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Absorption of radiation in a magnetoplasma and application to the laser-fusion process

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One mechanism for the generation of heat in the laser-fusion process is via inverse bremsstrahlung. Since it is desirable to maximize the amount of heat produced, it is thus of interest to investigate possibilities for increasing the absorption rate via the inverse bremsstrahlung process. Here we show that, in the presence of a magnetic field, the absorption rate can be either increased or decreased depending on the relative orientations of the polarization vector of the laser beam, the incident direction, and the magnetic field direction, and depending on the relative magnitudes of the electron's average energy due to thermal and electric-field effects. In particular, for radiation incident along the magnetic direction, we show that the *absorption* of *right*-circularly-polarized radiation is *decreased*, whereas that of left-circularly-polarized radiation is increased, if we are in the *weak-electric-field* regime (thermal effects dominate). On the other hand, in the strong-electric-field regime, the rate of absorption of right-polarized radiation is greater than that of left. In addition, the right-polarized radiation will penetrate to a region of greater plasma density than the left, in the direction of the magnetic field.

I. INTRODUCTION

The use of laser-induced fusion for the generation of controlled thermonuclear power[1-3] is now considered to have good prospects.[4,5] The heat required for fusion is generated via plasma instabilities[6] (which do not occur until several picoseconds after the laser beam is applied[6]) or via inverse bremsstrahlung[7]—although recent experiments[8,9] indicate that an important role is also played by Brillouin scattering and resonance absorption. Our efforts here will be concentrated on the inverse bremsstrahlung process (IB).

The main contribution to absorption is due to electron-ion collisions. In general, we would like to examine any possibilities that exist for increasing the absorption of heat. Here we concentrate our efforts in examining the effect of a magnetic field on the rate of absorption. However, it will be clear that the techniques we use have a wider applicability.

The efficiency of conversion of the incident laser energy into thermal energy is limited by the expansion of the plasma. This led to consideration of the possibility of obtaining increased absorption by utilizing magnetic confinement,[10] and it was shown that fields in the megagauss region could have a big influence on the *transport* properties of laser-fusion plasmas.[10,11] However, all of these investigations neglected the effect of the magnetic field on the IB process itself. Furthermore, it became apparent that megagauss fields are actually generated as a result of the interaction of the laser light with a plasma.[12-14]

A classical approach[15] to the analysis of bremsstrahlung in a static homogeneous magnetic field led to the conclusion that amplification of electromagnetic waves could occur, but the possibility of increased absorption due to the magnetic field was not considered nor were the implications for the laser-fusion process touched upon. On the other hand, Seely,[16] directly motivated by the implications for the laser-fusion process calculated the effect of a B field on the IB process. For the configuration of B-field and radiation-polarization directions which he considered, Seely concluded that the absorption coefficient is *decreased* as a laser frequency approaches the electron cyclotron frequency. It is our purpose to demonstrate here that configurations are possible for which one can obtain an *increase* in the absorption coefficient, so that the prospects of obtaining laser-induced fusion are increased. In addition, we consider that Seely's approach is unnecessarily complicated for the problem at hand, in that it describes the plasma electrons by the solution to the Schrödinger equation for an electron in the combined laser and B fields. This leads to complicated equations so that solutions eventually had to be obtained in the classical limit. Similar remarks apply also to the investigations of Pavlov and Panov,[17] but the latter authors were concerned with more general questions and not with the specific laser-fusion problem.

In Sec. II we consider the general problem of absorption of the laser light in a magnetoactive plasma. We derive a general result for the power absorbed in terms of three basic frequencies—ω (the laser frequency), ω_c (the cyclotron frequency), and ν (the collision frequency). The relative magnitudes of these frequencies play a decisive part in determining such questions as the different amounts of right- and left-circularly-polarized radiation which are absorbed. We demonstrate the inherent simplicity of our approach and show that it leads to results normally obtained with the use of more elaborate methods, both in the case where thermal effects dominate and in the case where strong laser fields dominate.

In Sec. III we discuss some of the parameters and numbers associated with the laser-fusion process. In Sec. IV we apply the general results obtained in Sec. II to the laser-fusion process. In particular, we emphasize the fact that there are two broad regimes to be considered depending on whether the electron motion is determined primarily by the temperature (weak-electric-field region) or by the electric field (strong-electric-field region). Then we concentrate our attention on the propagation of either right-circularly-polarized radiation or left-circularly-polarized radiation propagating along the magnetic-field direction. In the *weak-electric*-field regime, we find that the effect of the presence of the magnetic field is to *increase* (decrease) the absorption rate for *left* (right)-circularly-polarized radiation. On the other hand, in the strong-electric-field regime, we obtain the opposite result, viz., that the effect of the presence of a magnetic field is to decrease (increase) the absorption rate for left (right)-circularly-polarized radiation. In addition, we point out that the magnetic field not only has a direct effect (in the sense of modifying the directed motion of the electron and hence the basic absorption rate), but it also contributes indirectly by its effect on the plasma frequency (in the sense that photon propagation will depend on the direction of propagation relative to the magnetic field). In particular, we note that right-circularly-polarized radiation will penetrate to a region of greater plasma density than the left-circularly-polarized radiation, in the direction of the magnetic field.

II. ABSORPTION OF RADIATION IN A MAGNETOPLASMA

Consider a monochromatic plane electromagnetic wave propagating in a plasma, with electric vector given by

$$\vec{E}(t) = \mathrm{Re}\,\vec{E}_\omega e^{-i\omega t} . \quad (1)$$

The time-average power dissipated per unit volume is

$$P = \langle \vec{j}(t)\cdot\vec{E}(t)\rangle = \tfrac{1}{2}\mathrm{Re}(\vec{j}_\omega \cdot \vec{E}_\omega^*) , \quad (2)$$

where

$$\vec{j}(t) = \mathrm{Re}(\vec{j}_\omega e^{-i\omega t}) \quad (3)$$

is the electric current density resulting from the electric field. To determine $\vec{j}(t)$ we use the generalized Ohm law for a magnetoplasma[18]

$$d\vec{j}/dt + \nu\vec{j} - \omega_c \hat{z}\times\vec{j} = (ne^2/m)\vec{E}(t) . \quad (4)$$

Here $-e$, m, and n are the charge, mass, and number density of the electrons, and

$$\omega_c = eB/mc \quad (5)$$

is the electron cyclotron frequency with $\vec{B} = B\hat{z}$ the external (uniform) magnetic field. Since

$$\vec{j}(t) = -ne\vec{v} , \quad (6)$$

the generalized Ohm law (4) may be written in the alternative form

$$d\vec{v}/dt + \nu\vec{v} - \omega_c(\hat{z}\times\vec{v}) = -(e/m)\vec{E}_\omega \cos\omega t . \quad (7)$$

The quantity ν in (4) is the collision frequency, related to the mean force on the drifting electrons due to collisions. The dominant role is played by electron-ion collisions and thus we consider only this process, treating the ions as fixed centers of charge. The collision frequency is then[19]

$$\nu = N \left\langle \int d\Omega\, v(1-\cos\theta) d\sigma/d\Omega \right\rangle , \quad (8)$$

where N is the number density of ions and $d\sigma/d\Omega$ is the differential cross section for scattering of electrons with velocity v through angle θ. In (8) the braces $\langle\cdots\rangle$ denote an average over the electron velocity distribution. For nonrelativistic electrons, the cross section is given by the classical Rutherford formula

$$\frac{d\sigma}{d\Omega} = \left(\frac{Ze^2}{mv^2(1-\cos\theta)}\right)^2 , \quad (9)$$

where Z is the ionic charge ($n = NZ$). Putting this in (8), we obtain

$$\nu(v) = 2\pi N\left(\frac{Ze^2}{m}\right)^2 \langle L/v^3\rangle , \quad (10)$$

where

$$L = -2\ln\sin\tfrac{1}{2}\theta_m \quad (11)$$

is the Coulomb logarithm. The angle θ_m is the minimum scattering angle, corresponding to the maximum impact parameter.[20] Choosing the maximum impact parameter to be the Debye length yields[20]

$$\sin\tfrac{1}{2}\theta_m \approx \frac{Ze^2}{mv^2}\left(\frac{4\pi ne^2}{kT}\right)^{1/2} . \quad (12)$$

We have, of course, a distribution of velocities, but it is sufficient for our purposes to regard v in (10) as the square root of an ensemble average of v^2 (see below), and we will refer to the ensembleaveraged value of $\nu(v)$ simply as ν. Since L is a weakly dependent function of the velocity, we see in essence that ν is proportional to v^{-3}.

It should be noted that the velocity $\vec{v}$ consists of contributions from the random thermal motion and from the directed motion due to the electric field. In the case of a nondegenerate plasma, the thermal velocity $\vec{v}_{th}$, whose mean value is of course zero, has a root-mean-square value

$$(3kT/m)^{1/2} \equiv v_T \,. \tag{13}$$

Under the influence of a linearly polarized electric field $\vec{E} = \vec{E}_\omega \cos\omega t$, the electron acquires an additional directed velocity $\vec{v}_D$, which, in the absence of the magnetic field and collisions, is given by

$$\vec{v}_D = (-e\vec{E}_\omega / m\omega)\, \sin\omega t. \tag{14}$$

Thus, the total velocity $\vec{v}$ is simply given by

$$\vec{v} = \vec{v}_{th} + \vec{v}_D \,, \tag{15}$$

so that

$$\langle v^2 \rangle = v_T^2 + v_E^2\; 2\sin^2\omega t \,, \tag{16}$$

where $\langle\;\rangle$ denotes an ensemble average, and

$$v_E \equiv \frac{1}{2^{1/2}}\left(\frac{eE_\omega}{m\omega}\right). \tag{17}$$

In Eq. (16) we have assumed that we are dealing with a plane-polarized wave. In the case of a circularly polarized wave, the magnitude of the driven velocity remains constant so that the $\sin^2$ term is absent and the subsequent analysis is in fact simpler. Thus in the presence of right- or left-polarized radiation, propagating along the magnetic field

$$\vec{E} = E_\omega (1/\sqrt{2})(\hat{x}\cos\omega t \mp \hat{y}\sin\omega t)\,, \tag{18}$$

and in the absence of collisions, the corresponding result is

$$\langle v_{r,l}^2 \rangle = v_T^2 + v_E^2 \left(\frac{\omega}{\omega \pm \omega_c}\right)^2 , \tag{19}$$

where r and l refer to right and left, respectively.

In the weak-electric-field regime ($v_E \ll v_T$) we see that ν is essentially proportional to v_T^{-3} and thus can be regarded as a constant in (4) and (7). In this case, using (1) and (3), we find

$$(-i\omega + \nu)\vec{j}_\omega - \omega_c(\hat{z} \times \vec{j}_\omega) = (ne^2/m)\vec{E}_\omega \,, \tag{20}$$

or

$$\vec{j}_\omega = (\omega_P^2/4\pi)\left(\frac{1}{-i\omega + \nu}\hat{z}\cdot\vec{E}_\omega \hat{z} + \frac{1}{-i(\omega - \omega_c) + \nu}\tfrac{1}{2}(\hat{x} - i\hat{y})\cdot\vec{E}_\omega(\hat{x} + i\hat{y}) + \frac{1}{-i(\omega + \omega_c) + \nu}\tfrac{1}{2}(\hat{x} + i\hat{y})\cdot\vec{E}_\omega(\hat{x} - i\hat{y})\right), \tag{21}$$

where

$$\omega_P = (4\pi n e^2/m)^{1/2} \tag{22}$$

is the plasma frequency. From (2) it follows that the time-average power absorbed per unit volume is

$$P = \omega_P^2(\nu/8\pi)\left(\frac{|E_z|^2}{\omega^2 + \nu^2} + \frac{|E_l|^2}{(\omega - \omega_c)^2 + \nu^2} + \frac{|E_r|^2}{(\omega + \omega_c)^2 + \nu^2}\right), \tag{23}$$

where

$$E_z = \hat{z}\cdot\vec{E}_\omega, \quad E_{r,l} = \frac{\hat{x} \pm i\hat{y}}{\sqrt{2}}\cdot\vec{E}_\omega \,. \tag{24}$$

For propagation along the $\hat{z}$ direction, i.e., the direction of the external magnetic field, E_r and E_l are the amplitudes of the right- and left circularly-polarized components of the wave. The expression (23) is the general result in the weak-field limit for the power absorbed in terms of the three basic frequencies.

In the strong electric field regime ($v_E \gg v_T$) the problem is rather complicated because ν is now, in general, dependent on the directed velocity. However if we restrict ourselves to the (more physically interesting) situation, where the ν term in (7) is a small perturbation, then the problem is tractable. In addition, if we consider only right- and left-polarized radiation, then the latter assumption is equivalent to the condition

$$\nu \ll (\omega \pm \omega_c)\,. \tag{25}$$

This, of course, is the nonresonant case. Thus, to lowest order ($\nu = 0$), the magnitude of the driven velocity is a constant (but its direction changes with frequency $\omega \pm \omega_c$). Since $\nu \sim v^{-3}$ it follows that ν is a constant to lowest (zeroth) order. In other words, it may also be treated as a constant in (7) to first order. We conclude that, under assumption (25), (23) also holds in the strong-field limit for right- and left-polarized radiation.

In the case of linearly polarized radiation, the analysis is complicated by the fact that the driven velocity vector goes through zero during the periodic motion with the result that divergences occur in the collision frequency. This in turn gives rise to an additional $\ln(v_E/v_T)$ term in the power absorbed,[21,22] which is typically not large compared to unity.

Finally, in the case of resonance ($\omega \sim \omega_c$) for left-polarized radiation, (25) is no longer fulfilled. In such a case we obtain runaway electron velocities and strong absorption similar to what occurs in a static electric field ($\omega=0$) in the absence of a magnetic field ($\omega_c=0$). Our further considerations will be confined to situations where (25) and (23) hold and where the radiation is left- or right-polarized.

Before we apply our general formula (23) to the case of magnetoplasmas typical of those occurring in laser-fusion applications, it will be instructive to consider the case of absorption in an isotropic plasma with no magnetic field. In this case we set $\omega_c=0$ in (23) to get

$$P=\frac{\omega_p^2\nu}{8\pi}\;\frac{|E_\omega|^2}{\omega^2+\nu^2}\,. \tag{26}$$

If we consider now the case where the frequency of the radiation is high compared with the collision frequency, this becomes

$$P\sim(\omega_P^2\,\nu/8\pi\omega^2)\,|\vec{\mathrm{E}}_\omega|^2\,,\quad \omega\gg\nu\,. \tag{27}$$

We turn now to a consideration of explicit expressions for the collision frequency in the weak- and strong-field limits. For our purposes it will simply be sufficient to take the $\langle v^{-3}\rangle$ as being v_T^{-3} and $v_{r,l}^{-3}$ in the weak- and strong-field limits, respectively (r and l referring to right- and left-polarized radiation). The corresponding collision frequencies ν_T and $\nu_{r,l}$ are given explicitly by [see (10), (13), and (19)]

$$\nu_T=\frac{2\pi Z^2e^4NL}{m^{1/2}(3kT)^{3/2}} \tag{28}$$

and

$$\begin{aligned}\nu_{r,l}&=2^{5/2}\pi(Z^2em/E_\omega^3)(\omega\pm\omega_c)^3NL\\ &\equiv\nu_E(\omega\pm\omega_c)^3/\omega^3\,,\end{aligned} \tag{29}$$

respectively. These results are the same as those obtained by more sophisticated treatments.[21,23] Similar results were obtained by Bethe,[22] who also used an analysis based on the Rutherford cross-section. As emphasized by Bethe (29) is a striking result since [making use of (23) and (25) to conclude that P is proportional to ν to lowest order] we see that it implies that the stronger the field, the less energy is absorbed.

III. THE LASER-FUSION PROCESS

Research on laser-fusion has been conducted primarily with the 1.06-μm neodymium-glass laser and the 10.6-μm carbon-dioxide laser.[4,5,24] For simplicity of discussion we will concentrate our remarks on the former (while remarking that, contrary to expectations, it has been found that the carbon-dioxide laser is apparently as effective[5,25]).

Thus, the corresponding angular frequency is $\omega=1.8\times10^{15}$ rad/s, and $\hbar\omega=1.2$ eV. Thus, we see that the laser light will cease to propagate (i.e., ω_P becomes equal to ω) when n_e reaches a critical value $n_c=10^{21}/\mathrm{cm}^3$, corresponding to a critical density (taking $Z=1$) $\rho_c\sim4\times10^{-3}$ g/cm^3. The laser light is absorbed in the atmosphere of the pellet, generating hot electrons, eventually producing temperatures $T\simeq10^8$ K, so that $kT\simeq10$ keV $\sim2\times10^{-2}\,mc^2$. Thus, after the initial stage of plasma formation, $\hbar\omega\ll kT$, and the corresponding value of the thermal velocity $v_T\equiv(3kT/m)^{1/2}$ is $\simeq0.24c=7\times10^9$ cm/s. Typical values of the laser intensity and electric field amplitude are $I=3\times10^{15}$ W/cm^2 and $E_\omega=1.5\times10^9$ V/cm, respectively. Hence $v_E=(1/\sqrt{2})(eE_\omega/m\omega)=10^9$ cm/s. Also, the time-average oscillatory energy in the electric field, $\epsilon_{\mathrm{osc}}^{(0)}$ say (the superscript indicating the absence of a magnetic field), is

$$\epsilon_{\mathrm{osc}}^{(0)}=\tfrac{1}{2}mv_E^2=\tfrac{1}{4}(e^2E_\omega^2/m\omega^2)\,. \tag{30}$$

Thus, for the parameters chosen, we see that v_E is slightly smaller than v_T. However, for $T\sim2\times10^6$ K and all other parameters the same, v_E and v_T are equal and thus for the still lower temperatures existing at the initial stages of plasma formation we are in the strong-field region ($v_E\gg v_T$).

It is also of interest to note that the amplitude of oscillation in the above field is (in cm)

$$R_0=eE_\omega/m\omega^2=\sqrt{2}\,v_E/\omega=7.9\times10^{-7}\,. \tag{31}$$

Thus $R_0\ll\lambda$ (this condition is essentially equivalent to $v_E\ll c$) and so the spatial dependence of the electromagnetic wave may be neglected [as we had already anticipated in writing down (1)]. In other words, we are dealing with a so-called "cold plasma." This in fact provides the justification for our utilization of the single electron theory (Drude model) instead of the Boltzmann equation.

For orientation purposes, we will first of all consider the situation where the magnetic field is absent. In general, since the solution of (7) when $\vec{\mathrm{E}}=\vec{\mathrm{B}}=0$ is given by $\vec{v}(t)=\vec{v}(0)e^{-\nu t}$, we may write that the thermal (random) contribution to the total energy, ϵ_T say, is increasing according to the relation

$$d\epsilon_T/dt=-d\epsilon_{\mathrm{osc}}/dt=2\nu\epsilon_{\mathrm{osc}}\,, \tag{32}$$

where ϵ_{osc} is the time-averaged contribution to the energy (nonrandom or directed) due to external fields. In the case under consideration ϵ_{osc} is given by (30). Hence, in the weak-field limit, we use (28), (30), and (32) to obtain

$$\frac{d\epsilon_T}{dt}=\frac{\pi Z^2e^6NL}{m^{3/2}(3kT)^{3/2}}\,\frac{E_\omega^2}{\omega^2}\quad(v_E\ll v_T)\,. \tag{33}$$

In the strong-field limit, we use (29), (30), and

(32) to obtain

$$\frac{d\epsilon_T}{dt} = 2^{3/2}\pi \frac{Z^2 e^3 \omega}{E_\omega} NL \quad (v_T \ll v_E)\,. \tag{34}$$

Thus, in the weak-field limit, $(d\epsilon_T/dt)$ increases with increasing E_ω and decreasing ω whereas the reverse occurs in the strong-field limit. Our power dependences are in agreement with those obtained by Bethe.[22]

We turn now to a consideration of the magnetic field $\vec{B}$. Classically, an electron in a magnetic field executes a circular motion in a plane perpendicular to $\vec{B}$, with an angular frequency ω_c. The corresponding energy is (in eV)

$$\hbar\omega_c = 5.8 \times 10^{-3} B_6\,, \tag{35}$$

where $B_6 = B(\mathrm{G})/10^6$. Thus, for the magnetic fields and temperatures of interest for the laser-fusion process $\hbar\omega_c \ll kT$ and a quantum treatment (Landau energy levels, etc.) is unnecessary.

We also remark that $(\omega_c/\omega) = 4.8 \times 10^{-3} B_6$. Thus, for lasers of smaller frequency (the CO_2 laser has, of course, a frequency smaller by a factor of 10) and for B values greater than 10^6G, one has a situation where $\omega_c \sim \omega$.

Another quantity of interest is the Larmor radius $R_L = v/\omega_c$, where v is the velocity of the electron in the plane perpendicular to $\vec{B}$. Taking $v \sim v_T = 4 \times 10^9$ cm/s, and using the value for ω_c given in Eq. (5), we obtain $R_L \sim 2.3 \times 10^{-4} B_6^{-1}$ cm. Thus, for $B \sim 10^6$ G, we see that $R_L \sim \lambda$.

IV. ABSORPTION OF RADIATION IN THE LASER-FUSION PROCESS

We confine ourselves to a case of particular interest, viz., the laser beam propagating in the direction of the magnetic field (i.e., we take $E_Z = 0$). For the parameters currently under consideration we are in the weak-electric-field regime, i.e., $v_E \ll v_T$. However, we will later include in our development the opposite possibility $v_E \gg v_T$, which is simply achieved for temperatures $T \ll 10^6$ K (retaining the same value of $E_\omega = 1.5 \times 10^9$ V/cm). In the weak-field regime ($v \sim v_T$ and $\nu \sim \nu_T$), it follows from Eq. (28) that (with $Z = 1$) $\nu_T \lesssim 1.2 \times 10^{11}$ s^{-1}. Recalling that $\omega = 1.8 \times 10^{15}$ s^{-1} and $\omega_c \sim 10^{13}$ s^{-1} [see Eq. (5) and assume $B \sim 10^6$ G], we can thus conclude that the inequality given in (25) is in fact correct. For the parameters given above it is also true that

$$\omega \gg \omega_c\,. \tag{36}$$

Consider now the propagation of circularly polarized radiation along the Z direction. It follows from Eq. (22) that

$$P_{r,l} = \frac{\omega_p^2 \nu_T}{8\pi} \frac{E_\omega^2}{(\omega \pm \omega_c)^2 + \nu_T^2} \quad (\text{for } v_T \gg v_E)\,, \tag{37}$$

where ν_T is given by (28) and where $P_{r,l}$ refers to the power absorbed per unit volume from beams of right and left polarization, respectively, and where E_0 refers to the amplitude of either radiation. Thus we conclude that, in the *weak-electric-field regime*, the absorption rate is increased for left and decreased for right polarization, in the presence of a magnetic field. A convenient comparison of the respective rates is obtained by defining

$$\Delta(\omega) \equiv (P_l - P_r)/(P_l + P_r)\,. \tag{38}$$

Hence, in the case under discussion, we find, using (23), that

$$\Delta(\omega) = \frac{2\omega\omega_c}{\omega^2 + \omega_c^2 + \nu_T^2}\,. \tag{39}$$

It is of interest to note that similar results have been obtained by Kemp.[26] In the weak-field (nearly) collisionless regime ($\omega \gg \omega_c, \nu_T$),

$$P_{r,l} \approx \frac{\omega_p^2 \nu_T}{8\pi\omega^2} E_\omega^2 (1 \mp 2\,\omega_c/\omega)$$
$$= 2\nu_T n_e \epsilon_{\mathrm{osc}}^{(0)} (1 \mp 2\,\omega_c/\omega)\,, \tag{40}$$

similar to Eq. (32) and also[26]

$$\Delta(\omega) \approx 2\omega_c/\omega\,. \tag{41}$$

We end our discussion of the weak-electric-field regime by using (22) and (28) in (37) to give

$$P_{r,l} = \frac{\pi Z^2 e^6 n NL}{m^{3/2}(3kT)^{3/2}} \frac{E_\omega^2}{(\omega \pm \omega_c)^2 + \nu_T^2} \quad (v_T \gg v_E)\,. \tag{42}$$

For $\omega_c = \nu_T = 0$, we remark that this result is consistent with Eq. (33). We also note the E_ω^2 dependence, which is characteristic of the weak-electric-field regime.

Next, we turn our attention to the strong-electric-field regime ($v_E \gg v_T$) so that $\nu \sim \nu_E$. It follows from (29) that (again taking $Z = 1$) $\nu_E \leqslant 8 \times 10^{12}$ s^{-1}. Thus $\nu \ll \omega$, ω_c still holds, providing no resonance.

From (23) it follows that

$$P_{r,l} = \frac{\omega_p^2 \nu_{r,l}}{8\pi} \frac{E_\omega^2}{(\omega \pm \omega_c)^2 + \nu_{r,l}^2}\,. \tag{43}$$

Next, making use of (25) and (29), we may put our result in the form

$$P_{r,l} = 2^{3/2}\pi (Z^2 e^3/E_\omega) n NL (\omega \pm \omega_c) \quad (\text{for } v_E \gg v_T)\,. \tag{44}$$

For $\omega_c = 0$, we remark that this result is consistent with (34). We again draw attention to the characteristic E_ω^{-1} behavior in the strong-electric-field regime. Thus, in contrast to what we found in the weak-electric-field regime, we conclude that, in the *strong-electric-field regime*, the absorption rate is *increased for right and decreased for left-polarized radiation* in the presence of a magnetic field. Our result in the case of left polarization is the same as that obtained by Seely in the case of right polarization. Also, Seely did not consider the other polarization state nor did he consider the weak-electric-field regime. In addition, our derivation is much simpler and is classical throughout whereas Seely commenced with a more elaborate quantum approach and later took a classical limit.

As a final consideration, we turn to another difference between right- and left-polarized radiation, connected with the fact that they do not penetrate to the same depth in a plasma, along the magnetic-field direction. Now the dielectric constant ϵ has the values (Ref. 19, p. 228)

$$\epsilon_{r,l} = 1 - \omega_P^2/[\omega(\omega \pm \omega_c)], \tag{45}$$

where we have assumed that we are away from resonance and that $\omega \gg \nu$. The solution of the equation $\epsilon = 0$ gives the critical frequencies, viz.,

$$\omega_{r,l} = \tfrac{1}{2}[(\omega_c^2 + 4\omega_P^2)^{1/2} \mp \omega_c]. \tag{46}$$

Alternatively, for fixed frequency $\omega_r = \omega_l = \omega$, we may write

$$(\omega_P)_{r,l} = \omega(\omega \pm \omega_c), \tag{47}$$

so that

$$(n_c)_r > (n_c)_l. \tag{48}$$

In other words, for propagation along the magnetic-field direction, the right-polarized radiation will penetrate to a region of greater plasma density than the left.

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