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Stopping power of matter for alpha particles at extreme relativistic energies

M. L. Rustgi and J. E. Turner
Health and Safety Research Division, Oak Ridge National Laboratory, Oak Ridge, Tennessee 37830

P. T. Leung
Physics Department, State University of New York, Buffalo, New York 14260
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The stopping power of matter for alpha particles at extreme relativistic energies has been calculated by incorporating the charge form factor. A table is presented for aluminum, copper, and lead. It is found that at the highest energies considered here, inclusion of form factor reduces the mass stopping power by about 6–8 % in comparison with that predicted by Bethe's relativistic formula.

The stopping power of matter for heavy charged particles (muon, proton, etc.) is described by Bethe's relativistic formula[1,2]

$$-dE/ds=\kappa[ln(2\gamma^2mv^2/I)-\beta^2]. \quad (1)$$

Here $\kappa=4\pi z^2e^4NZ/mv^2$, where ze and v are the charge and speed of the incident particle, $-e$ is the electronic charge, NZ represents the number of electrons per unit volume in the medium, $\beta=v/c$ is the speed of the particle relative to the speed of light, $\gamma^2=1/(1-\beta^2)$, and I is the mean excitation energy of the medium. In deriving Eq. (1), the theory is simplified by assuming that

$$\gamma m/M \ll 1, \quad (2)$$

where M is the rest mass of the incident particle. In this case the heavy particle can lose only a small fraction of its energy in a single atomic collision. The dependence of the stopping power on speed is then the same for all particles, as expressed by Eq. (1). Condition (2) breaks down at very high energies when γ becomes large. The stopping power then depends on other factors such as M, the particle's spin and internal structure. The last property can be expressed by means of the form factors for the distribution of charge and magnetic moment. At extreme relativistic energies the stopping-power formula depends on the particular particle under consideration. In this paper we calculate the stopping power for energetic alpha particles without employing restriction (2). Earlier companion papers have treated the muon and proton[3] and the deuteron.[4]

The differential cross section for the scattering of an electron at an angle θ from an α particle at rest may be written as

$$\frac{d\sigma}{d\Omega}=\left(\frac{ze^2}{2\gamma mv^2}\right)^2\frac{\cos^2\frac{1}{2}\theta}{\sin^4\frac{1}{2}\theta}\,\frac{F_E^2(q^2)}{[1+(2\gamma m/M_\alpha)\sin^2\frac{1}{2}\theta]}, \quad (3)$$

where $\hbar q$ is the magnitude of the change in the electron's (= alpha particle's) energy-momentum four vector, M_α is the mass of the alpha particle, and F_E is the charge form factor of the alpha particle. This factor is related to the bare form factor $F_B(q^2)$ through the equation

$$F_E(q^2)=F_B(q^2)\times F_{ES}(q^2), \quad (4)$$

where

$$F_{ES}=F_{Ep}+F_{En} \quad (5)$$

is the isoscalar form factor. The bare form factor is related to the Fourier transform of the squared α-particle wave function, i.e.,

$$F_B=\frac{1}{2(1+C^2)}\int|\psi_s+C\psi_D|^2\times\left(\exp\frac{i}{2}\vec{q}\cdot(\vec{u}-\sqrt{2}\,\vec{w})+\exp\frac{i}{2}\vec{q}\cdot(\vec{u}+\sqrt{2}\,\vec{w})\right)d\vec{u}\,d\vec{v}\,d\vec{w}, \quad (6)$$

where

$$\vec{u}=\tfrac{1}{2}(\vec{r}_3+\vec{r}_4-\vec{r}_2-\vec{r}_1), \quad (7)$$

and

$$\vec{v}=(\vec{r}_2-\vec{r}_1)/\sqrt{2}, \qquad \vec{w}=(\vec{r}_4-\vec{r}_3)/\sqrt{2}. \quad (8)$$

Here $\vec{r}_1$ and $\vec{r}_2$ denote the neutron and $\vec{r}_3$ and $\vec{r}_4$ the proton position vectors, and ψ_S and ψ_D are the wave functions of the admixture of the $^1S_0$ and $^5D_0$ states of the alpha particle.

By carrying out the integrations, it has been shown by Singh *et al.*[5] that

$$F_B=\frac{1}{(1+C^2)}\left(\frac{1}{(1+3q^2/64\alpha^2)^5}+\frac{C^2(1-\frac{19}{192}q^2/\beta^2)}{(1+3q^2/64\beta^2)^8}\right), \quad (9)$$

TABLE I. Values of $F_B^2 F_{ES}^2$ for various values of Q.

Q	Fitted	Calculated from Eqs. (9) and (10)
0	1.0	1.0
100	0.1254	0.1254
200	2.0787×10^{-2}	2.0771×10^{-2}
300	4.2608×10^{-3}	4.2565×10^{-3}
400	1.0314×10^{-3}	1.0314×10^{-3}
500	2.8546×10^{-4}	2.8618×10^{-3}
600	8.8203×10^{-5}	8.8815×10^{-5}
700	2.9882×10^{-5}	3.0280×10^{-5}

where $\alpha = 0.841\times 10^{13}$ cm^{-1}, $\beta = 1.365\times 10^{13}$ cm^{-1}, and $C = -0.153$. According to Dudelzak[6]

$$F_{ES} = \frac{2.28}{(1+0.063\,45q^2)} - \frac{1.28}{(1+0.037\,39q^2)}\,. \quad (10)$$

In order to calculate the extreme relativistic contributions to this stopping-power formula, let us denote the energy lost by the alpha particle in a single collision by

$$Q = \hbar^2 q^2/2m\,. \quad (11)$$

It is easily shown that the maximum energy Q_m that can be lost is given by ($m/M_\alpha \ll 1$):

$$Q_m = 2\gamma^2 mv^2/(1+2\gamma m/M_\alpha)\,. \quad (12)$$

For $q \leqslant 4F^{-1}$ ($F = 10^{-13}$ cm), $Q_m \lesssim 610$ GeV, and $\gamma \leqslant 10^4$. We now express each of the factors in Eqs. (3), (9), and (10) in terms of Q, the alpha particle's energy loss. It is found that

$$d\sigma = 2\pi\frac{z^2e^4}{mv^2}\frac{dQ}{Q^2}\left(1-\frac{Q\beta^2}{Q_m}\right)\sum_{i=1}^{2}\frac{A_i}{(1+a_iQ)^{12}}\,. \quad (13)$$

In writing (13) we have fitted $F_B^2F_{ES}^2$ by

$$\sum_i \frac{A_i}{(1+a_iQ)^{12}}\,,$$

with $a_1 = 1.787\times 10^{-3}$ GeV^{-1}, $a_2 = 1.7\times 10^{-3}$ GeV^{-1}, $A_1 = 9.2099$, and $A_2 = 1 - A_1$. The accuracy of the fit can be seen from Table I.

The contribution of distant collisions to the stopping power of a medium is given by

$$-\left(\frac{dE}{ds}\right)_{Q<\eta} = \tfrac{1}{2}\kappa[\ln(2\gamma^2mv^2\eta/I^2) - \beta^2]\,, \quad (14)$$

where η is an intermediate value of energy loss.[2,7] The contribution of close collisions is given by

$$-\left(\frac{dE}{ds}\right)_{Q>\eta} = NZ\int_\eta^{Q_m} Q\,d\sigma\,.$$

Multiplying both sides of (13) by NZQ, integrating, and adding the result to $-(dE/ds)_{Q<\eta}$, we find for the stopping power ($a_i\eta \ll 1$)

$$-\left(\frac{dE}{ds}\right) = \kappa\left[\ln\frac{2\gamma^2mv^2}{I} - \frac{\beta^2}{2} - \frac{1}{2}\sum_i A_i\ln(1+a_iQ_m) + \frac{1}{2}\ln\frac{M_\alpha}{M_\alpha+2\gamma m} - \frac{1}{2}\left(\sum_{r=1}^{11}\frac{1}{r} + \frac{\beta^2}{11Q_m}\sum_i\frac{A_i}{a_i}\right) + \sum_{r=1}^{11}\sum_i\frac{A_i}{2r(1+a_iQ_m)^r} + \sum_i\frac{\beta^2}{22Q_m}\frac{A_i}{a_i}\frac{1}{(1+a_iQ_m)^{11}}\right]. \quad (15)$$

This formula is valid when $\gamma \lesssim 10^4$ and can be simplified if $a_iQ_m \ll 1$. In this approximation ($a_iQ_m \ll 1$), (15) becomes

$$-\left(\frac{dE}{ds}\right) = \kappa\left(\ln\frac{2\gamma^2mv^2}{I} - \beta^2 + \frac{1}{2}\ln\frac{M_\alpha}{M_\alpha+2\gamma m} - 6\sum_i A_ia_iQ_m\right). \quad (16)$$

Equations (15), (16), and (1) were used to calculate the mass stopping power $(-1/\rho)dE/ds$ for Al, Cu, and Pb. The mean ionization potentials I for these elements were taken to be 163, 316, and 825 eV, respectively. The results are listed in Table II. It is seen that at higher energies ($\gamma \sim 1000$), the form-factor effects decrease the mass stopping power by about 6–8%. The above calculations were also carried out with the wave function of Jain and Srivastava[8] without any significant difference in the results.

TABLE II. Mass stopping power of Al, Cu, and Pb for alpha particles at extreme relativistic energies.

γ	Alpha energy (GeV)	Al Eq. (15)	Al Eq. (1)	Al Eq. (16)	Cu Eq. (15)	Cu Eq. (1)	Cu Eq. (16)	Pb Eq. (15)	Pb Eq. (1)	Pb Eq. (16)
10	37.3	7.385	7.386	7.385	6.620	6.621	6.620	5.268	5.270	5.268
50	186	9.210	9.222	9.202	8.353	8.364	8.344	6.775	6.785	6.768
100	373	10.001	10.041	9.967	9.102	9.139	9.069	7.425	7.458	7.397
250	932	10.945	11.125		9.996	10.166		8.201	8.348	
500	1864	11.503	11.945		10.525	10.944		8.659	9.022	
750	2796	11.777	12.426		10.783	11.399		8.883	9.417	
1000	3728	11.959	12.767		10.956	11.722		9.033	9.697	

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