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Electrodynamical properties of two-dimensional classical electron systems

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The electrodynamic properties of a classical one-component two-dimensional plasma are studied on the basis of a self-consistent-field approximation including the short-range correlations between particles. The static screening, the plasma dispersion relation for arbitrary wave number, and the dynamic structure factor are determined in a self-consistent scheme. The results are compared with the earlier calculations based on the random-phase approximation. Sizable differences are found revealing accurately the effects associated with correlations between particles. Our results mark a definite improvement over those calculations.

I. INTRODUCTION

Recently, experimental and theoretical investigations of a classical one-component two-dimensional plasma have attracted considerable amount of interst.[1,2] Electrons trapped on the liquid helium surfaces form the cleanest example of a strongly coupled classical two-dimensional system. Experiences with this electron gas have been carried out with electron densities between 10^4 and 10^9 cm^{-2}, corresponding to the Fermi temperature, T_F, between 10^{-7} and 10^{-2} K. Since the accessible experimental range of temperature, $T\sim 1$ K, is much bigger than T_F, this electron gas is in fact a classical plasma.

The two-dimensional classical electron system is characterized by the dimensionless plasma parameter $\alpha = 2\pi m e^4/T^2$ or the parameter $\Gamma = (\pi n)^{1/2} e^2/T = (\frac{1}{2}\alpha)^{1/2}$, where n is the density, T the temperature in energy units, and e the effective electronic charge incorporating the effects of the substrate. The first theoretical discussion in such a classical system was given by Fetter[3] within the random-phase approximation (RPA). However, the RPA is inadequate in treating this system since the electron-correlation effects are quite important. In order to include the short-range correlations, several authors[4-9] have studied this system, showing the necessity for improving the RPA.

The principal purpose of the present work is to investigate some electrodynamic properties of the two-dimensional classical electron systêm through a self-consistent-field approximation[10] (SCFA) which takes into account the short-range correlation effects. This method, besides improving the RPA, has the advantage of being a dynamic one contrasting other elaborate approaches applied to this system.

In this paper we apply the SCFA to calculate the screening density around a fixed charged impurity, the plasma dispersion relation for arbitrary wave number, and the dynamic structure factor for a classical two-dimensional one-component plasma, and compare the results with those of the earlier theories.

In Sec. II we shall briefly present the SCFA. In Sec. III the screening of a static impurity is analyzed. In Sec. IV the plasma dispersion relation is presented. The dynamic structure factor is calculated in Sec. V and a concluding remarks of the results is given in Sec. VI.

II. SELF-CONSISTENT-FIELD METHOD

Since in an earlier paper[6] we have discussed the self-consistent-field approximation (SCFA) for a classical two-dimensional electron system we shall briefly write down the following set of equations which describes the SCFA:

$$\chi(\vec{k},\omega) = \frac{\chi_0(\vec{k},\omega)}{1-\phi(\vec{k})[1-G(\vec{k})]\chi_0(\vec{k},\omega)}, \tag{1}$$

$$G(\vec{k}) = -\frac{1}{n}\int \frac{\vec{k}\cdot\vec{q}}{kq}[S(\vec{k}-\vec{q})-1]\frac{d^2q}{(2\pi)^2}, \tag{2}$$

$$S(\vec{k},\omega) = -\frac{T}{\pi n\omega}\operatorname{Im}\chi(\vec{k},\omega). \tag{3}$$

$\phi(\vec{k})$ is the bare particle-particle interaction, n is the density, T is the temperature in energy units, and $\chi_0(\vec{k},\omega)$ is the density-density response function of the noninteracting electron system given by

$$\chi_0(\vec{k},\omega) = -\frac{n}{T}W\left[\frac{\omega}{k}\left(\frac{m}{T}\right)^{1/2}\right], \tag{4}$$

where

$$\operatorname{Re}W(x) = 1 - xR(x), \tag{5a}$$

$$\operatorname{Im}W(x) = xI(x), \tag{5b}$$

with

$$R(x) = e^{-x^2/2}\int_0^x dy\, e^{y^2/2}, \tag{6a}$$

$$I(x) = (\pi/2)^{1/2}e^{-x^2/2}. \tag{6b}$$

We notice that the expression for the density-density response function in the random-phase approximation[3] (RPA) or Debye–Hückel approximation is recovered if in Eq. (1) we neglect the local-field corrections, i.e., if we set $G(\vec{k})=0$.

III. SCREENING OF A FIXED CHARGED IMPURITY

As our first application of the SCFA we shall study the effects of a static charged impurity, located at the origin on the classical two-dimensional electron system. We consider an electron system placed at the interface between two semi-infinite media with dielectric constants ϵ_1 and ϵ_2 neutralized by a rigid uniform background of opposite charges. The electrodynamic response of an external scalar potential $\phi_{\text{ext}}(\vec{k},\omega)$ arising from a fixed-point charge Ze_0 may be characterized by an induced particle density given by[11]

$$\delta n(\vec{k},\omega)=-\chi(\vec{k},\omega)e_0\phi_{\text{ext}}(\vec{k},\omega)\,, \tag{7}$$

where

$$\phi_{\text{ext}}(\vec{k},\omega)=\frac{2Ze_0}{(2\pi)^2(\epsilon_1+\epsilon_2)}\,\frac{1}{k}\,\delta(\omega)\,, \tag{8}$$

and $\chi(\vec{k},\omega)$ is given by the self-consistent solution of Eqs. (1), (2), and (3).

The inverse Fourier transform of Eq. (7) gives the following expression for the screening density at a distance R,

$$\delta n(R)=-Ze^2\int dk\,J_0(kR)\chi(\vec{k},0)\,, \tag{9}$$

where $e=e_0[2(\epsilon_1+\epsilon_2)^{-1}]^{1/2}$ is the renormalized charge and $J_0(x)$ is the Bessel function of order zero. From the density-density response function, Eq. (1) in the static limit $\omega=0$, we may rewrite Eq. (9) as

$$\delta n(R)=\frac{Zk_D}{2\pi}\int dk\,\frac{kJ_0(kR)}{k+k_D[1-G(k)]}\,, \tag{10}$$

where $k_D=2\pi ne^2/T$ is the two-dimensional analog of the Debye screening constant.

In order to obtain the screening density results in the RPA method we neglect all correlations, i.e., putting $G(k)=0$ in Eq. (10). The, it takes the form

$$\delta n(R)=\frac{Zk_D}{2\pi}\int dk\,\frac{kJ_0(kR)}{k+k_D} \tag{11a}$$

$$=\frac{Zk_D}{2\pi}\left(\frac{1}{R}-\frac{\pi}{2}\,k_D[\bar{H}_0(k_DR)-Y_0(k_DR)]\right)\,, \tag{11b}$$

where $\bar{H}_0(x)$ and $Y_0(x)$ are Struve and Neumann functions. This result was earlier obtained by Chalupa[9] through the linearization of Poisson's equation for the screening potential. It also corresponds to the semiclassical model of Stern and Howard[12] applied to the electrons in the inversion layers problem, but with a modified screening constant.

The induced total charge Q, is given by

$$Q=-e_0\int d\vec{R}\,\delta n(\vec{R})$$

$$=-Ze_0k_D\int dk\,\frac{k_D\delta(k)}{k+k_D[1-G(k)]}=-Ze_0\,, \tag{12}$$

which means that the charged impurity is completely screened at large distances.

Unfortunately, the screening-density expression, Eq. (10), has an undesirable feature, i.e., it diverges logarithmically at $R=0$, yielding an infinite induced charge density there. This divergence arises because the linearized equation of motion for the classical one-particle distribution function which is the starting point of this approximation, is invalid near the charged impurity. In order to prevent this divergence, the integral in Eq. (9) at $R=0$ is cut off due to the quantum effects.[13] This is what happens in some specific situations in classical plasmas[14] where the semiclassical expressions including the $\hbar$-dependent terms are taken into account to avoid the divergences caused by large-angle collisions. Then, this divergence can be avoided if the static quantum density-density response function of the system is taken into account in the classical limit, instead of the strictly classical limit ($\hbar\to 0$), that is,

$$\chi(\vec{k},0)=-(n/T)\zeta(k/k_T)\,, \tag{13}$$

where $k_T=(2mT/\hbar^2)^{1/2}$ is the thermal wave vector and

$$\zeta(x)=(1/\sqrt{2}\,x)R(\sqrt{2}\,x)\,. \tag{14}$$

As a result, the screening density may be written as

$$\delta n(R)=\frac{Zk_D}{2\pi}\int dk\,kJ_0(kR)\,\frac{\zeta(k/k_T)}{k+k_D[1-G(k)]\zeta(k/k_T)}\,, \tag{15}$$

which is finite even at $R=0$, since the asymptotic behavior of the functions ζ and G yields the following realtions

$$\zeta(x)\cong 1/x^2,\quad x\gg 1$$

$$G(k)\to 1,\quad k\gg k_D\,.$$

Although Eq. (15) is finite at $R=0$ the numerical results we obtained are extremely large. Nevertheless it is interesting to notice that the calculation very near the impurity center is not reliable since the linear theory is not valid in this region.

For distances, far from the impurity center the behavior of the integrand at small values of the wave vector $\vec{k}$ dominates the integral in Eq. (15). Then, in this case the function $\xi(\vec{k})$ and the local-field correction $G(\vec{k})$ can be written in the long-wavelength approximation as[6]

$$\xi(k) = 1 ,$$

$$G(k) = kG'(0) = \alpha\gamma k/k_D ,$$

where

$$\gamma = -\tfrac{1}{2}\int dk[S(k) - 1] . \qquad (16)$$

Then, in this limit Eq. (15) is rewritten as

$$\delta n(R) = \frac{Zk_D^*}{2\pi}\int dk\,\frac{kJ_0(kR)}{k + k_D^*} , \qquad (17)$$

where $k_D^* = k_D/(1-\alpha\gamma)$ is the renormalized Debye screening constant. As we can see this expression is formally identical to Eq. (11a) given by the RPA approximation. By taking the asymptotic limit of Eq. (11b) we get the following expression for the induced density:

$$\delta n(R) = \frac{Zk_D^{*2}}{2\pi}\left(\frac{1}{(k_D^*R)^3} - \frac{9}{(k_D^*R)^5} + \frac{225}{(k_D^*R)^7} - \cdots\right) . \qquad (18)$$

This equation shows the existence of a critical value for the plasma parameter $\alpha_c = 1/\gamma$; beyond that the system becomes electrodynamically unstable. At the same α_c we have earlier[6] found that this system also becomes thermodynamically unstable. The nature of such an instability in a three-dimensional classical plasma has been discussed by Totsuji and Ichimaru[15] on the basis of the fluctuation analysis of the positive uniform background. The critical value of the plasma parameter α_c may be interpreted with the point at which the onset of the short-range order appears. For $\alpha \geqslant \alpha_c$, the electronic motions become more correlated and liquidlike behavior is expected. On the other hand, as the plasma parameter increases, the structure factor $S(k)$ [see, for instance, Eq. (8) from Ref. 6] will diverge at $k = k_s$ such as

$$k + k_D[1 + G(k)] = 0 ,$$

determining then another critical value α_s. The system will undergo a phase transition and crystallization with lattice constant corresponding to k_s should take place. For electrons on liquid-helium surfaces, Grimes and Adams[16] have experimentally observed that the fluid-solid transition occurs at $\alpha_s = 3.7\times10^4$. Our calculation, however, is inadequate in obtaining this experimental result because we are dealing with a uniform and isotropic system.

It is interesting to notice that in the limit of large R, we obtained an algebraic expression for the screening density in contrast to the exponential form of the Debye–Hückel three-dimensional version. Equation (15) has been evaluated numerically as a function of R for various values of α. In Fig. 1 our self-consistent results for $\alpha = 1$ and $\alpha = 20$ are compared with those obtained from the RPA. It is seen that the present results give significant differences in the screening density as the plasma parameter increases, showing the presence of the short-range correlations in the system.

IV. PLASMA DISPERSION RELATION

The SCFA is now used to describe a time-dependent perturbation with frequency ω. The plasma dispersion relation $\omega(\vec{k})$ and the damping $\Gamma(\vec{k})$ of the plasma oscillation are determined from the poles of the density-density response function $\chi(\vec{k}, \omega)$. We then have to find the solution

$$\omega = \omega(\vec{k}) + i\Gamma(\vec{k})$$

from the equation

$$F(\vec{k}, \omega) \equiv 1 - \phi(\vec{k})[1 - G(\vec{k})]\chi_0(\vec{k}, \omega) = 0 . \qquad (19)$$

For small damping, $\Gamma(\vec{k}) \ll \omega(\vec{k})$ and in the long-wavelength limit we get, after some rearrangement, the following expressions

$$\omega(\vec{k}) = \omega_0(k/k_D)^{1/2}[1 - (\tfrac{3}{2} - \tfrac{1}{2}\alpha\gamma)k/k_D] , \qquad (20)$$

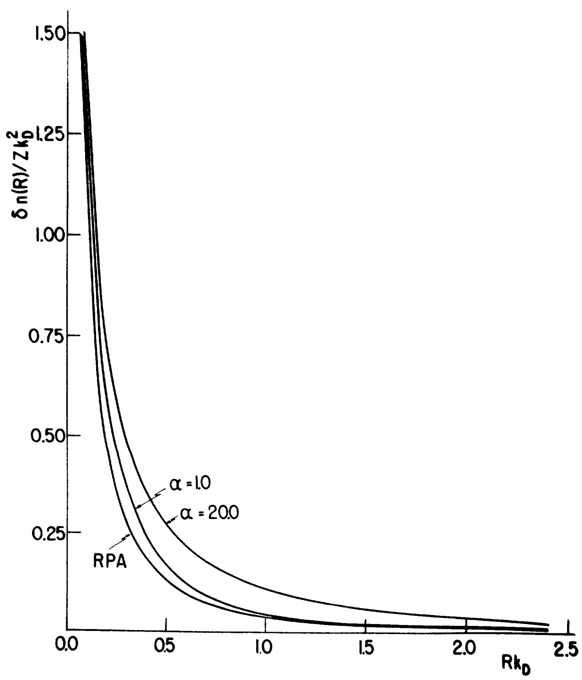

FIG. 1. Screening density $\delta n(R)/Zk_D^2$ as a function of Rk_D. The RPA curve corresponds to that given earlier (Ref. 3).

$$\Gamma(\vec{k}) = -(\tfrac{1}{8}\pi)^{1/2}(k_D/k - \alpha\gamma)^{3/2} \times \exp[-(k_D/2k + \tfrac{3}{2} - \tfrac{1}{2}\alpha\gamma)]\omega(\vec{k}) , \tag{21}$$

where γ is given by Eq. (16), and

$$\omega_0 = (2\pi n e^2 k_D/m)^{1/2} . \tag{22}$$

The present many-body approximation which includes the short-range correlations between particles gives a correction to the RPA results by decreasing the coefficient of the $k^{3/2}$ term in the plasma dispersion relation. This is one of the major points given by the SCFA because it takes into account the correlation effects which are relevant in this system. The first attempt to investigate correlations in such a two-dimensional electron gas was recently made by Beck and Kumar.[5] Their results showed a decrease in the coefficient of the $k^{3/2}$ term of the order of λ_T, the termal wavelength, which obviously vanishes in the strictly classical limit ($\hbar \to 0$).

In order to determine the plasma dispersion relation at arbitrary k, Eq. (19) has been numerically solved for almost real roots. In Fig. 2 the results for the real part of the plasmon modes for $\alpha = 1$ and $\alpha = 20$ are shown in comparison with those obtained earlier by Platzman and Tzoar[17] from

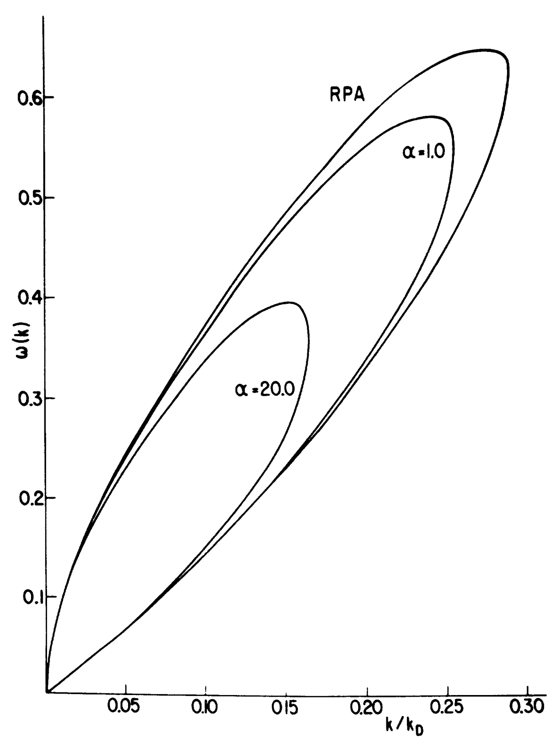

FIG. 2. Plot of the real part of the plasma dispersion curves for two values of the plasma parameter α, in units of $\omega_0 = (2\pi n e^2 k_D/m)^{1/2}$. The RPA curve corresponds to that given earlier (Ref. 17).

the RPA. From Eq. (19) one can immediately find that there are no real roots for

$$k > 0.284[1 - G(k)] .$$

As can be seen from Fig. 2 the plasmon dispersion yields two different branches. The upper one corresponds to the plasmon branch very closely to the zero-temperature results where the Coulomb interactions are the major contributions to the system (cold plasma). The lower branch is related to a soundlike mode with a phase velocity near the electron thermal velocity. In this case the thermal energy contribution dominates the electrostatic energy part (hot plasma).

V. DYNAMIC STRUCTURE FACTOR

As another application of the SCFA we have calculated the dynamic structure factor $S(\vec{k}, \omega)$ for the two-dimensional classical electron system. As it is known, the dynamic structure factor is the Fourier transform of the time-dependent density-density correlation function and plays the central part in formulating a theory of many-body systems. It is, through inelastic scattering experiments, a directly observable quantity.

By the fluctuation-dissipation theorem,[18] the dynamic structure factor for a two-dimensional classical electron gas is given by

$$S(\vec{k}, \omega) = -(T/n\pi\omega)\,\mathrm{Im}\chi(\vec{k}, \omega) , \tag{23}$$

which, in the SCFA takes the following form:

$$S(\vec{k}, \omega) = \frac{-T}{\pi n \omega} \frac{\mathrm{Im}\chi_0(\vec{k}, \omega)}{[1 - \psi(\vec{k})\,\mathrm{Re}\chi_0(\vec{k}, \omega)]^2 + [\psi(\vec{k})\,\mathrm{Im}\chi_0(\vec{k}, \omega)]^2} , \tag{24}$$

where $\psi(\vec{k}) = \phi(\vec{k})\,[1 - G(\vec{k})]$ is the self-consistent effective potential. We recall that in the RPA, $G(k) = 0$ and $\psi(\vec{k})$ reduces to the bare particle-particle interaction.

For a classical ideal electron gas, i.e., $\phi(\vec{k}) = 0$, the dynamic structure factor $S^0(\vec{k}, \omega)$ assumes, at fixed k, a Gaussian form around $\omega = 0$ given by

$$S^0(\vec{k}, \omega) = -\frac{T}{\pi n \omega}\,\mathrm{Im}\chi_0(\vec{k}, \omega) = \left(\frac{m}{2\pi T}\right)^{1/2} \frac{1}{k} \exp\left(-\frac{m\omega^2}{2k^2 T}\right) . \tag{25}$$

This behavior is significantly different for an interacting classical electron system, where $S(\vec{k}, \omega)$ can assume large values as $\mathrm{Im}\chi_0(\vec{k}, \omega)$ becomes large as

$$\phi(\vec{k})[1 - G(\vec{k})]\,\mathrm{Re}\chi_0(\vec{k}, \omega) = 1 ,$$

corresponding to the zeros of $F(\vec{k}, \omega)$, Eq. (19).

For $k \ll k_D$ the relevant contribution to $S(\vec{k}, \omega)$ arises from the collective modes of the plasma,

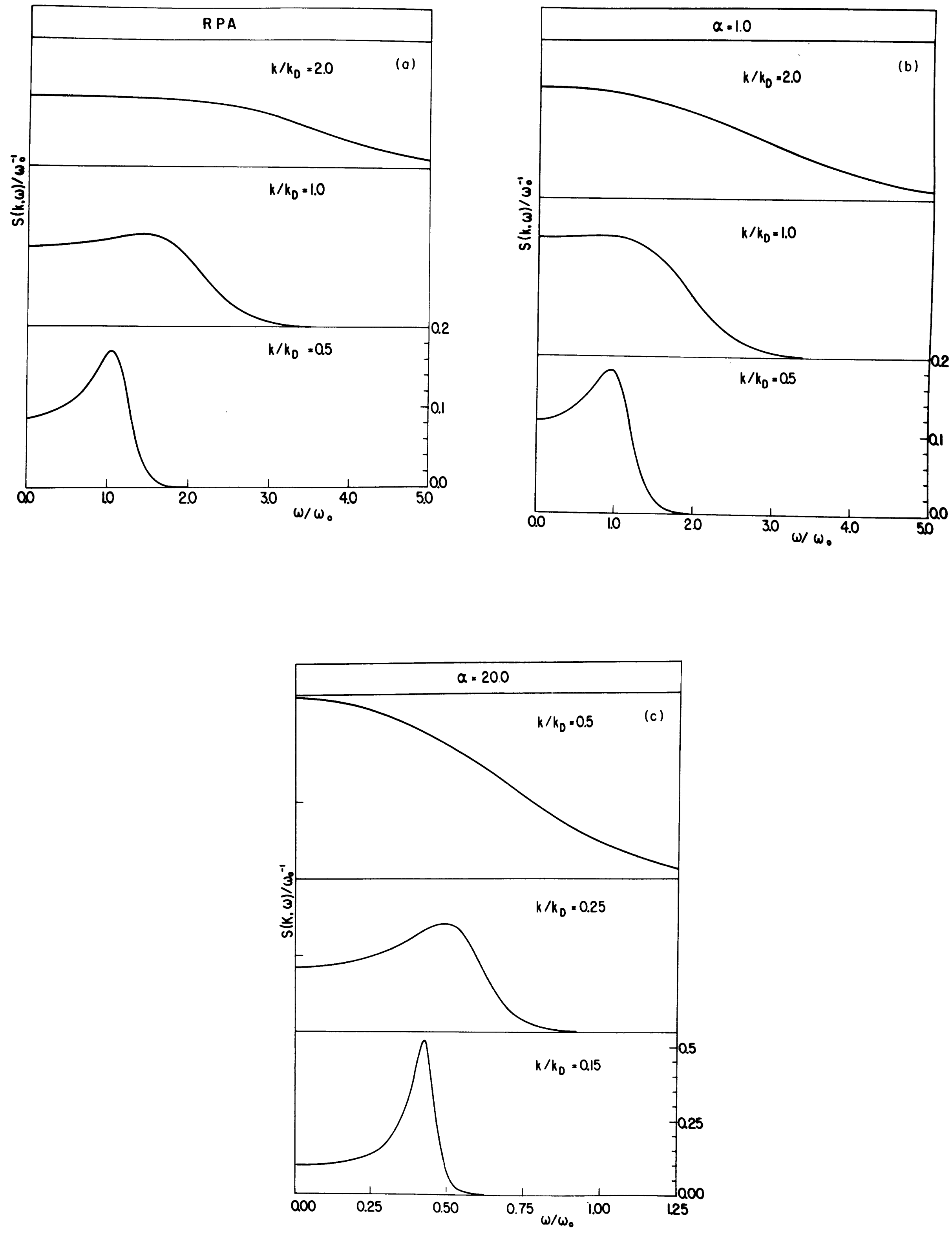

FIG. 3. Plot of the dynamic structure factor $S(\vec{k},\omega)/\omega_0^{-1}$ vs ω/ω_0 for fixed values of k/k_D at two values of the plasma parameter $\alpha = 1.0$ and $\alpha = 20.0$, and the RPA curve.

acquiring a peak at the plasma frequency $\omega(\vec{k})$. Expanding Eq. (24) near $\omega=\omega(\vec{k})$ we get the Lorentzian form for $S(\vec{k},\omega)$,

$$S(\vec{k},\omega)=-\frac{k}{2\pi k_D}\frac{\Gamma(\vec{k})}{[\omega-\omega(\vec{k})]^2+\Gamma^2(\vec{k})}, \tag{26}$$

where $\omega(\vec{k})$ and $\Gamma(\vec{k})$ are given by Eq. (20) and Eq. (21), respectively.

In the limit $k\gg k_D$ the classical electron plasma behaves like a system of free individual particles with $S(\vec{k},\omega)\approx S^0(\vec{k},\omega)$.

In Fig. 3 we plot $S(\vec{k},\omega)$ as a function of ω, for several values of the wave number k, at $\alpha=1.0$ and $\alpha=20.0$. For comparison we have also plotted the RPA results. As it is seen at low densities, i.e., small α, our results agree with those of the RPA. We also can see that $S(\vec{k},\omega)$ is dominated by the contribution of the collective mode for small k. The plasma oscillation decreases with increasing k. Unfortunately, there are no experiments carried out for a two-dimensional classical electron system for confirmation of our results which has been deduced from purely theoretical considerations.

VI. CONCLUSIONS

In the present paper we have applied the self-consistent-field approximation which includes the short-range correlation effects, to investigate the electrodynamic properties of a classical two-dimensional electron gas. The numerical results for the screening density around a fixed charged impurity in such a classical system were obtained showing significative differences from those given by the RPA. In particular, we found that the system is electrodynamically unstable for the plasma parameter $\alpha=\alpha_c\gtrsim 1/\gamma$. As mentioned earlier this system becomes also thermodynamically unstable at the same critical value of α. Far away the charged impurity the screening density is formally the same as obtained by the RPA with a renormalized Debye screening constant. It should be stressed that none of the calculations is reliable very close the impurity since the linear theories are not valid in this region. Moreover, our results for the plasma dispersion relation for a classical two-dimensional plasma oscillations mark a definite improvement over the expressions given by earlier methods. It is interesting to notice that our dispersion relation calculation includes for the first time the effects of the short-range correlation between the particles of this classical system. Numerical calculations of the dynamic structure factor were performed and the results agree with those of the RPA at very low densities.

The plasma oscillation and the importance of the collective excitation decreases with increasing the wave number. As it was pointed out by Grimes,[19] any experiment for a classical two-dimensional one-component plasma, that could measure the structure factor would be very useful to confirm the theoretical predictions.

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