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Comment on "Electric dipole interaction in quantum optics"

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It is shown that both Hamiltonians, one with $-\vec{\mu}\cdot\vec{E}$ and the other with $-\vec{\mu}\cdot\vec{A}$ as interaction energy between an atom and the electromagnetic field, lead to Maxwell's equations. In contrast to the viewpoint in Mandel's paper, $\vec{E}$ is considered to be proportional to the canonically conjugate field of the vector potential.

In a recent paper,[1] it has been suggested that the minimal coupling Hamiltonian

$$H'' = \tfrac{1}{2}\int\left(\epsilon_0\vec{E}^2 + \frac{1}{\mu_0}\vec{B}^2\right)d\vec{x} + H_A - \dot{\vec{\mu}}(t)\cdot\vec{A}(\vec{r}_0, t) \quad (1)$$

is to be preferred over the point-electric-dipole Hamiltonian

$$H' = \tfrac{1}{2}\int\left(\epsilon_0\vec{E}^2 + \frac{1}{\mu_0}\vec{B}^2\right)d\vec{x} + H_A - \vec{\mu}(t)\cdot\vec{E}(\vec{r}_0, t) \quad (2)$$

because of the claim that the former leads to Maxwell's equations for the electromagnetic field in contrast to the latter. However, in our previous work,[2] we have shown that both Hamiltonians give rise to Maxwell's equations. In this comment, we give an explicit demonstration that (2) does indeed give Maxwell's equations.

For both Hamiltonians, the conjugate field variables are $\vec{A}$ and $-\epsilon_0\vec{E}/c$ which are subject to the commutation rules,

$$\left(A_i(\vec{r}, t), -\frac{\epsilon_0}{c}E_j(\vec{r}', t)\right) = i\hbar\delta^T_{ij}(\vec{r} - \vec{r}') \quad (3)$$

and the electric radiation field $\vec{E}^{\mathrm{rad}}(\vec{r}, t)$ is given by

$$\vec{E}^{\mathrm{rad}}(\vec{r}, t) = -\frac{1}{c}\dot{\vec{A}}(\vec{r}, t). \quad (4)$$

Now, for Hamiltonian (1),

$$E_i^{\mathrm{rad}}(\vec{r}, t) = -\frac{1}{i\hbar c}[A_i(\vec{r}, t), H''] = E_i(\vec{r}, t), \quad (5)$$

while for (2),

$$\begin{aligned} E_i^{\mathrm{rad}}(\vec{r}, t) &= -\frac{1}{i\hbar c}[A_i(\vec{r}, t), H'] \\ &= E_i(\vec{r}, t) - \frac{1}{i\hbar c}[A_i(\vec{r}, t), -\vec{\mu}(t)\cdot\vec{E}(\vec{r}, t)] \\ &= E_i(\vec{r}, t) - \frac{1}{\epsilon_0}\mu_j(t)\delta^T_{ij}(\vec{r} - \vec{r}_0). \end{aligned} \quad (6)$$

Thus, from the point-electric-dipole Hamiltonian (2), we have the equations of motion

$$\begin{aligned} \frac{\partial E_i^{\mathrm{rad}}(\vec{r}, t)}{\partial t} &= \frac{1}{i\hbar}[E_i^{\mathrm{rad}}(\vec{r}, t), H'] \\ &= \frac{1}{i\hbar}[E_i(\vec{r}, t), H'] \\ &\quad + \frac{1}{i\hbar\epsilon_0}[\mu_j(\vec{r}, t), H']\delta^T_{ij}(\vec{r} - \vec{r}_0) \\ &= c^2[\vec{\nabla}\times\vec{B}(\vec{r}, t)]_i - \frac{\dot{\mu}_j(t)}{\epsilon_0}\delta^T_{ij}(\vec{r} - \vec{r}_0) \end{aligned} \quad (7)$$

and

$$\begin{aligned} \frac{\partial B_i(\vec{r}, t)}{\partial t} &= \frac{1}{i\hbar}[B_i(\vec{r}, t), H'] = -[\vec{\nabla}\times\vec{E}(\vec{r}, t)]_i - \frac{1}{\epsilon_0}\epsilon_{ijk}\mu_j(t)\nabla_k\delta(\vec{r} - \vec{r}_0) \\ &= -[\vec{\nabla}\times\vec{E}^{\mathrm{rad}}(\vec{r}, t)]_i - \frac{1}{\epsilon_0}[\vec{\nabla}\times\vec{\mu}(t)\delta(\vec{r} - \vec{r}_0)]_i - \frac{1}{\epsilon_0}\epsilon_{ijk}\mu_j(t)\nabla_k\delta(\vec{r} - \vec{r}_0) \\ &= -[\vec{\nabla}\times\vec{E}^{\mathrm{rad}}(\vec{r}, t)]_i. \end{aligned} \quad (8)$$

Equations (7) and (8) are the Maxwell's equations for the radiation field with the transverse current density $\dot{\mu}_j\delta^T_{ij}(\vec{r} - \vec{r}_0)$. As expected, these are identical with those derived from Hamiltonain (1). Based on these considerations, both Hamiltonians are equally acceptable for the study of interaction of light with atoms or molecules. In fact, perturbation calculations of quantities such as energy shifts and transition rates give identical results.[2] Finally, we point out that the demonstration that both Hamiltonians lead to Maxwell's equations is independent of the electric dipole approximation; the general development is given in Ref. 2.

[1]L. Mandel, Phys. Rev. A 20, 1590 (1979).
[2]M. Babiker, E. A. Power, and T. Thirunamachandran, Proc. R. Soc. London A338, 235 (1974); see also, E. A. Power and T. Thirunamachandran, Am. J. Phys. 46, 370 (1978); E. A. Power, in *Multiphoton Processes*, edited by J. H. Eberly and P. Lambropoulos (Wiley, New York, 1978), p. 11.