Electron momentum-transfer cross section in cesium: Fit to the experimental data

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Experiments related to the electron–cesium-atom momentum-transfer cross section $Q(v)$ in the range of energies $\epsilon = mv^2/2$ from 0.05 to 2 eV are considered in which the following quantities are measured: (a) width of the electron cyclotron resonance, (b) attenuation of microwaves, (c) electrical conductivity in both equilibrium and nonequilibrium pure Cs or Ar − Cs plasmas, (d) electron thermal conductivity, (e) perpendicular electrical conductivity in a strong magnetic field, and (f) electron drift velocity. A convenient algorithm is proposed to vary $Q(v)$ until the best fit to all data is obtained. As a result an almost unique curve $Q(v)$ is found so that the rms deviation of the experimental values from the calculated ones is 20%. The recommended dependence $Q(v)$ can be used for calculations in the range of electron temperatures (or equivalent mean energies) from 500 to 2500 K within an error less than 20%.

I. INTRODUCTION

Reliable data on the electron-atom (or molecule) momentum-transfer cross section $Q(v)$ for low energies $\epsilon = mv^2/2$ from 0.01–10 eV have been obtained mostly from "swarm" measurements.[1,2] For this purpose, the well-known technique of Frost and Phelps[3,4] has been used to analyze measurements of electron-transport coefficients. Postma[5] and Nighan and Postma[6] made a successful effort to take into account some of the available experimental works and to find $Q(v)$ in cesium. They used a trial-and-error method and did not propose straightforward algorithm to optimize $Q(v)$. Therefore, the uniqueness of their results is questionable. The purpose of this work is to summarize all the available experimental measurements of different quantities depending on $Q(v)$ in cesium, to propose and analyze an algorithm for determination of $Q(v)$, and to find out the best curve $Q(v)$ consistent with the experimental data in the range 0.05–2 eV.

II. FORMULATION AND DISCUSSION OF THE PROBLEM

The problem of determining $Q(v)$ using measured values of some transport coefficient k at different electron temperatures T_e in some extent is equivalent to solving a Fredholm equation of the first kind:

$$k(T_e) = \int_0^\infty K(T_e, v)Q(v)dv \qquad (1)$$

(or instead of Q, we may need to take Q^{-1} as can be seen further), where the kernel K is specific for a given transport coefficient and depends on the electron distribution function f_e, which in most cases considered below, is Maxwellian.

One must clearly recognize that any procedure of solving Eq. (1) is ill posed.[7] Even if the exact analytical form of $k(T_e)$ is known, small changes in $k(T_e)$ can lead to rapid changes in $Q(v)$ (instability of the solution). The situation is worse when $k(T_e)$ contains an experimental error which may depend on T_e. In principle, the solution of Eq. (1) is not unique—an infinite set of functions could satisfy it within the limits of a prescribed accuracy. For example, an addition of a quickly oscillating function to a solution $Q(v)$ will also be a solution.

To convert this ill-posed problem to a well-posed one in the sense of Tikhonov,[7] one needs supplementary information, sometimes rather subjective, to narrow the class of possible solutions. In the particular case here, a curve $Q(v)$ will be looked for, which does not belong to the class of all functions but to the limited class of functions determined by cubic splines[8] with few knots[9] so that $Q(v)$ must be smooth enough and must have as few minima and maxima as possible. To avoid the uncertainty of the problem arising from the experimental error in $k(T_e)$, one has to take into account as many experimental measurements (preferably of different transport properties, i.e., with different kernels K) as possible but, at the same time, only accurate enough measurements have to be accepted.

The method proposed here to determine $Q(v)$ from measured values of transport coefficients can be outlined as follows. Sets of different k in some range of T_e will be considered with known kernels $K(T_e, v)$. For a given representation of $\log_{10}Q(v)$ (using cubic splines with knots prescribed by us), characterized by several constants C_j, the functional F will be formed:

$$F = \Sigma R_i^2, \quad R_i = \ln\left(k(T_e)/\int_0^\infty K(T_e, v)Q(v)dv\right) = \ln(k_{\text{expt}}/k_{\text{calc}}), \qquad (2)$$

TABLE I. Characteristics of the experimental work used to find $Q(v)$.

No.	References	Number of experimental points considered	Overall weight of the points	f_e is Maxwellian	Coulomb interactions	Eq. (2) used to find T_e	T_e range (K)	N_{Ar}/N_{Cs}	Equation used to integrate $Q(v)$	Parts of $Q(v)$ related to the measured quantity
1	10	4	8	yes	no	no	600–1200	0	(3)	min
2	11	3	8	yes	no	no	450–550	0	(5)	max
3	12, 17	6	6	yes	yes	no	1240–1700	0	(6)	min
4	13	9	9	yes	yes	no	1600–2000	10^2–10^4	(6)	min and max
5	14	4	4	yes	yes	yes	2100–2400	10^2–10^4	(6, 9)	all (weak dependence)
6	15	7	7	yes	yes	yes	1700–2300	0	(11), (6), and (9)	all (strong dependence)
7	16	1	4	yes	yes	no	1300	10^2	(6)	min
8	17	12	12	yes	small	no	1000–1300	0	(5)	max
9	18	4	8	no	no	no	1400–2400 [a]	0	(12)	all (strong dependence)

[a] As f_e is not Maxwellian, the values of T_e are calculated from the mean logarithmic slope of f_e (obtained using curve *A* of Fig. 1) in the range 0.1–1 eV.

where the index i refers to the number of the experimental point. Then the minimum of $F(C_j)$ regarding C_j as independent variables has to be found. The metric in Eq. (2) is chosen so that relative values of k are considered. At last when a curve $Q(v)$ which minimizes F is found, its uniqueness and sensitivity to the choice of a particular set of experimental points in Eq. (2) will be analyzed.

III. EXPERIMENTAL DATA USED TO FIND $Q(v)$

In Table I, a short description of all of the experimental work[10–18] used is given.

No. 1. Electron-cyclotron resonance measurements of Meyerand and Flavin.[10] The half-width ν_h of the plasma absorption line has been determined in Cs vapor with $n/N < 10^{-6}$ (n and N are the densities of electrons and neutral atoms) so that the Coulomb collisions can be neglected. Using Eq. (1) of Meyerand and Flavin and comparing the propagation coefficients β at the maximum of the absorption line (β_m) and at the half-width (β_h) as $2\beta_h = \beta_m$, one obtains

$$2\int_0^\infty \frac{z^3 e^{-z^2} dz}{Q(v)[1+(\nu_h/\nu)^2]} = \int_0^\infty \frac{z^3 e^{-z^2} dz}{Q(v)},$$

where $z = (mv^2/2k_BT_e)^{1/2}$ and $\nu = vNQ(v)$. Meyerand and Flavin took into account the dependence $\nu(v)$ so that their procedure to find

$$\langle Q\rangle^\sigma = \left(2\int_0^\infty \frac{z^3 e^{-z^2} dz}{Q(v)}\right)^{-1} \tag{3}$$

as function of T_e was correct.[19] Therefore one can use Eq. (3) to calculate R_i in Eq. (2):

$$R_i = \ln(\langle Q\rangle^\sigma_{\text{calc}}/\langle Q\rangle^\sigma_{\text{expt}}) . \tag{4}$$

No. 2. Microwave interferometry measurements of Chen and Raether.[11] The high-frequency complex electrical conductivity in an afterglow of a Cs discharge has been measured and

$$\langle Q\rangle^\nu = \int_0^\infty z^5 e^{-z^2} Q(v) dz \tag{5}$$

has been found.[20] Here R_i are determined by Eq. (4), where indices σ are changed to ν.

No. 3. Electrical conductivity measurements in pure Cs vapor.[12,17] The mixture rule of Frost[21] was used to include the contribution of the Coulomb interactions when calculating the electrical conductivity

$$\sigma = \frac{8ne^2}{3\pi^{1/2}m}\int_0^\infty \frac{z^4 e^{-z^2} dz}{\nu(v) + a\nu_c(v)}, \tag{6}$$

where $a = 0.476$ and $\nu_c(v) = (8\pi n/m^2v^2)(e^2/4\pi\epsilon_0)^2 \times (m/2k_BT_e)^{1/2}(\ln\Lambda - 0.52)$, $\ln\Lambda$ being the Coulomb logarithm. The deviations are

$$R_i = \ln(\sigma_{expt}/\sigma_{calc}) . \quad (7)$$

The data of Bohn *et al.*[12] only for $T \geq 1300$ K were considered because low-temperature measurements are not reliable due to electrode-plasma phenomena.[17] For the same reason, only one experimental point of Cox *et al.*,[17] i.e., that with the largest N for $B = 0$, was considered.

No. 4. Electrical conductivity measurements of Harris[13] in mixtures Ar + Cs. Again Eq. (6) was used with $\nu = \nu_{eCs} + \nu_{eAr}$, $Q_{eAr}(v)$ taken from Milloy *et al.*,[22] and Eq. (7) determining R_i. Three sets of experimental points were considered with $p_{Cs} = 0.1$, 1, and 10 Torr ($p_{Ar} = 760$ Torr). It should be pointed out that at $p_{Cs} = 0.1$ Torr, σ is affected by the maxima in $Q(v)$ while at 1 and 10 Torr, σ is determined mostly by the minimum of $Q(v)$.

No. 5. Electrical conductivity measurements of Bernard et al.[14] in Ar + Cs *plasma with* $T_e \neq T_a$. The procedure was the same as in No. 4 with T_e calculated by means of the electron energy equation

$$\frac{j^2}{\sigma} - 3mnk_B\left(\frac{\bar{\nu}_{eAr}}{M_{Ar}} + \frac{\bar{\nu}_{eCs} + \bar{\nu}_{ei}}{M_{Cs}}\right)(T_e - T_a) - R = 0 , \quad (8)$$

where j is the current density and M_{Ar} and M_{Cs} are the atom masses. The radiation losses R are few times less than the second term in Eq. (8). They are evaluated according to Eq. (18) of Ref. 23. The mean electron-atom collision frequencies are given by

$$\bar{\nu} = \frac{8N}{3}\left(\frac{2k_BT_e}{\pi m}\right)^{1/2} \int_0^\infty z^5 e^{-z^2} Q(v) dz , \quad (9)$$

and the electron-ion collision frequency is

$$\bar{\nu}_{ei} = \tfrac{4}{3}(2\pi/m)^{1/2} n (k_BT_e)^{-3/2} \times (e^2/4\pi\epsilon_0)^2 (\ln\Lambda - 1.37) . \quad (10)$$

Only data with $j = 1$ A/cm^2 were taken into account, but even they do not depend very much on $Q(v)$ because of the relatively high degree of ionization. The data with $j = 10$ A/cm^2 correspond to a fully ionized plasma and were used to confirm the accuracy of calculating T_e from Eq. (8), the coincidence between σ_{expt} and σ_{calc} being within 12%.

No. 6. Electron thermal conductivity measurements of Stefanov[15] with $T_e \neq T_a$. In the procedure of processing the experimental data, the main contribution of $Q(v)$ in λ_e is given by

$$\langle Q \rangle^\lambda = \left(\frac{8}{9}\int_0^\infty \frac{z^3(z^2 - 2.5)^2 e^{-z^2} dz}{Q(v)}\right)^{-1} , \quad (11)$$

with the Coulomb interactions included in the denominator as in Eq. (6), but with different value of a.[24] Since Eq. (8) is used to interpret the experimental data, the integrals appearing in Eqs. (6) and (9) are also important. Only data with $p_{Cs} > 4$ Torr were used to avoid, as much as possible, errors connected with a diffusion of charged particles and with plasma currents. The deviations were calculated as $R_i = \ln(\lambda_{expt}/\lambda_{calc})$.

No. 7. Electrical conductivity measurements of Beynon and Brooker[16] where $p_{Cs} = 6$ Torr *and* $p_{Ar} = 760$ Torr *are similar to No. 4.* The primary result on σ is not given in the paper so that it was calculated starting with the proposed cross section and going back through the formulas used there. The result was $\sigma = 6.31 \times 10^{-2}\ \Omega^{-1}\,\text{m}^{-1}$.

No. 8. Electrical conductivity measurements of Cox et al.[17] in a strong magnetic field seem to be among the most reliable ones. For $\omega \gg \nu$ ($\omega = eB/m$ being the electron-cyclotron frequency) the perpendicular component of the conductivity $\sigma_\perp$ depends on the mean cross section defined by Eq. (5). The Coulomb interactions were taken into account according to the extended Frost rule.[25] Twelve experimental points in the linear parts of the logarithmic dependences $\sigma_\perp(B)$ (see Figs. 6 and 7 of Cox *et al.*) for $N = 6.1 \times 10^{13}$ to 1.1×10^{15} cm^{-3} were chosen. Because $\sigma_\perp$ is proportional to Q, the deviations were defined as $R_i = \ln(\sigma_{calc}/\sigma_{expt})$.

No. 9. Drift velocity measurements of Chanin and Steen.[18] Here the procedure of Postma[5] was used in the calculations. First the electron distribution function f_e had to be found for a given $Q(v)$ using Eq. (8) of Postma together with the data for the excitation cross section.[26] Then the drift velocity was calculated:

$$v_D = \frac{8k_BT_eeE}{3m^2N}\int_0^\infty \frac{z^2}{Q(v)}\frac{\partial f_e}{\partial z} dz \quad (12)$$

and $R_i = \ln\{v_{D\,expt}/v_{D\,calc}\}$. The value of $v_{D\,calc}$ is sensitive not only to the minimum of $Q(v)$, but through f_e to all parts of $Q(v)$.

In the calculations of Nos. 3–8, the electron densities were found from the Saha equation. A short analysis[15] confirms the presence of a Saha equilibrium determined by T_e.

Experimental data not taken into account.[27–34] In the following discussion in this section, the deviations R_i of the various experiments are calculated using the $Q(v)$ given by curve A of Fig. 1 (see Sec. IV). These deviations are almost the same for any $Q(v)$ which is in accordance with experiments Nos. 1–9.

The measurements of Ingraham[27] of ν_h were found to be factor of 2.5 to 3 low compared with the computed values based on experiments Nos. 1–9. When these data were included in the minimization of F in Eq. (2), a strong stratification of all R_i values was observed, the data of Ingraham

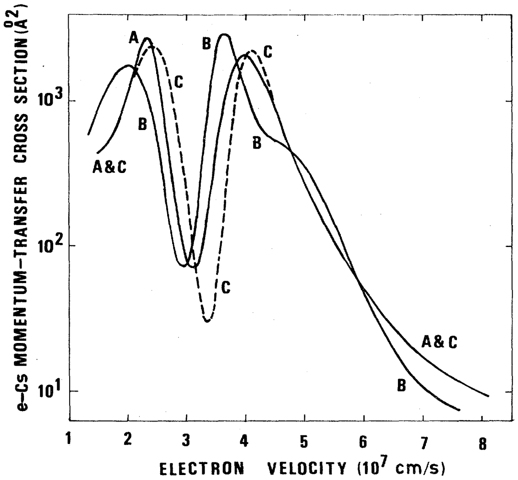

FIG. 1. Electron–cesium-atom momentum-transfer cross section obtained from the experimental values of different transport coefficients. Internal knots (values of the electron velocity in 10^7 cm/s for the points of smooth transition from one to another cubic polynomial) as follows: Curve A (best solution): 2, 2.4, 2.8, 3.2, 3.6, 4.5, and 6; another curve practically coinciding with curve A: 1.5, 2, 2.5, 3, 3.75, 4.5, 6; curve B: 2, 2.5, 3, 3.5, 4, 4.75, and 6.25; Curve C: 1.8, 2.6, 3, 3.4, 3.8, 4.5, and 6 (curve C is obtained with the electrical conductivity data of Mirlin *et al.*, which are 2 to 3 times higher than the data of experiment No. 3 accepted for the other curves).

opposing all the data of Nos. 1–9.

The microwave data of Balfour *et al.*[28,29] do not seem to be accurate enough. The slopes of the dependences of $\bar{\nu}$ on the pressure which determine $\langle Q\rangle^{\nu}$ are uncertain within a factor of 2 due to the scattering of the measured points. Nevertheless, the reported values of $\langle Q\rangle^{\nu}_{\rm expt}$ at 1600 K do not deviate substantially from $\langle Q\rangle^{\nu}_{\rm calc}$ based on experiments Nos. 1–9: $\langle Q\rangle^{\nu}_{\rm expt}/\langle Q\rangle^{\nu}_{\rm calc}=0.93$ and 0.61 for the two experimental tubes,[28] while at 680 K (Ref. 29) this ratio is 0.44.

The electrical conductivity measured by Roehling[30] is 4 to 9 times higher, and that measured by Mirlin *et al.*[31] is 2 times (at 1700 K) to 3 times (at 1300 K) higher than the data of Bohn *et al.*[12] The discrepancy arises mainly from the high probing current density of the order of 10^{-1} A/cm^2 which causes an elevation of T_e over T_a and increases σ. This can be clearly seen in Fig. 5 of Bohn[35] or Fig. 2 of Bohn *et al.*[12] The plasma equilibrium is affected when $j>10^{-1}$ A/cm^2 at 1700 K or $j>4\times10^{-4}$ A/cm^2 at 1300 K for $p=4$ Torr. For lower pressures, this critical current density is even smaller. It should be pointed out that in experiments Nos. 3–5 and 7 and 8, either the probing current has been well below the critical value or the dependences $\sigma(j)$ have been measured. Mullaney *et al.*[32] did not give the pressure so that $\sigma_{\rm expt}$ cannot be compared with $\sigma_{\rm calc}$; the measurements of $\sigma_\perp$ seem to be misinterpreted because the proposed cross section is at least one order of magnitude less than that derived from experiments Nos. 1–9.

The measurements of the drift velocities in cesium discharges[33,34] were not considered here because of the difficulties arising when solving the Boltzmann equation with combined electron-atom and Coulomb interactions.[36] Postma[37] and Nighan and Postma[6] did not include the electron-electron interactions in all parts of their analysis[38] and this is probably the reason why $Q(v)$ proposed by them has a maximum near 2 eV. This result is in contrast with that of Postma[5] who found $Q(v)$ having no maximum in this region from the measurements of Chanin and Steen[13] where the Coulomb interactions are negligible. As is shown by Nighan and Postma[6] and Postma,[37] the part of their data[33,34] with a low degree of ionization is in good agreement with the data of Chanin and Steen.

IV. RESULTS AND DISCUSSION

The procedure of determining $Q(v)$ was outlined in Sec. II. We minimize F in Eq. (2) using different numbers of knots N ranging from zero (i.e., one cubic polynomial over the whole interval of integration $v=0.375$ to 9.75×10^7 cm/s) to eight. The number of the coefficients in the B-spline representation,[8] which are independent variables (C_j), is $K=N+4$. However, in the case of large K, the value of F does not depend on C_1, C_{K-1}, and C_K because they prescribe $Q(v)$ near the ends of the integration interval where $K(T_e, v)$ in Eq. (1) is small. Therefore, for large N, these coefficients were fixed (and then $K=N+1$), i.e., fixed values of $Q(v)$ at $v=0.375$ and near 9.75×10^7 cm/s were used. The calculations were performed on the computer B7700 at the Eindhoven University of Technology. Over 100 solutions were found for different choices of knots, number of variables, fixed end values of $Q(v)$ and starting points (initial values of all C_j). Simple functions such as $Q=$ const or monotonically decreasing $Q(v)$ failed to fit the experimental data as was also noted by Nighan and Postma.[6] As the number of knots was increased up to $N=6$ or 7, the resulting minimum value of $F=\Sigma R_i^2$ decreased. For $N=8$, $Q(v)$ became unstable and no improvement was observed. In most cases the solution $Q(v)$ did not depend on the initial set of C_j. In the very few cases when two local minima of F were found, one of them had substantially lower value and was believed to

TABLE II. Recommended values of the electron–cesium-atom momentum transfer cross section (curve A in Fig. 1). The velocity may be converted to energy units by means of the relation $\epsilon(\text{eV}) = [v(10^7\ \text{cm/s})/5.93]^2$.

$v(10^7$ cm/s)	$Q(v)$ (Å^2)	$v(10^7$ cm/s)	$Q(v)$ (Å^2)
1.25	448	3.75	1509
1.5	448	3.875	1922
1.75	576	4.0	2021
1.875	750	4.125	1827
2.0	1096	4.25	1479
2.125	1754	4.375	1118
2.25	2550	4.5	882
2.375	2677	4.625	608
2.5	1684	4.75	455
2.625	725	5.0	265
2.75	281	5.5	104
2.875	128	6.0	49
3.0	80	6.5	27
3.125	72	7.0	17
3.25	100	7.5	12
3.375	199	8.0	10
3.5	454	8.5	8
3.625	938	9.0	7

be an absolute minimum.

Some of the results are shown in Fig. 1. Curve A represents the best solution with $F = 2.84$ and and is given also in Table II. Curve B with different knots represents another solution with the same value of F, but it has a more complicated form. A third solution with another choice of knots was obtained. It was almost the same as curve A and therefore is not shown in Fig. 1. The deviations R_i for all the three solutions $Q(v)$ almost coincide and are plotted against T_e in Fig. 2. The numbers on the lines (or points) correspond to the numbers of the experiments considered in Sec. III. It can be seen that the main inconsistency of all the data comes from the fact that experiments Nos. 1 and 3 are opposing Nos. 4b, 4c, and 7, all of them being related to the minimum of $Q(v)$. The deviations were analyzed by

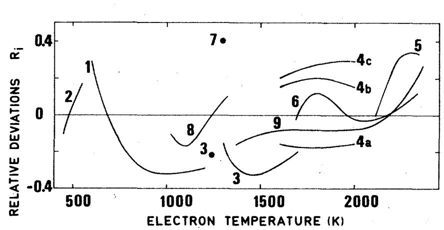

FIG. 2. Relative deviations $R_i = \ln(k_{\text{exp}}/k_{\text{calc}})$ or $\ln(\langle Q\rangle_{\text{calc}}/\langle Q\rangle_{\text{exp}})$ of different experiments listed in Table I. Curve A from Fig. 1 is used in the calculations. Three curves representing experiment No. 4 correspond to different cesium pressures: (a) 0.1 Torr, (b) 1 Torr, (c) 10 Torr.

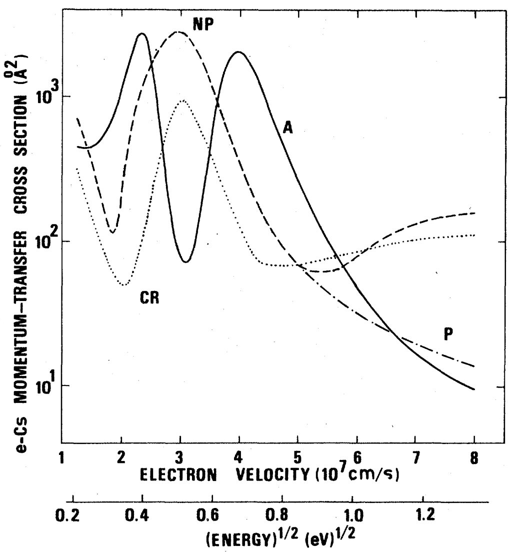

FIG. 3. Comparison of curve A with Nighan and Postma (Ref. 6) (NP), Postma (Ref. 5) (P), and Crown and Russek (Ref. 39) (CR), results. Note that curve NP is chosen from the class of functions similar to curve CR. The calculated curve CR shown here corresponds to polarizability $\alpha_\infty = 51.3$ Å^3; for the set $\alpha_\infty = 0.0$, 51.3, and 60.3 Å^3 Crown and Russek found the first minimum of the nonexchange total cross section to be situated at 6.6, 1.4, or 2×10^7 cm/s and the maximum at 10.3, 3, or 8.4×10^7 cm/s correspondingly.

means of the χ^2 criterion and found to have a Gaussian distribution with a probability 0.50, this low value was probably caused by the small number of the considered experimental points. The dashed curve C in Fig. 1 is the result of minimizing F with the data No. 3 replaced by the data of Mirlin *et al.*,[31] which are 2 to 3 times higher.

A comparison could be made with $Q(v)$ proposed by Nighan and Postma[6] (their curve A) and Postma[5] (see Fig. 3), the two curves being different only at $v > 5\times10^7$ cm/s. As one can see in Fig. 4, some of the experiments are in good agreement with these curves. However, it should be noted that the procedure used by Postma and Nighan and Postma did not make it possible to locate the minimum and the maximum and the solution was not unique. The position of the maximum was chosen by them to be in accordance with the calculations of Crown and Russek,[39] also shown in Fig. 3. Moreover, $\Sigma R_i^2 = 8$, even for the curve of Postma.

The nonmonotonic character of $Q(v)$ is due to the fact that measured values of $\langle Q\rangle^\nu$ are much larger than $\langle Q\rangle^\sigma$ or $\langle Q\rangle^\lambda$; i.e., to fit the experimental data both large and small values of $Q(v)$ are needed in a comparatively narrow range of v.

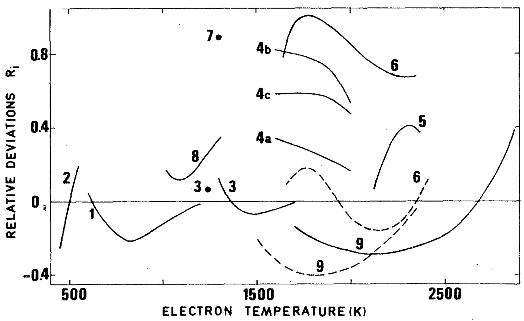

FIG. 4. Relative deviations of 9 different experiments listed in Table I from the calculated values. $Q(v)$ of Nighan and Postma (Ref. 6) (their curve A) is used. For $Q(v)$ proposed by Postma (Ref. 5) only the deviations of experiments Nos. 6 and 9 are substantially changed (dashed lines).

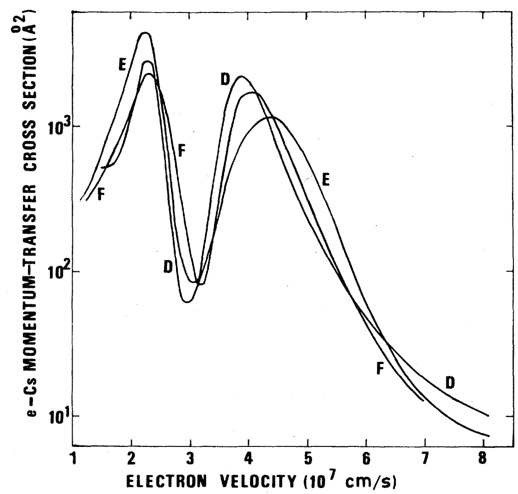

FIG. 5. Variation of curve A from Fig. 2 when some of the experimental data are withdrawn from the procedure of minimization of the deviations. D: Without experiment No. 1, E: Without experiment No. 2, F: Without experiment No. 7.

This could be a curve with any number of minima and maxima as mentioned in Sec. II. When too many knots were introduced or some of them were too close together, then one more additional maximum and minimum appeared with no substantial decrease of ΣR_i^2.

Sensitivity analysis for curve A. The first check was to vary $Q(v)$ and to observe how the quantities R_i changed. Parts of $Q(v)$ along intervals with lengths $\Delta v = 0.5$ to 1×10^7 cm/s were multiplied by factor of 2 or 0.5. The computations showed that the R_i's are almost insensitive to $Q(v)$ at $v < 1.5\times 10^7$ cm/s and $v > 8\times 10^7$ cm/s. Below $v = 2\times 10^7$ cm/s, $Q(v)$ is determined mainly by experiments Nos. 1 and 2, the left maximum by Nos. 2, 6, and 9, the minimum by Nos. 1, 3, 4, 6, and 7, the right maximum by Nos. 6, 8, and 9, and the falling part at the right by Nos. 6 and 9. It can be seen that $Q(v)$ is more sensitive to the experiment No. 6.

The second check was to withdraw different experimental data and then to minimize again F in Eq. (2) using the knots of curve A. Withdrawing Nos. 3, 4, 5, 6, 8, and 9 did not change curve A substantially. In Fig. 5, the curves D, E, and F are solutions where experimental data Nos. 1, 2, and 7, correspondingly, have not been taken into account. We paid special attention to curve F because there the most deviating data (No. 7) have been withdrawn. For this curve $\Sigma R_i^2 = 1.87$ and the rms deviation of all the remaining data is 0.174. Nevertheless, $\langle Q\rangle^\sigma$ was changed by less than 12% and this change reflects the accuracy of the integrated values of $Q(v)$ for the recommended curve A. Another test for accuracy is curve C in Fig. 1, for which $\langle Q\rangle^\sigma$ was changed by less than 25%.

The withdrawal of experiment No. 9 did not change substantially curve A. But for different sets of knots, a family of curves was obtained with quite different positions of the two maxima and the minimum. The reason is that, in fact, the knots are implicit variables in minimizing F in Eq. (2). For fixed knots, such as in the case of sensitivity analysis of curve A, any variation of the set of C_j when instead of experiments Nos. 1–9 only experiments Nos. 1–8 are considered, leads to an increased value of ΣR_i^2; i.e., $Q(v)$ is very near to curve A and represents the best solution for experiments Nos. 1–8 for this particular choice of knots. If experiments Nos. 1–8 are processed using different sets of knots, then different $Q(v)$ are obtained. In contrast with this, experiments Nos. 1–9 result in almost coinciding curves $Q(v)$ even when different sets of knots are used (see Fig. 1). Therefore experiment No. 9 (but only together with Nos. 1–8) is crucial for the determination of the location of the maxima and the minimum in $Q(v)$.

The last check was part of the second one and consisted of comparing for a particular experiment its R_i values using curve A with R_i calculated using another curve $Q(v)$ obtained when this particular experiment was withdrawn from the procedure. The largest change was when experiment No. 2 was withdrawn. Then the maximum value of R_i was increased from 0.17 to 0.67. This is due to the fact that experiment No. 2 alone provides information about $\langle Q\rangle^\nu$ in the range of T_e

near 500 K. The withdrawal of experiment No. 7 increased its R_i value from 0.40 to 0.55. For No. 9, the change in R_i was up to 0.20 but the maximum deviation was not changed. For all the other experiments the changes of R_i were less than 0.11.

Comparison with direct measurements and theoretical calculations. Visconti *et al.*[40] measured the total cross section $Q^{(0)}(v)$ for the scattering of 0.3- to 9-eV electrons by Cs and found it to be monotonically decreasing. There are two possible reasons for a qualitative discrepancy between $Q^{(0)}(v)$ and the momentum-transfer cross section $Q(v)$. The first is that $Q(v)$ is much more sensitive to large-angle scattering than $Q^{(0)}(v)$. Therefore, an energy-dependent peak on the differential cross section for angles between $\pi/2$ and π could produce oscillations of $Q(v)$, even if $Q^{(0)}(v)$ is monotonous. The second is connected with the large energy spread of the beam in the experiment: $\Delta\epsilon = 0.225$ eV, which is equivalent to a velocity spread $\Delta v = 1$ to 1.5×10^7 cm/s in the region where both maxima and the minimum of $Q(v)$ proposed here are situated. Therefore, averaged values were measured and $Q^{(0)}(v)$ was smoothed substantially.

In the range $v = 6$ to 8×10^7 cm/s, the data of Visconti *et al.* for $Q^{(0)}$ are 6 to 20 times higher than Q given by curve A. A partial explanation of this discrepancy could be based on two circumstances (a) The ratios experimental/calculated values do not exceed 50% deviation when our $Q(v)$ is changed by a factor of two in this range because there are few electrons at these velocities (b) $Q^{(0)}/Q$ could be as large as 5 as shown in Ref. 39.

Two different theoretical calculations of $Q(v)$ for Cs are known and they disagree completely. Karule[41] calculated the differential cross section for low energies. The appropriate integration shows that both $Q^{(0)}(v)$ and $Q(v)$ are monotonically decreasing. In contrast with it, Crown and Russek[39] found $Q(v)$ to have two maxima and two minima in the interval $v = 1$ to 10×10^7 cm/s. Curve A proposed here is not inconsistent with their results because large shifts of the positions of the minima and the maxima are observed when changing the polarizability of Cs in their calculations.

V. CONCLUSION

The recommended curve A is almost unique in the sense discussed in Sec. II, i.e., if it is supposed to be smooth enough and to have not more than two maxima and one minimum. The possible variations of its form are given by the other curves in Figs. 1 and 5. In the future, a direct beam-type measurement of the differential cross section for at least one value of v with a spread $\Delta v < 0.2\times 10^7$ cm/s ($\Delta\epsilon < 0.04$ eV at $\epsilon = 0.4$ eV) is highly desirable. An alternative is to have measurements of the same type as Nos. 1–9, but with higher accuracy. The data of Table II representing curve A can be used in all kinds of calculations of transport coefficients in the range $T_e = 500$–2500 K within an error probably not exceeding 20%. They also can be used to solve kinetic problems such as finding the electron distribution function.

The concept of mean cross section which is universal for different transport coefficients can be wrong by one order of magnitude. For example, the ratio $\langle Q\rangle^{\lambda} : \langle Q\rangle^{\sigma} : \langle Q\rangle^{\nu}$ for Cs at 2000 K is 1 : 3 : 10.

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