alpha A)_{e-h} = |M|^2 N_0^2$ where N_0 is the density of s-wave states at the Fermi level and M is the S-wave matrix element of the (screened) Coulomb potential of the entering (exiting) current carrier. Physically, every entering or exiting carrier will excite an infinite number of arbitrarily weak electron-hole excitations. We can express the Anderson exponent in terms of the S-wave phase shift δ at the Fermi energy: $(\alpha A)_{e-h} = 2\delta^2/\pi^2$. A calculated value for lithium[49] is $\delta = 0.954$, which would yield a $1/f$ noise almost two orders of magnitude too large. A large reduction of this effect is to be expected due to the gradual character of the actual entrance and exit process in which an image charge is induced before the carrier reaches the surface. Finally, electron-hole excitations may be generated mainly from scattering, in which the sudden change in the Fermi-liquid backflow field around the carrier excites the electron-hole pairs, i.e., the infrared-divergent response.

The anisotropy and lack of $\vec{k}$-space inversion-center symmetry present in indirect gap semiconductors generates a nonvanishing coupling of the carriers to transversal phonons. This coupling, piezoelectric coupling in general, leads to matrix elements for scattering with energy loss ϵ to photons proportional to $\epsilon^{-1/2}$, just as photons, and electron-hole excitation coupling, do. We obtain a $1/f$ contribution $(\alpha A)_{ph}$ in a similar way. In cases other than the photons (nonelectromagnetic electric $1/f$ noise) we expect a lower frequency limit of the $1/f$ behavior at the lowest excitation frequency. For electron-hole pairs and for phonons this is higher than 1 Hz. However, for some "hydrodynamic" modes and for quasidegenerated, or correlated (rearrangement) states[54-57] lower frequencies cannot be excluded. Experimentally, one would see the $1/f$ noise fall to a lower level as one goes down in frequency, every time the lowest frequency of a certain group of excitations is left behind. Such structure is not uncommon in experimental $1/f$ plots.[58]

Only the electromagnetic and gravitonic parts are known to be present in the limit $f\to 0$. Screening is very inefficient in eliminating the very low frequency electromagnetic modes, due to the $f^{-1/2}$ dependence of the skin depth. Only by enclosing the whole experiment in a superconducting cavity would the low end of the electromagnetic spectrum be affected.

The essence of the theory presented here is the interpretation of $1/f$ noise as an infrared phenomenon arising form all "environment-forming" quanta coupled to the current. These are the quanta which yield infrared- divergent response in the time domain. In the case of photons and gravitons, the environment is the gauge-invariant field of the universe (including the metric) and therefore the lower frequency limit is zero. In the case of phonons, the lattice geometry is at stake. Thus, $1/f$ noise appears as the consequence of the long-range interaction with the rest of the world, i.e., as a contribution to shaping it. While the infrared divergences are very familiar, the "bridge" to $1/f$ noise, i.e., the self-interference effect (first-order $1/f$ noise[43]) is new, and may transform $1/f$ noise into a means to study infraquanta. This same bridge can be presented in various different forms, by using e.g., the Kubo-Greenwood formula.[59-60] Another, slightly different, bridge is second order $1/f$ noise,[43] to be discussed elsewhere. There may be more bridges leading to electric $1/f$ noise.

In essence, the infrared divergence is a consequence of the nonlinear coupling of field modes due to the presence of matter. Various forms of nonlinear coupling, i.e., generalized turbulence,[61-67] are the more general cause of $1/f$ noise, e.g., in the flow of cars on expressways.[66]

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