Some aspects of the Aharonov-Bohm effect

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It is shown that the Aharonov-Bohm scattering cross section diverges. This presents a problem since the range of the scattering force (Lorentz force) is finite. The cross section found by Aharonov and Bohm (AB) results whether one takes an incident wave function corresponding to particles moving in the incident direction, as was done by AB, or just takes the incident state to be a plane wave. The mechanism by which a localized force results in an infinite cross section is not clear. This is an apparent contradiction to recent proofs that electrodynamics is a local theory.

I. INTRODUCTION

In their well known papers that were written twenty years ago,[1] Aharonov and Bohm (AB) attributed the scattering of charged particles by a whisker of magnetic flux to the vector potential itself. The effect could be explained on this basis, and it was considered to be understood, even if it was somewhat unusual. However, within the past several years electrodynamics has been shown to be a strictly local theory.[2,3] This requires that the AB scattering be produced directly by the magnetic flux in the whisker. This requirement unfortunately cannot be reconciled with a result recently found by Purcell and Henneberger.[4] They considered the energy eigenstates of a free electron confined to the interior of a cylindrical volume. Transitions from one superposition of states to another induced by a whisker of magnetic flux along the Z axis were computed in first Born approximation. The paper gives a perturbative treatment of the AB effect. Free-particle solutions which are superposed have the form

$$\psi_{k,k_z,l}=J_l(k\rho)e^{il\varphi}e^{ik_zZ}. \qquad (1)$$

Transitions from the state

$$(1/\sqrt{2})(|k,k_z,l_a\rangle+|k,k_z,l_b\rangle)$$

to the state

$$(1/\sqrt{2})(|k',k_z',l_a\rangle-|k',k_z',l_b\rangle)$$

are considered, and the S matrix elements are found to be

$$S_{fi}=\frac{ie\hbar\Phi\pi^2k}{2mcRL}\left(\frac{l_a}{|l_a|}-\frac{l_b}{|l_b|}\right)\delta(k_z-k_z')\delta(E_f-E_i). \qquad (2)$$

Thus transitions are induced only between superpositions of eigenstates of L_z having eigenvalues of opposite sign. A more remarkable result, however, is the fact that S_{fi} depends only upon the relative sign of the angular momenta. States involving arbitrarily large angular momenta give the same result as states of small angular momenta.

This result cannot be reconciled with the explanation that the AB effect is caused by the penetration of the whisker of flux by the tail of the wave packet as proposed by Strocchi and Wightman,[2] since the Bessel functions are proportional to ρ^l near the Z axis, so that states of large angular momentum should be affected less than states of small angular momentum. This, however, is not the case, as one sees from Eq. (2).

The result of Eq. (2), therefore, provides the stimulus for further investigations of the AB effect as a scattering problem. It is interesting to consider the result that would be obtained by a person who wished to draw a conclusion about the range of the scattering force using only data obtained from scattering. To this end, we first review the method of partial waves in two dimensions.

II. TWO-DIMENSIONAL SCATTERING THEORY

We begin by considering scattering by a potential having cylindrical symmetry. The stationary scattering states have the asymptotic form

$$\psi_m=\left(\frac{2}{\pi kr}\right)^{1/2}\cos\left(kr-\frac{m\pi}{2}-\frac{\pi}{4}+\delta_m\right)e^{im\theta}, \qquad (3)$$

where δ_m is a phase shift that gives information about the scattering potential. Although the AB effect is not due to potential scattering, it is interesting to note that the stationary states found by AB have this asymptotic form. The scattering solutions in which a plane wave is incident in the x direction are given asymptotically by

$$\psi_{\text{asympt}}\sim e^{ikx}+f(\theta)e^{ikr}/\sqrt{r}, \qquad (4)$$

and these solutions are expressible as linear combinations of the solutions given in Eq. (3). By the usual argument, the differential scattering cross section (which has the dimension of length) is given by

$$\frac{d\sigma}{d\theta} = |f(\theta)|^2. \tag{5}$$

We define θ by $x = r\cos\theta$, so that $\theta = 0$ is in the forward direction, and make use of the relation

$$e^{ikr\cos\theta} = \sum_{m=-\infty}^{\infty} i^m J_m(kr) e^{im\theta}. \tag{6}$$

It is required to find scattering states

$$\psi^{(+)}_{(r,\theta)} = \sum_{m=-\infty}^{\infty} a_m \left(\frac{2}{\pi k r}\right)^{1/2} \cos\left(kr - \frac{m\pi}{2} - \frac{\pi}{4} + \delta_m\right) e^{im\theta} \tag{7}$$

that have the asymptotic form of Eq. (4). The requirement that there be no ingoing cylindrical waves leads to the condition

$$a_m = i^m e^{i\delta_m}, \tag{8}$$

and straightforward computation leads to the result

$$f(\theta) = \sum_{m=-\infty}^{\infty} \frac{1}{\sqrt{2\pi k}} e^{im\theta} e^{-i\pi/4}(e^{2i\delta_m} - 1). \tag{9}$$

This gives

$$\frac{d\sigma}{d\theta} = \left| \sum_{m=-\infty}^{\infty} \frac{1}{\sqrt{2\pi k}} e^{im\theta}(e^{2i\delta_m} - 1) \right|^2 \tag{10}$$

and

$$\sigma_{\text{total}} = \sum_{m=-\infty}^{\infty} \frac{4}{k} \sin^2\delta_m. \tag{11}$$

It is of some interest to investigate the form of the optical theorem in two dimensions. Unitarity of the S matrix, which has elements

$$S_{ij} = \delta_{ij} - 2\pi i \delta(E_j - E_i) T_{ij}, \tag{12}$$

leads to the relation

$$2\pi \sum_n \delta(E_n - E_i) T^*_{ni} T_{nj} = i(T_{ij} - T^*_{ji}). \tag{13}$$

It is now necessary to relate the scattering amplitude to the elements of the T matrix. The stationary scattering states are given by

$$\psi^{(+)}_{(\vec{r})} = -\frac{m}{2\pi\hbar^2} \int G(\vec{r}, \vec{r}\,') V(\vec{r}\,') \psi^{(+)}_{(\vec{r}\,')} d^2_{r'} + \psi_0(\vec{r}), \tag{14}$$

where $G(\vec{r}, \vec{r}\,')$ is a Green function satisfying

$$(\nabla^2 + k^2) G(\vec{r}, \vec{r}\,') = -4\pi\delta(\vec{r} - \vec{r}\,'), \tag{15}$$

$k^2 = 2E/\hbar^2$ and $\psi_0(\vec{r})$ is the incident plane wave. It is convenient to solve Eq. (15) for $G(\vec{r}, \vec{r}\,')$ by setting

$$\delta(\vec{r} - \vec{r}\,') = \frac{1}{r'} \delta(r - r') \sum_{m=-\infty}^{\infty} \frac{1}{2\pi} e^{im(\theta-\theta')} \tag{16}$$

and

$$G(\vec{r} - \vec{r}\,') = \sum_{m=-\infty}^{\infty} g_m(r, r') e^{im(\theta-\theta')}. \tag{17}$$

Insertion of Eq. (16) and Eq. (17) into Eq. (15) leads to the result that $g_m(r, r')$ satisfies Bessel's equation when $r \neq r'$. Hence $g_m(r, r')$ has the form

$$g_m(r, r') = A_m J_m(kr_<) H^{(1)}_m(kr_>). \tag{18}$$

Equation (18) follows from the required symmetry of $g_m(r, r')$ in the variables r and r', together with the boundary conditions, that g_m be finite at the origin and have the form of an outgoing cylindrical wave at large distances. In Eq. (18), $r_<$ and $r_>$ mean the smaller and larger of r and r', respectively. Computation of the discontinuity in

$$(d/dr) g_m(r, r')$$

at the point $r = r'$ gives the result

$$A_m = i\pi, \tag{19}$$

so that the required Green's function is

$$G(\vec{r}, \vec{r}\,') = \sum_{m=-\infty}^{\infty} i\pi J_m(kr_<) H^{(1)}_m(kr_>) e^{im(\theta-\theta')}. \tag{20}$$

Equation (4) with the usual normalization is

$$\psi^{(+)}_{(\vec{r})} \sim \frac{1}{2\pi}\left(e^{ikx} + \frac{f(\theta)}{\sqrt{r}} e^{ikr}\right). \tag{21}$$

Making use of the asymptotic form

$$H^{(1)}_m(kr) \sim (2/\pi k r)^{1/2} e^{ikr} e^{-i(m\pi/2 + \pi/4)},$$

one obtains from Eq. (14),

$$f(\theta) = -\left(\frac{2}{\pi k}\right)^{1/2} \frac{m}{\hbar^2} \sum_{m=-\infty}^{\infty} e^{-i(m\pi/2+\pi/4)} i\pi \int J_m(kr')\, e^{im(\theta-\theta')} \psi^{(+)}_{(\vec{r}\,')} V(\vec{r}\,') r' dr' d\theta'. \tag{22}$$

The relation

$$\sum_{m=-\infty}^{\infty} e^{-im\pi/2} J_m(kr') e^{im(\theta-\theta')} = e^{-ikr'\cos(\theta-\theta')} \tag{23}$$

yields

$$f(\theta) = -\left(\frac{2}{\pi k}\right)^{1/2} \frac{m}{\hbar^2} e^{-i\pi/4} i\pi \int e^{-ikr'\cos(\theta-\theta')} \times \psi^{(+)}_{(\vec{r}\,')} V(\vec{r}\,') r' dr' d\theta'. \tag{24}$$

The relation

$$T_{ks} = \langle \psi_r, V\psi_s^{(+)} \rangle$$

implies

$$T_{ks} = \frac{2\pi}{L^2} \int e^{-i\vec{k}\cdot\vec{r}'} V(\vec{r}')\psi_s{}^{(+)}_{(\vec{r}')} d^2r' . \tag{25}$$

The quantity L^2 is the quantization area, $\vec{k}$ is a vector whose azimuthal direction is θ (scattered momentum) and $\vec{k}\cdot\vec{r}' = kr'\cos(\theta - \theta')$. Comparison of Eqs. (25) and (24) yields

$$T_{kS} = (\tfrac{1}{2}\pi k)^{1/2} e^{i\pi/4} i(2\hbar^2/mL^2) f(\theta) , \tag{26}$$

where the state $\psi_s^{(+)}$ has its incident wave parallel to the x axis. When the incident wave is in the direction $\vec{k}'$ and θ is in the direction of $\vec{k}$, Eq. (26) is equivalent to

$$T_{\vec{k}\vec{k}'} = (\tfrac{1}{2}\pi k)^{1/2} e^{i\pi/4} i(2\hbar^2/mL^2) f_{\hat{k}'}(\hat{k}) . \tag{27}$$

With this relation, Eq. (13) becomes

$$\frac{L^2}{4}\left(\frac{2\hbar^2}{mL^2}\right)^2 \frac{m}{\hbar^2} \iint \frac{k''}{k} \delta(k - k'') f^*_{\hat{k}}(\hat{k}'') f_{\hat{k}'}(\hat{k}'') k'' dk'' d\theta'' = i(T_{\vec{k}\vec{k}'} - T^*_{\vec{k}'\vec{k}}) . \tag{28}$$

Setting $\vec{k} = \vec{k}'$ yields

$$\frac{k}{2}\int_0^{2\pi} |f(\theta)|^2 d\theta = -\sqrt{\pi k}[\mathrm{Re} f(0) - \mathrm{Im} f(0)] . \tag{29}$$

and the two dimensional optical theorem is

$$\sigma_T = -2(\pi/k)^{1/2}[\mathrm{Re} f(0) - \mathrm{Im} f(0)] . \tag{30}$$

From Eq. (9),

$$f(0) = \sum_{m=-\infty}^{\infty} \frac{1}{(2\pi k)^{1/2}} e^{-i\pi/4}(e^{2i\delta_m} - 1) . \tag{31}$$

Substitution of this into Eq. (30) yields Eq. (11). This serves as a check on the results and at the same time shows that unitarity places no restriction on the phase shifts.

III. AHARONOV-BOHM SCATTERING

We now return to the problem of Aharonov-Bohm scattering. AB found the stationary states

$$\psi_m(r,\theta) = J_{|m+\alpha|}(kr) e^{im\theta}, \quad m = 0, \pm 1, \pm 2, \cdots , \tag{32}$$

where $\alpha = -e\Phi/ch$. In what follows, the quantity α is assumed to satisfy $0 < \alpha < 1$. The states of Eq. (32) then have the asymptotic form

$$\psi_m(r,\theta) \sim \begin{cases} \left(\dfrac{2}{\pi kr}\right)^{1/2} \cos\left(kr - \dfrac{m\pi}{2} - \dfrac{\alpha\pi}{2} - \dfrac{\pi}{4}\right) e^{im\theta} , & m \geq 0 \\ \left(\dfrac{2}{\pi kr}\right)^{1/2} \cos\left(kr + \dfrac{m\pi}{2} + \dfrac{\alpha\pi}{2} - \dfrac{\pi}{4}\right) e^{im\theta} , & m < 0 . \end{cases} \tag{33}$$

These are of the same form as the solutions obtained in potential scattering given by Eq. (3). This is not surprising since Eq. (3) is general; the concept of potential scattering was not introduced until the discussion of the optical theorem. The optical theorem itself is a consequence of the unitarity of the S matrix. It could also be derived without reference to a scattering potential.

The phase shifts δ_m are thus given by

$$\delta_m = \begin{cases} -\tfrac{1}{2}\alpha\pi , & m \geq 0 \\ \tfrac{1}{2}\alpha\pi , & m < 0 . \end{cases} \tag{34}$$

It is interesting to note that all phase shifts have the same magnitude. Thus Eq. (11) would give an infinite total cross section for AB scattering if it were applied blindly. This phenomenon is related to the difficulty connected with Eq. (2), as has been discussed in the introduction. However, the usual scattering treatment fails in this problem, since the phase shifts δ_m do not go to zero with increasing $|m|$. The treatment of the previous section can easily be modified to treat the case in which the incident wave is

$$\psi_{\mathrm{inc}} = e^{ikr\cos\theta - i\alpha\theta} , \tag{35}$$

which corresponds to the incident wave considered by AB; this gives a particle flux in the x direction. A more serious problem is the fact that the asymptotic form is valid for the Bessel functions only if $|m| \ll kr$. However, this inequality is always violated for infinitely many of the terms in a Bessel function expansion, even at a detector that is a meter from the scatterer. This problem is generally ignored in the partial-wave treatment of scattering found in most textbooks. The consequences are not serious since the phase shifts are assumed to go to zero with increasing angular momentum, so that the spherical counterpart of Eq. (11) gives a finite result. This does not happen in AB scattering. It is this difficulty that caused AB to carry out infinite sums before going to asymptotic forms. Their rigorous treatment is thus much more cumbersome than the recent one by Corinaldesi and Rafeli.[5]

The present approach is similar to the one by Corinaldesi and Rafeli, with the modification that it takes into account the finite width of the incident beam. This is assumed to be of the form

$$\psi_{\mathrm{inc}} = \sum_{m=-\infty}^{\infty} i^m J_m(kr) e^{im\theta} e^{-i\alpha\theta} e^{-|m|\epsilon} , \tag{36}$$

where ϵ is a small positive quantity satisfying $\epsilon \gg 1/kR$, where R is the distance from the scatterer to the detector. Thus if $|m|$ is $O(kR)$, $e^{-|m|\epsilon}$ is effectively zero. Thus the use of asymptotic forms for the Bessel functions is always justified. The condition $1/kR \quad \epsilon \ll 1$ implies that $|m| \ll kR$ for all terms in the sum not cut off by the exponential. Thus we have $|m|\hbar \ll \hbar kR = pR$. But $|m|\hbar$ is roughly the classical momentum of the incident particle multipled by the classical impact parameter. Therefore, the condition on ϵ merely assumes that the largest impact parameter (i.e., the beam half-width) be much smaller than the distance from the scatterer to the detector. This condition is satisfied in every scattering experiment.

A requirement that is generally agreed upon in quantum theory is that the wave function be single valued[6] and continuous. This requirement can be satisfied by demanding that the discontinuity in the incident wave (let us take $0 \leq 0 < 2\pi$) be cancelled by the discontinuity in the scattered wave. With this convention, these discontinuities occur behind the scatterer (i.e., in the forward direction). The incident and scattered waves cannot be distinguished in the laboratory in the forward direction. Thus the treatment is consistent.

One therefore requires a superposition of stationary states that have the asymptotic form

$$\sum_{m=-\infty}^{\infty} i^m\left(\frac{2}{\pi kr}\right)^{1/2} \cos\left(kr - \frac{m\pi}{2} - \frac{\pi}{4}\right)e^{im\theta}e^{-i\alpha\theta}e^{-|m|\epsilon} + \tilde{f}(\theta)\frac{e^{ikr}}{\sqrt{r}} = \sum_{m=-\infty}^{\infty} a_m\left(\frac{2}{\pi kr}\right)^{1/2} \cos\left(kr - \frac{m\pi}{2} - \frac{\pi}{4} + \delta_m\right)e^{im\theta}. \quad (37)$$

Thus

$$\tilde{f}(\theta)\frac{e^{ikr}}{\sqrt{r}} = \left(\frac{2}{\pi kr}\right)^{1/2}\left[\sum_{m=-\infty}^{\infty} a_m \cos\left(kr - \frac{m\pi}{2} - \frac{\pi}{4} + \delta_m\right)e^{im\theta} - \sum_{m=-\infty}^{\infty} i^m e^{-|m|\epsilon}\cos\left(kr - \frac{m\pi}{2} - \frac{\pi}{4}\right)e^{im\theta}e^{-i\alpha\theta}\right]. \quad (38)$$

The condition that the coefficient of $e^{-ikr}/\sqrt{r}$ on the right side of Eq. (38) be zero is

$$\sum_{m=-\infty}^{\infty} [a_m i^m e^{-i\delta m} - i^{2m}e^{-|m|\epsilon}e^{-i\alpha\theta}]e^{im\theta} = 0. \quad (39)$$

Therefore,

$$a_m = i^m e^{i\delta m}e^{-i\pi\alpha}e^{-|m|\epsilon} \quad (40)$$

is a suitable choice for the coefficients. Equation (39) then becomes

$$(e^{-i\pi\alpha} - e^{-i\alpha\theta})\sum_{m=-\infty}^{\infty} e^{-|m|\epsilon}e^{im(\theta-\pi)} = 0. \quad (41)$$

If $|\theta - \pi| \gg \epsilon$, we may put $e^{-\epsilon} = 1$ after the summation has been carried out. In this approximation, the second factor in Eq. (41) vanishes. If $\theta - \pi$ is $0(\epsilon)$, then the square bracket is also of order ϵ, while the sum is of order $1/\epsilon$. Thus the incoming cylindrical wave vanishes everywhere except in the backward direction where it is indistinguishable from the incident wave.

Straightforward algebra then gives

$$\tilde{f}(\theta) = \frac{e^{i\pi/4}}{(2\pi k)^{1/2}}\sum_{m=-\infty}^{\infty}(e^{2i\delta m}e^{-i\pi\alpha} - e^{-i\alpha\theta})e^{im\theta}e^{-|m|\epsilon}. \quad (42)$$

The phase shifts of Eq. (34) yield

$$\tilde{f}(\theta) = \frac{-e^{i\pi/4}e^{-i\pi\alpha}}{(2\pi k)^{1/2}}2i\sin(\pi\alpha)\sum_{m=0}^{\infty} e^{im\theta}e^{-m\epsilon}. \quad (43)$$

The series in Eq. (43) converges uniformly to give

$$\sum_{m=0}^{\infty} e^{im\theta}e^{-m\epsilon} = \frac{1}{1 - e^{i(\theta+i\epsilon)}}. \quad (44)$$

If $\theta \gg \epsilon$, and $(2\pi - \theta) \gg \epsilon$, we may approximate $e^{-\epsilon}$ by 1, and obtain

$$\tilde{f}(\theta) = \frac{e^{i\pi/4}e^{-i\pi\alpha}}{(2\pi k)^{1/2}}\sin(\pi\alpha)\frac{e^{-i\theta/2}}{\sin\frac{1}{2}\theta}. \quad (45)$$

This gives for the differential cross section the well-known result

$$\frac{d\sigma}{d\theta} = |\tilde{f}(\theta)|^2 = \frac{\sin^2(\pi\alpha)}{2\pi k\sin^2\frac{1}{2}\theta}, \quad (46)$$

where $\theta = 0$ defines the forward direction. In the forward direction, Eq. (44) shows that $\tilde{f}(\theta)$ is of order $1/\epsilon$ and $d\sigma/d\theta$ is of order $1/\epsilon^2$. Thus the total cross section can be made extremely large by employing a wide incident beam and placing the detector far from the scatterer. This result is consistent with that of Eq. (2). The total cross section may be considered to diverge.

A final question to be explored is the importance of having the particles (rather than the waves) incident in the x direction. If the mechanism of scattering were not understood, one might assume the incident wave to be of the form e^{ikx} rather than $e^{ikx-i\alpha\theta}$. The cross section would then be obtained directly from Eq. (9) suitably modified to take into account the finite beam width as before. The phase shifts are given by Eq. (34). To be sure, these phase shifts could not be due to scattering by an axially symmetric potential. Such a potential gives stationary states that are related by

$$\psi_{k,m}(r,\theta) = \psi_{k,-m}(r,-\theta).$$

This is not the case for AB scattering, since phase shifts for positive and negative m values have the opposite sign. On the other hand, the scattering could not be due to an asymmetric potential because the Z component of the angular momentum is a good quantum number.

Suppose that in spite of these difficulties, one takes the scattering cross section to be that of Eq. (9), modified to take into account the finite beam width. One then finds

$$f(\theta)=\frac{e^{-i\pi/4}}{(2\pi k)^{1/2}}\left(\sum_{m=0}^{\infty}(e^{-i\pi\alpha}-1)e^{im\theta}e^{-m\epsilon}+\sum_{m=-1}^{-\infty}(e^{i\pi\alpha}-1)e^{im\theta}e^{m\epsilon}\right)$$

$$=\frac{e^{-i\pi/4}}{(2\pi k)^{1/2}}\left(e^{-i\pi\alpha}+2\,\mathrm{Re}\sum_{m=1}^{\infty}e^{-i\alpha\pi}e^{im\theta}e^{-m\epsilon}-\sum_{m=-\infty}^{\infty}e^{im\theta}e^{-|m|\epsilon}\right). \quad (47)$$

The third term on the right of Eq. (47) vanishes for angles deviating from the forward direction by much more than ϵ. This term contributes only to forward scattering. The effect of such a term could not be distinguished from the incident beam in a laboratory experiment; the term may therefore be dropped. Such a forward scattering term was dropped in the recent treatment by Corinaldesi and Rafeli.[5] This term, however, is necessary for the unitarity of the S matrix.

The remaining sum in Eq. (47) may now be carried out rigorously and for angles deviating from the forward direction by more than ϵ, one obtains

$$f(\theta)=\frac{e^{i\pi/4}}{2\pi k}\left(e^{-i\pi\alpha}+\frac{\cos(\frac{1}{2}\theta-\pi\alpha+\frac{1}{2}\pi)}{\sin\frac{1}{2}\theta}\right). \quad (48)$$

A bit of algebra then gives

$$|f(\theta)|^2=\frac{1}{2\pi k}\frac{\sin^2(\pi\alpha)}{\sin^2\frac{1}{2}\theta}, \quad (49)$$

which is identical with Eq. (46), the result found by AB. This result is certainly comforting, even to those who do not find it surprising. Had Eq. (49) given a result at variance with Eq. (46), one would be faced with the problem of having to understand the nature of a scattering force before one can analyze the scattering. But the purpose of a scattering experiment is precisely to gain information about the nature of the force.

IV. CONCLUSION

In a scattering process, the momentum of the scattered particle is changed, and this momentum change must be caused by a force. The only available force in AB scattering is the Lorentz force which is nonzero everywhere except in a small neighborhood of the Z axis. One purpose of a scattering experiment is to obtain information about the range of the scattering force. In the case of AB scattering, the theoretical result already implies that we are dealing with a force of infinite range. However, the diameter of the region in which the (Lorentz) force is nonvanishing was assumed to be quite small. We therefore have a contradiction that is not easily explained. The problem is compounded by the fact that electrodynamics has been shown to be a local theory,[2,3] so that the only mechanism available for AB scattering would seem to be a penetration by the wave packet of the region in which the magnetic field is nonvanishing. This mechanism has been suggested by Strocchi and Wightman.[2]

It may be that a modification of scattering theory is required, but this is far from clear. Certainly, one is encouraged by the fact that an incident plane wave leads to the AB differential cross section, as seen from Eq. (49), just as the incident state taken by AB does. If this were not the case, one would have real cause for concern, since the incident state is always taken to be a plane wave in cases where the nature of the scattering force is not clearly understood.

From a mathematical point of view, the AB effect is characterized by phase shifts all of the same magnitude but having a sign that depends on the sign of the Z component of angular momentum. It is this sign difference that causes the AB effect; without it, all scattering would be forward scattering.

The manner in which this problem will be resolved is at present unclear. There may yet be hope for the point of view expressed by DeWitt[7] that electrodynamics should be viewed as a nonlocal theory.

Note added in proof. The problem discussed here has since been resolved by the author. Its resolution has been submitted to the J. Math. Phys. under the title "Aharonov-Bohm Scattering and the Velocity Operator."

[1]Y. Aharonov and D. Bohm, Phys. Rev. 115, 485 (1959); 123, 1511 (1961); 130, 1625 (1963). These authors are hereafter referred to as AB.

[2]F. Strocchi and A. S. Wightman, J. Math. Phys. 15, 2198 (1974).

[3]R. Meinikoff and D. H. Sharp, J. Math. Phys. 18, 471

(1977).
[4]Kay M. Purcell and W. C. Henneberger, Am. J. Phys. 46, 1255 (1978).
[5]E. Corinaldesi and F. Rafeli, Am. J. Phys. 46, 1187 (1978).
[6]E. Merzbacher, Am. J. Phys. 30, 237 (1962).
[7]B. S. DeWitt, Phys. Rev. 125, 2189 (1962).