PHYSICAL REVIEW A VOLUME 22, NUMBER 5 NOVEMBER 1980

Electronic K x-ray energies in heavy muonic atoms

H. Schneuwly
Institut de Physique de l'Université, CH-1700 Fribourg, Switzerland

P. Vogel
California Institute of Technology, Pasadena, California 91125
(Received 30 May 1980)

Experimentally determined energy shifts of the electronic K x rays in heavy muonic atoms are compared with calculations. These shifts may be caused by the incomplete screening of the nuclear charge by the muon and, in addition, by electron vacancies produced in higher shells by preceding Auger transitions. The first mechanism alone explains the experimental data within their uncertainties. This agreement and the absence of a noticeable difference between the shifts of the electronic $K\alpha$ and $K\beta$ x rays, show that the inner shells are essentially instantaneously refilled during the muonic cascade. Precision measurements of this type would give supplementary information on the initial muon distribution over l states.

I. INTRODUCTION

In muonic atoms as well as in all exotic atoms the interplay of the electrons and the negatively charged particle is very important. Not only is the very existence of such exotic atoms dependent on the inelastic meson-electron collisions, but virtually all features of the mesic atom are affected by the behavior of the electron cloud surrounding the bound meson-nucleus system. The intensities of the exotic x rays depend on the competition between the radiative and the Auger processes; their energies are shifted due to the screening of the electron cloud; the muon polarization is affected by unsaturated electron spins, etc. Therefore, a detailed understanding of the exotic atoms is impossible without a rather detailed treatment of the surrounding electron cloud.

In this paper we analyze a particular aspect of this problem, the refilling of the inner-electron-shell vacancies created by preceding Auger transitions. It is generally agreed that the muon is captured into a state with large main quantum number. It proceeds then to states with smaller quantum numbers, initially almost exclusively by Auger transitions with small change ($\Delta n = 1, 2$) of the principal quantum number. The electron shells with binding energies nearest to the transition energy contribute most to the Auger transition rate. Thus electrons from different shells are ejected at the various stages of the muon cascade: the M electrons near $n=25$, the L electrons near $n=15$ and the K electrons near $n=7$. The refilling rate of, say, an electron K vacancy is obviously proportional to the number of $2p$ electrons present at that instant, which in turn depends on the previous history of the L shells, etc.

The refilling rate for the various electron shells may be deduced from experiment in several indirect ways. Using the existing cascade programs one can try to fit the electron K-refilling rate to the measured mesic x-ray intensities. For low-Z muonic atoms, such fits give, typically, electron K-refilling rates substantially smaller than the corresponding rate in a normal atom.[1] Another method is based on accurate measurements of the mesic x-ray energies and deduced electron screening corrections. A reduction of the electron screening is interpreted as an absence of electrons. In such a way it has been shown that the electron shells of muonic lead are virtually undisturbed.[2,3] On the other hand, only one K electron seems to be present in muonic silicon.[4] A special case of particular interest is the pressure dependence of muonic x-ray intensities in gas targets. There the refilling rate of the electron shells is correlated to the collision frequency.[5]

In this paper we report results of the experimental determination of shifts of the prompt electron K x rays in muonic atoms. Together with the previously reported results,[6,7] we now have data for seven atoms with $Z > 75$. We analyze the implications of these shifts for the refilling process described above.

The prompt electronic K x rays, which are the subject of this paper, are emitted when an electron K vacancy, created in the preceding Auger transition, is refilled. The muon is then still in orbit and screens almost completely one unit of nuclear charge. The energy of such x rays is, therefore, close to the energy of the ordinary electronic K x rays of the $(Z-1)$ atom. The difference between the energies of these prompt K x rays and the electronic K x rays of the ordinary $(Z-1)$ atom, contained in the delayed spectrum, is the observable quantity. There are two possible sources of such a difference. First, the screening of the nuclear charge by the muon is not complete. The $1s$ electron partially penetrates the muon orbit and its binding energy is, therefore, larger than in the $(Z-1)$ atom. All electronic K x rays are then shifted by the same amount toward higher energies. The amount of shift determines the muon orbit at the instant of the electronic x-ray emission, and thus the electron K-shell refilling rate. Second, besides the $1s$ electron vacancy, there might be other inner-shell electronic vacancies left from the preceding cascade. They will affect the self-consistent atomic field and cause an additional shift. Such shifts will be different for the electronic $K\alpha$ and $K\beta$ x rays. Thus, comparison of these two x-ray energies gives direct evidence about the L- and M-shell refilling rates.

II. EXPERIMENTAL RESULTS

A. Measurement

The measurements have been performed at the muon channel of the synchro-cyclotron (SC) at CERN by the Berlin-Darmstadt-Fribourg-ETHZ collaboration. The main purpose of the experiments on muonic Os, Ir, Hg, and Tl was to determine other properties of these muonic atoms,[8-11] and the electronic x rays were a by-product. Prompt, delayed, and calibration spectra were registered simultaneously with a conventional setup. The detector for the electronic x rays was a planar 1 cm^3 Ge(Li) detector. Its energy resolution was about 700 eV at 120 keV. Two time windows on the time to amplitude converter (TAC) spectra selected "prompt" events (30 ns) and "delayed" events (100–200 ns) with respect to a muon stop event defined by the telescope counters. The time resolution of the system was about 15 ns. As a consequence about 5% of the prompt events appeared in the delayed spectra. Due to the finite time window, part of the delayed events appeared also in the prompt spectra (Figs. 1 and 2). The prompt spectra contain electronic K x rays of the ordinary Z atom, excited by the beam, and the prompt electronic K x rays of the muonic μZ atom, emitted during the cascade. The delayed spectrum contains the electronic K x rays of the ordinary $Z-1$ atom, emitted during the deexcitation following nuclear muon capture. It also contains electronic K x rays of the ordinary Z atom externally excited by various products of muon capture.

For identification and rough energy calibration of the interesting lines, we used the $\mu C(np\text{-}1s)$ transitions, which appeared always as background lines from muons stopped in the telescope counters, and the $\mu N(2p\text{-}1s)$ transition due to muons stopped in the air.[12]

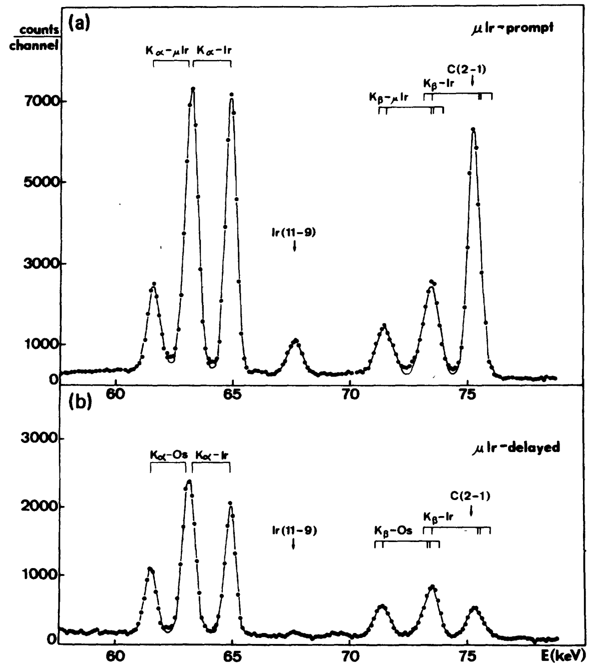

FIG. 1. Part of the spectra of prompt and delayed events in muonic Ir containing the electronic x rays. In the prompt spectrum appear the K lines of Ir and μIr, in the delayed spectrum those of Ir and its daughter atom Os. With the prompt muonic transitions Ir(11-9) and C(2-1) appearing in both spectra one gets an idea of the supression of prompt events in the delayed spectrum.

B. Analysis

In the detailed analysis of the data with a computer-fit program, one encounters several problems. The electronic K x rays of the Z atom partially overlap with the electronic K x rays of the daughter $(Z-1)$ atom in the delayed spectra and with the electronic K x rays of the muonic μZ atom in the prompt spectra. For iridium, e.g., the $K\alpha_2$ line of Ir(Z) has an energy of 63.29 keV and the $K\alpha_1$ line of Os$(Z-1)$ an energy of 63.00 keV. These lines are not resolved by the Ge(Li) detector (Fig. 1). In addition, the exact line shape is not precisely known and the parameters of the line shape,[8-13] which can be determined using other lines, e.g., from the calibration spectra, cannot be unconditionally used for these energetically lower lines because, among other reasons, of different absorption in the target and the detector window. The influence of the natural linewidth upon the line shape can be neglected, because the energy resolution of the detector is approximately ten times larger. However, our goal was to determine only the relative positions of lines of the same shape near each other, so that the exact line shape was

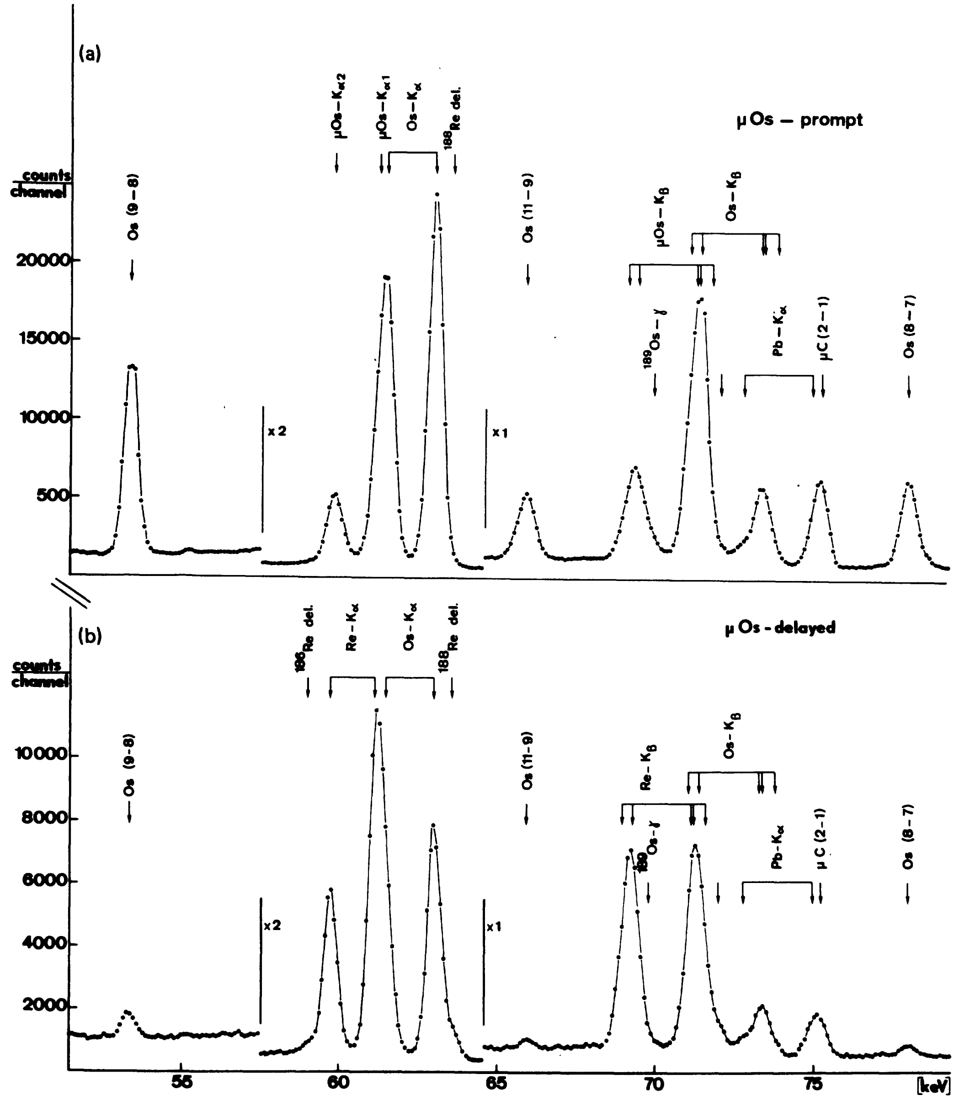

FIG. 2. Part of the prompt and delayed spectra of muonic Os containing the electronic x rays. In addition to the K lines of Os, μOs, and Re there appear also delayed γ rays from Re which were used, in addition to the muonic transitions in C, as energy calibration lines. Pb-$K\alpha$ lines from the shielding are also visible. An unknown prompt line at about 70 keV might be an isomer shifted nuclear transition in ^{189}Os.

only of minor importance.

Because some of the electronic K x rays of element Z were not resolved by the detector from the corresponding electronic K x rays of element $(Z-1)$ and the electronic K x rays of muonic atom Z, one had to correlate positions of lines in the fitting procedure. The slope of the energy calibration (energy/channel) was determined by the muonic x-ray transitions from carbon and nitrogen, and the relative positions were correlated to the energies. In the fit, the energy splittings of the $K\alpha$ and $K\beta$ lines were fixed for the transitions in atom Z, $(Z-1)$, and μZ. These energy splittings were taken from the tables of Storm and Israël.[14] For the μZ atom, the energy splittings of the $(Z-1)$ atom were used. With this fitting procedure, the experimental energy difference between the electronic K x rays of atom Z and those of atom $(Z-1)$ agreed, in the delayed spectra, within less than 13 eV, with the tabulated ones.[14]

In addition to the correlation in energy, the two $K\alpha$ lines and, separately, the seven main $K\beta$ lines of each element were correlated in intensity according to the same tables.[14] For the muonic atom μZ, the intensities were correlated according to the element $(Z-1)$ even if the intensity ratios might well be slightly different because of vacancies in the higher electron shells. In order to test the accuracy of the determination of the energy shifts, we have fitted the two $K\alpha$ lines of μZ without correlating them either in energy or in intensity, while keeping the correlations of $K\alpha$ lines of Z. The fitted intensities of μZ were then not very different from the tabulated ones and the energy shift of the $K\alpha$ lines (the lower of the two) agreed within 10 eV with that determined in a correlated fit. The uncertainty of 20 eV in the fitted energy shifts for the $K\alpha$ lines in muonic μZ seems, therefore, to be rather conservative.

The linewidths of the fine-structure components of the $K\alpha$ and $K\beta$ lines of each element were correlated in the fits. The analysis showed that for the $K\alpha$ lines the linewidths for elements Z and $(Z-1)$ were the same, but for the muonic μZ they were systematically broader by about 80 eV for μOs and μIr and about 130 eV for μHg and μTl. Because of the complexity of the spectra and insufficient statistics, a broadening of the corresponding $K\beta$ lines could not be seen in μOs and μIr. In μHg and μTl, however, the broadening could well be of the order of 200–250 eV; the χ^2 value of the fits is actually improved by 30% when one does not correlate these linewidths to the widths of the other lines. However, the complexity of the spectra did not allow us to determine the precision of this broadening. The cause of the broadening has not been determined.

C. Results

The energy differences of the electronic K x rays of the element Z and the muonic μZ were determined by the fits. These differences were subtracted from the differences in the tabulated energies[14] of the electronic K x rays between element Z and the daughter element $(Z-1)$ produced by nuclear muon capture. The energy shifts ΔE, for Os, Ir, Hg, and Tl are, therefore, differences of differences of energies. As an example, in Os the measured energy difference of the electronic $K\alpha_2$ x rays between Os and μOs was 1652 eV, whereas the tabulated energy difference for the same x rays in Os(Z) and Re($Z-1$) is 1767 eV. The difference between these two, 115 eV, appears in Table I.

III. THEORETICAL ANALYSIS

In this section we essentially follow the approach developed earlier in Ref. 15. In the muonic atom with nuclear charge Ze, the potential felt by the electrons is the combined nuclear plus muon potential of the form

$$V(r) = -\frac{Ze}{r_+} e\left(\frac{1}{r}\int_0^r \rho_\mu(t)t^2\,dt + \int_r^\infty \rho_\mu(t)t\,dt\right), \qquad (1)$$

TABLE I. Experimental and calculated electronic K x-ray shifts ΔE.

		ΔE (eV), Meas.	ΔE (eV), Calc., Eq. (5)
$_{76}$Os	$K\alpha$	115 ± 20	159
	$K\beta$	131 ± 40	
$_{77}$Ir	$K\alpha$	118 ± 20	165
	$K\beta$	139 ± 40	
$_{80}$Hg	$K\alpha$	206 ± 20	185
	$K\beta$	254 ± 40	
$_{81}$Tl	$K\alpha$	206 ± 20	192
	$K\beta$	229 ± 40	
$_{82}$Pb	$K\alpha_2$	286 ± 43[a]	200
	$K\beta$	310 ± 100[a]	
$_{90}$Th	$K\alpha_2$	334 ± 66[a]	270
$_{92}$U	$K\alpha_1$	329 ± 76[b]	290
	$K\alpha_2$	382 ± 51[b]	

[a] Data of Ref. 7.
[b] Data of Ref. 6.

where $\rho_\mu(r)$ is the spherically symmetric muon density. Note that in Eq. (1) we assume for simplicity a point nucleus; in our numerical calculations, however, we use a proper nuclear charge distribution. The second term in Eq. (1) can be transformed to the form

TABLE II. Shifts (in eV) of the electronic 1s state and of the electronic K x rays, Eq. (4), emitted while the muon is in various n, l states.

Z	73	76	77	80	81	90	92
n, l							
1s	1	1	1	1	1	3	3
3d	10	12	13	15	15	24	27
4f	25	29	30	34	36	55	61
5g	50	57	59	67	70	104	114
5f	63	71	74	83	87	127	139
6h	86	96	100	113	117	171	186
6g	105	117	122	137	142	203	221
6f	120	133	138	155	160	228	247
7i	134	150	155	174	180	256	277
7h	160	177	184	205	212	297	321
7g	180	199	206	230	237	330	356
7f	196	216	224	248	257	355	383
8k	195	216	224	249	258	358	387
8i	226	250	258	286	296	406	437
8h	252	278	287	317	327	446	480
8g	273	300	310	342	353	479	514
9l	268	295	305	337	349	476	512
9k	304	334	344	379	392	529	567
9i	334	366	378	415	428	575	615
9h	359	394	406	445	459	613	655
9g	380	416	429	469	484	644	688
10l	385	424	438	481	496	662	707
10k	408	454	469	518	534	711	758
10i	419	473	491	546	564	752	802

$$\frac{e}{r}+e\int_r^\infty \rho_\mu(t)\left(t-\frac{t^2}{r}\right)dt \equiv \frac{e}{r}+V_\mu(r). \tag{2}$$

The attractive potential $V_\mu(r)$ characterizes the deviation of the muon potential from the potential of the point charge at the origin. Potential $V_\mu(r)$ obviously vanishes outside the muon orbit.

Neglecting the small corrections due to the self-consistency requirements, it is this potential $V_\mu(r)$ which distinguishes the potential of an ordinary $(Z-1)$ atom from the potential of a muonic μZ atom. We have verified numerically that the perturbation treatment of $V_\mu(r)$ agrees with the complete self-consistent treatment to an accuracy better than 5 eV in the electronic K x-ray energy.

For muon quantum numbers $n<14$ the potential $V_\mu(r)$ affects primarily the electronic s states. Consequently, all electronic K x rays emitted by the muonic atom (for a given set of muon quantum numbers n, l, j) are shifted by the same amount:

$$\Delta E_{n,l,j}=E_K^\mu - E_K^{Z-1} \simeq -e\int_0^\infty V_\mu(r)\rho_{1s}(r)r^2dr>0\,. \tag{3}$$

For practical calculations it is advantageous to use Green's theorem and change Eq. (3) to the form

$$\Delta E_{n,l,j}=-e\int_0^\infty \rho_\mu(r)V_{1s}(r)r^2dr\,. \tag{4}$$

Here, $\rho_\mu(r)$ is the corresponding muon density and $V_{1s}(r)$ is the screening potential of a single $1s$ electron. To a high degree of accuracy one may simply use the $1s$ wave function of the $(Z-1)$ atom in evaluating $V_{1s}(r)$ and, respectively, $E_{n,l,j}$. Table II shows the resulting shifts for a variety of muonic states.

In order to compare the observed electronic K x-ray shifts with calculations one has to consider the following three questions:

(a) When in the muonic cascade are the electron K vacancies produced?

(b) How fast are they refilled?

(c) Are there any additional electron vacancies present, and if yes, how much do they affect the electronic K x rays?

We used the muonic cascade program[16] to solve problem (a). Typical results are illustrated in Fig. 3. The electron K vacancies are produced

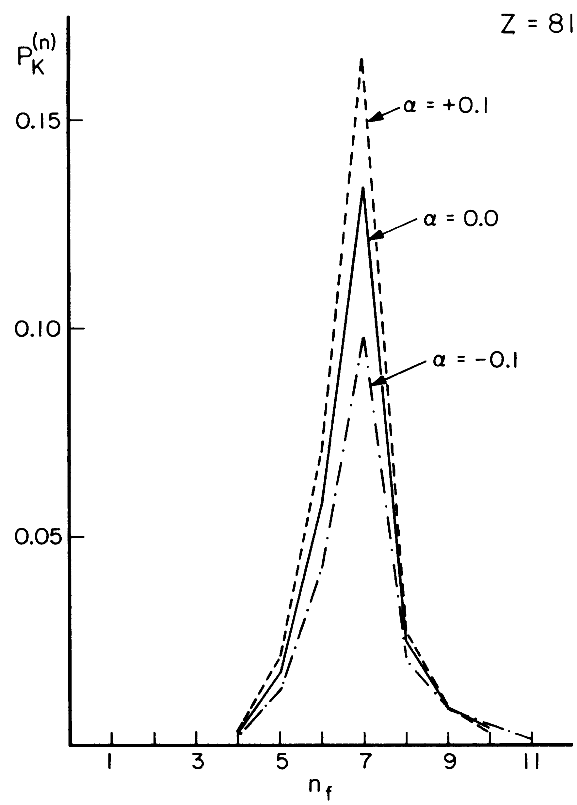

FIG. 3. Dependence of the K-electron Auger emission probability on the final muon principal quantum number n_f. The contributions of various l states have been added. The three lines correspond to different initial l distributions $(2l+1)e^{\alpha l}$ at $n=18$. The total number of ejected K electrons is calculated to be $P_K=0.32$ ($\alpha=+0.1$), 0.28 ($\alpha=0.0$), and 0.23 ($\alpha=-0.1$).

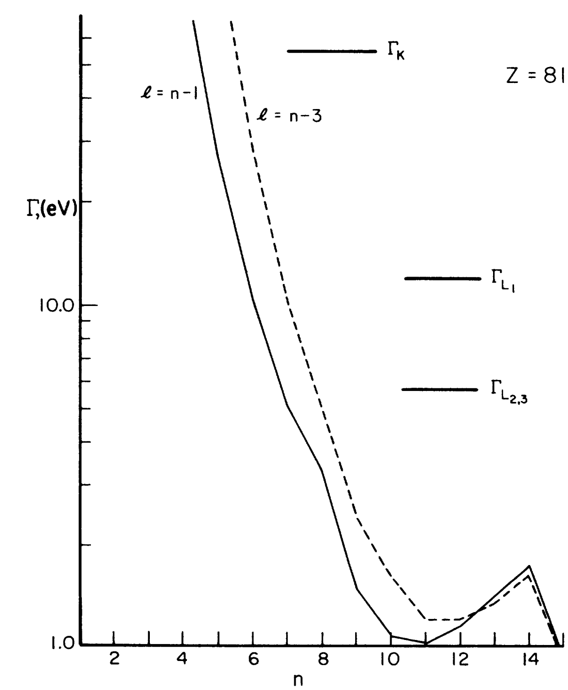

FIG. 4. Widths of several muonic (n,l) states in the $Z=81$ atom. For comparison, the normal atom widths of single electron vacancies[17] are shown as thick horizontal lines.

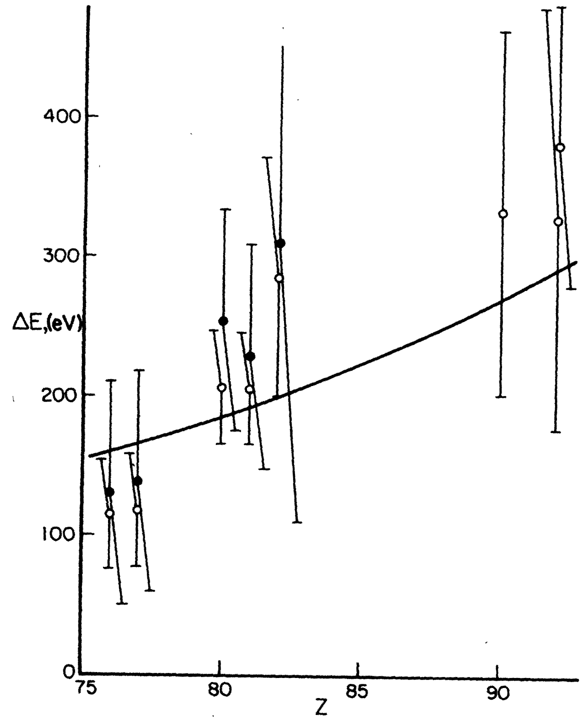

FIG. 5. The experimental x-ray shifts of electronic $K\alpha$ lines (empty circles) and $K\beta$ lines (filled circles) with their uncertainties. The solid line shows the calculated shifts [Eq. (5)].

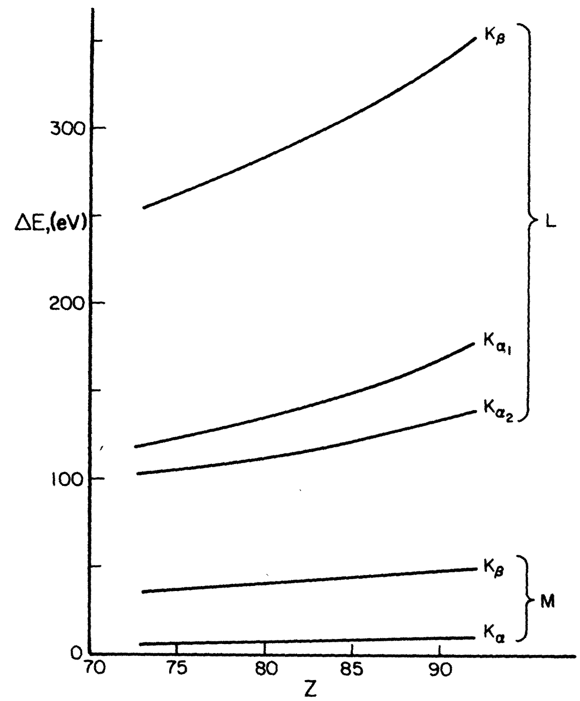

FIG. 6. The shift of various electronic K x rays caused by a single additional electron L or M vacancy.

mainly in the vicinity of $n=7$. The shape of the distribution in n_f only weakly depends on the shape of the initial muon l distribution. Note, however, that the total probability of the K-electron ejection increases quite steeply with increasing steepness of the initial muon l distribution, characterized by the parameter α ($P_l \sim (2l+1)e^{\alpha l}$). For $\alpha=0$ (statistical l distribution), the total K-vacancy production probability P_K decreases with increasing Z, from $P_K=0.38$ for $Z=73$, to $P_K=0.19$ for $Z=92$. Absolute experimental determination of the quantity P_K is obviously difficult, but if possible it would give information on the initial l distribution

To answer question (b) one has to compare the rates of muonic transitions with the rates of the electron refilling processes, i.e., x-ray emission. Figure 4 illustrates the situation. Note the irregularities in the muonic widths related to the opening of new channels near $n=14$ (L emission by $\Delta n=1$) and $n=8$ (K emission by $\Delta n=1$). The electron widths shown[17] are those of a normal atom; they are about ten times larger than the muonic widths for states where K or L electrons are ejected. Thus one may expect that in the heavy muonic atoms the electronic K x ray is emitted before the muon makes a subsequent transition. The expected K x-ray shift is then

$$\Delta E_K = \sum_{n,l} (\Delta E_{n,l} P_{n,l}) \Big/ \sum_{n,l} P_{n,l}\,, \qquad (5)$$

where $\Delta E_{n,l}$ is the shift of Eq. (4) (Table II), and $P_{n,l}$ is the K-electron vacancy probability, summed over all transitions having the quantum numbers (n, l) as the final state (see Fig. 3). The resulting shifts are shown in the last column of Table I and in Fig. 5. The calculation was performed for the initial muonic statistical l distribution. To judge the dependence on this assumption let us note that for $Z=81$ we obtain $\Delta E_K=205$ eV ($\alpha=-0.1$), 192 eV ($\alpha=0.0$), and 185 eV ($\alpha=0.1$).

Finally, we have to consider problem (c). One has to remember that in heavy muonic atoms two to three L electrons are ejected in the cascade between $n=16$ and $n=11$; even a larger number of M electrons is emitted earlier. Thus, remnants of these vacancies may still be present when the muon has reached $n=7$. Such vacancies change the self-consistent Hartree-Fock field in which the remaining electrons are moving. To calculate the effect we used the Dirac-Hartree-Slater program. The results are shown in Fig. 6. The effective vacancy used there simply means that each electron subshell is weighed with its statistical weight $(2j+1)$. The main feature of the results is the large difference between the $K\alpha$ and $K\beta$ shifts. The average experimental difference (see Table I) is (in eV)

$$\langle \Delta E_{K\beta} - \Delta E_{K\alpha} \rangle = 27 \pm 22 .$$

The near equality of $E_{K\alpha}$ and $E_{K\beta}$, therefore, means that no more than ~0.2 L vacancies (or 1 M vacancy) are present when the muon reached the $n = 7$ state.

IV. CONCLUSIONS

The experimental data and the theoretical analysis presented here show the usefulness of the prompt electronic K x rays as a probe of the muonic cascade. We have demonstrated that in the region spanning 16 units of nuclear charge the refilling times of the electron K, L, and M shells are not much longer than in corresponding normal atoms. The good agreement between the measured and calculated electronic $K\alpha$ x-ray shifts, and the absence of a noticeable difference between the shift of the electronic $K\alpha$ and $K\beta$ x rays, show that the inner-electron shells in these solid and metallic (with the exception of HgO) targets are essentially instantaneously refilled during the muonic cascade. This finding considerably simplifies analyses of the muonic x-ray intensities, electron screening, and other related quantities. It remains to be seen whether a similar treatment can be extended to somewhat lower Z values.

ACKNOWLEDGMENTS

This work was partially supported by U. S. DOE Contract No. DE-AC-03-76-ER00063 and the Swiss National Science Foundation. One of the authors (P.V.) would like to thank Professor O. Huber for hospitality extended to him during his stay in Fribourg.

[1]R. Bergmann, H. Daniel, P. Ehrhart, F. J. Hartmann, H. J. Pfeiffer, J. J. Reidy, T. von Egidy, and W. Wilhelm, contribution to the Eighth International Conference on High Energy Physics and Nuclear Structure, Vancouver, 1979, 1B3 (unpublished).

[2]P. Vogel, A. Zehnder, A. L. Carter, M. S. Dixit, E. P. Hincks, D. Kessler, J. S. Wadden, C. K. Hargrove, R. J. McKee, H. Mes, and H. L. Anderson, Phys. Rev. A 15, 76 (1977).

[3]E. Bovet, F. Boehm, R. J. Powers, P. Vogel, and K.-C. Wang, Phys. Lett. (in press).

[4]W. Ruckstuhl, B. Aas, W. Beer, I. Beltrami, P. Ebersold, R. Eichler, M. Guanziroli, Th. V. Ledebur, H. J. Leisi, and W. W. Sapp, contribution to The Eighth International Conference on High Energy Physics and Nuclear Structure, Vancouver, 1979, 1B22 (unpublished).

[5]J. D. Knight, C. J. Orth, M. E. Schillaci, R. A. Naumann, F. J. Hartmann, J. J. Reidy, and H. Schneuwly, contribution to The Eighth International Conference on High Energy Physics and Nuclear Structure, Vancouver, 1979, 1B17 (unpublished).

[6]R. Arlt, Dz. Gansorig, T. Krogulski, H. G. Ortlepp, S. M. Polikanov, B. M. Sabirov, W. D. Fromm, U. Schmidt, H. Schneuwly, and R. Engfer, Zh. Eksp. Teor. Fiz. Pis'ma Red. 20, 635 (1974) [JETP Lett. 20, 291 (1974)].

[7]W. D. Fromm, Dz. Gansorig, T. Krogulski, H. G. Ortlepp, S. M. Polikanov, B. M. Sabirov, V. Schmidt, R. Arlt, R. Engfer, and H. Schneuwly, Phys. Lett. 55B, 377 (1975).

[8]H. Backe, R. Engfer, U. Jahnke, E. Kankeleit, R. M. Pearce, C. Petitjean, L. Schellenberg, H. Schneuwly, W. V. Schröder, H. K. Walter, and A. Zehnder, Nucl. Phys. A189, 472 (1972).

[9]R. Link, L. Schellenberg, H. Backe, R. Engfer, E. Kankeleit, R. Michaelsen, H. Schneuwly, W. V. Schröder, J.-L. Vuilleumier, H. K. Walter, and A. Zehnder, Phys. Lett. 42B, 57 (1972).

[10]H. Backe, R. Engfer, E. Kankeleit, R. Link, R. Michaelsen, C. Petitjean, L. Schellenberg, H. Schneuwly, W. V. Schröder, J.-L. Vuilleumier, H. K. Walter, and A. Zehnder, Nucl. Phys. A234, 469 (1974).

[11]J.-L. Vuilleumier, W. Dey, R. Engfer, H. Schneuwly, H. K. Walter, and A. Zehnder, Z. Phys. 278, 109 (1976).

[12]T. Dubler, L. Schellenberg, H. Schneuwly, R. Engfer, J.-L. Vuilleumier, H. K. Walter, A. Zehnder, and B. Fricke, Nucl. Phys. A219, 29 (1974).

[13]T. Dubler, K. Kaeser, B. Robert-Tissot, L. A. Schaller, L. Schellenberg, and H. Schneuwly, Nucl. Phys. A294, 397 (1978).

[14]E. Storm and H. I. Israël, At. Data Nucl. Data Tables A7, 641 (1970).

[15]P. Vogel, Phys. Lett. 58B, 52 (1975).

[16]V. R. Akylas and P. Vogel, Comput. Phys. Commun. 15, 291 (1978).

[17]O. Kesti-Rathoven and M. O. Krause, At. Data Nucl. Data Tables A14, 139 (1974).