Radiative decay of the $2p\ ^2P_{3/2,1/2}$ states of lithiumlike krypton ($Z=36$) and the $2s2p\ ^3P_1$ state of berylliumlike krypton

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The beam-foil time-of-flight technique has been used to measure the wavelengths and lifetimes of the $\Delta n=0$ transitions to the ground state $2p\ ^2P_{3/2,1/2}\rightarrow 2s\ ^2S_{1/2}$ in lithiumlike Kr^{33+} and the intercombination transition $2s2p\ ^3P_1\rightarrow 2s^2\ ^1S_0$ in berylliumlike Kr^{32+}. The following results are obtained for the transition, wavelength (Å), and lifetime (nsec), respectively: Kr^{33+}: $^2P_{1/2}\rightarrow\ ^2S_{1/2}$, 174.15 ± 0.26, 0.32 ± 0.03; Kr^{33+}: $^2P_{3/2}\rightarrow\ ^2S_{1/2}$, 91.08 ± 0.10, 0.052 ± 0.003; Kr^{32+}: $^3P_1\rightarrow\ ^1S_0$, 169.9 ± 0.5, 2.3 ± 0.3. These results are compared with calculations.

I. INTRODUCTION

The study of relativistic effects in three- and four-electron atoms has been a subject of considerable theoretical effort with the goal of developing a framework for accurate calculation of transition energies and decay rates in high-Z systems.[1-22,42] A variety of approaches applied to the $\Delta n=0$ transitions in these lithiumlike and berylliumlike high-Z ions has, however, yielded discrepant results.

Measurements of wavelengths and lifetimes of transitions between low-lying states in which the principal quantum number does not change ($\Delta n=0$) provide a sensitive test of relativistic calculations. The large but easily calculated Rydberg term is absent, while the relativistic corrections to the Coulomb interaction between electrons are a significant and rapidly increasing fraction of the transition energy with increasing Z. Radiative corrections also make a measurable contribution to the binding energy of low-lying S states. In this paper we report measured values of the wavelengths and mean lives of states which decay through $\Delta n=0$ transitions to the ground state: $2p\ ^2P_{3/2,1/2}\rightarrow 2s\ ^2S_{1/2}$ in lithiumlike Kr^{33+} ($Z=36$) and the $2s2p\ ^3P_1\rightarrow 2s^2\ ^1S_0$ intercombination transition in berylliumlike Kr^{32+}. Krypton is the highest-Z system for which measurements of these transitions have been made, and our measurements of the transition wavelengths clearly distinguish between the discrepant calculations.

II. THEORY

Calculations of the transition energies and decay rates have been of two types: *ab initio* and semiempirical. The semiempirical approach used by Edlén[15,16] to calculate wavelengths in the lithium and beryllium isoelectronic sequences, gives excellent agreement with experiments.

Ab initio calculations are based upon several approaches. One widely used method is to calculate Schrödinger radial wave functions in a multiconfigurational Hartree-Fock (HF) approach and to then treat the relativistic corrections in the Pauli approximation as first-order perturbations of the nonrelativistic energy.[8-14,42] Another approach is to form multiconfigurational products from relativistic single-particle (Dirac) wave functions. A relativistic Hartree-Fock code (RHF) developed by Desclaux[6,7] has been the basis of several such calculations.[1-3]

The relativistic random-phase approximation has also been applied to calculations of wavelengths and decay rates in the beryllium sequence.[18] However, difficulties with two-particle–two-hole correlations in the ground state seriously affect the accuracy of the calculated $^3P_1\rightarrow\ ^1S_0$ transition energy.

A perturbation expansion[23] in the parameters $1/Z$ and $Z\alpha$ provides a useful technique for evaluating the relative importance of various relativistic and nonrelativistic terms as a function of Z. The binding energies of the $n=2$ states in the lithium sequence are in units of $mc^2=4.121\,484\times10^9$ cm^{-1} where m is the electron mass and c is the speed of light:

$$\begin{aligned}
E_{2S_{1/2}} &= W_{1/2}+1.022\,805Z^{-1}(Z\alpha)^2+0.680\,282Z^{-1}(Z\alpha)^4 \\
&\quad -0.404\,87Z^{-2}(Z\alpha)^2-0.0457Z^{-3}(Z\alpha)^2 \\
E_{2P_{1/2}} &= W_{1/2}+1.093\,526Z^{-1}(Z\alpha)^2+0.788\,955Z^{-1}(Z\alpha)^4 \\
&\quad -0.517\,25Z^{-2}(Z\alpha)^2-0.146Z^{-3}(Z\alpha)^2 \qquad (1) \\
E_{2P_{3/2}} &= W_{3/2}+1.093\,526Z^{-1}(Z\alpha)^2+0.570\,846Z^{-1}(Z\alpha)^4 \\
&\quad -0.517\,25Z^{-2}(Z\alpha)^2-0.146Z^{-3}(Z\alpha)^2,
\end{aligned}$$

where $\alpha = 1/137.036$, and W_j is the Dirac hydrogenic binding energy[24] of an electron in a state of principal quantum number n and angular momentum $j = k - \frac{1}{2}$:

$$(W_j + 1)^{-1} = (1 + \{Z\alpha/(n - k + [k^2 - (Z\alpha)^2]^{1/2})\}^2)^{1/2}.$$

In Eq. (1) the terms $Z^{-i}(Z\alpha)^2$, the nonrelativistic Coulomb interaction for the exchange of i virtual photons between electrons, are obtained from Ref. 25. The term $Z^{-1}(Z\alpha)^4$, the lowest order relativistic correction to single photon exchange, is obtained from Doyle.[19] Higher-order relativistic corrections are not yet available.

The transition rates are given by

$$A_{2P_{1/2}} = 12\alpha \frac{mc^2}{\hbar}\left(\frac{E_{1/2}}{mc^2}\right)^3 (Z\alpha)^{-2}[1 + 1.647/Z - 0.417(Z\alpha)^2]^2,$$
$$A_{2P_{3/2}} = 12\alpha \frac{mc^2}{\hbar}\left(\frac{E_{3/2}}{mc^2}\right)^3 (Z\alpha)^{-2}[1 + 1.647/Z - 0.167(Z\alpha)^2]^2, \quad (2)$$

where $E_{1/2}$, $E_{3/2}$ are the appropriate transition energies. The zeroth-order term in the matrix element is the nonrelativistic hydrogenic approximation. The term of relative order Z^{-1}, the first correction in the nonrelativistic Z expansion of the matrix element, is obtained from Dalgarno and Parkinson.[26] The leading relativistic correction to the hydrogenic matrix element, of relative order $(Z\alpha)^2$ is obtained from Refs. 27 and 28.

In the beryllium isoelectronic sequence, mixing between the $1s^22s^2$ and $1s^22p^2$ configurations and mixing of the $2s2p\ ^1P_1$ and $^3P_1$ states requires that the energy matrix be diagonalized for each value of Z. Nonrelativistic Z expansion terms through order $Z^{-3}(Z\alpha)^2$ and the $Z^{-1}(Z\alpha)^4$ relativistic term are obtained from Refs. 19, 20, and 29, and the $jj - LS$ transformations are obtained from Condon and Shortley.[30] Unfortunately, the off-diagonal elements of the ground-state energy matrix have only been calculated for the single-photon contribution.[19] The wavelength and decay rate of the $2s2p\ ^3P_1 \rightarrow 2s^2\ ^1S_0$ transition in berylliumlike Kr are included in Tables I and II. To calculate transition rates we take as the intermediate coupled states appropriate linear combinations of LS coupled states: $(2s^2\ ^1S_0)$, $(2p^2\ ^1S_0)$, $(2p^2\ ^3P_0)$ for $j = 0$, and $(2s2p\ ^1P_1)$, $(2s2p\ ^3P_1)$ for $j = 1$. The zeroth-order nonrelativistic matrix element and the Z^{-1} correction are obtained from Ref. 31. The relativistic correction to the $2s2p \rightarrow 2s^2$ portion of the transition rate is calculated using the radial integrals evaluated for Ref. 28.

III. EXPERIMENT

Studies of the $^2P_{3/2,1/2} \rightarrow {}^2S_{1/2}$ and the $^3P_1 \rightarrow {}^1S_0$ transitions have previously been made in lower-Z systems by the beam-foil technique.[32,33] A critical compilation of wavelengths for these transitions can be found in Refs. 15 and 16.

We observed photons from decays in flight of 8.5 MeV/amu ^{84}Kr ions ($v = 4\times10^9$ cm/sec) after passage of the ions through a 600-μg/cm^2 carbon foil. The Kr ions were obtained from the Lawrence Berkeley Laboratory's Super-Heavy-ion linear accelerator (HILAC). The maximum flux was 5×10^{11} particles/sec. The ion beam path was defined by a pair of 0.64-cm collimators, three meters apart, located several centimeters upstream from the region viewed. Photons were analyzed in a 2.2-meter grazing-incidence monochromator (McPherson model 247) used in the usual side-on viewing configuration. The operating parameters of the monochromator and the detector are identical to those specified in Ref. 33. A spectral scan of the 165–190 Å region which includes lines from all three transitions is shown in Fig. 1.

Accurate wavelength measurement of spectral lines from a rapidly moving beam-foil source requires the use of calibration lines from ions traveling at the same velocity and along the same path as the ions emitting the lines whose wavelength measurement is desired. These lines provide a calibration of the instrumental line shape from a moving source and determine a linear Doppler-shift correction due to any angular misalignment of the spectrometer axis from perpendicularity to the beam axis. However, the mean ion state of the 8.5 MeV/amu Kr ions after passage through foil is +32.5. Thus by the nature of this experiment there are no lines of experimentally established wavelengths from the Kr beam-foil source with sufficient accuracy to serve as calibrations. Instead, lines of known wavelength[34] at 192.03 and 255.10 Å from the $^2P_{3/2,1/2} \rightarrow {}^2S_{1/2}$ transitions in lithiumlike Fe^{23+} produced by passing 8.5 MeV/amu ^{56}Fe through the same foil and collimators were used for calibration. The experimental setup, ion velocity, and accelerator tune for the Fe ions were identical to those used in the Kr measurements.[35]

Calibration of the spectrometer dispersion was obtained from scans over the 225–600-Å region containing known lines[36] from He, He^+, and Ne^+, produced by a stationary ion gauge source mounted across the beam line from the spectrometer entrance slit. Wavelength measurements at 91 and 174 Å of the $^2P_{3/2,1/2} \rightarrow {}^2S_{1/2}$ transition in lithiumlike Kr^{33+} were performed between two Fe calibrations. The wavelength measurement of the

TABLE I. Wavelengths (Å) of the $2p\ ^2P_{3/2,1/2} \rightarrow 2s\ ^2S_{1/2}$ transition in lithiumlike Kr^{33+} and the $2s2p\ ^3P_1 \rightarrow 2s^2\ ^1S_0$ transition in berylliumlike Kr^{32+}.

Transition	Experiment(Å)	Theory *ab initio* (with hydrogenic Lamb shift of 13 940 cm^{-1}) Relativistic Hartree-Fock		Hartree-Fock with relativistic corrections		Perturbation expansion	Semiempirical
		KD[a]	AFL[b]	Weiss[c]	Cowan[d]	This work	Edlén[e]
$Kr^{33+}\ 2p\ ^2P_{3/2} \rightarrow 2s\ ^2S_{1/2}$	91.08±0.10	91.1	91.2	93.6	93.2	91.0	91.0
$2p\ ^2P_{1/2} \rightarrow 2s\ ^2S_{1/2}$	174.15±0.26	174.0	173.6	182.7	184.6	173.1	173.7
		KD[a]	AFL[b]/CJ[f]	Weiss[g]	Cowan[d]	This work	Edlén[h]
$Kr^{32+}\ 2s2p\ ^3P_1 \rightarrow 2s^2\ ^1S_0$	169.9 ±0.5	169.6	169.4	177.3	193.1	167.2	169(est)
				SCG[i]	NS[j]		
				169.0	176.8		

[a] Y.-K. Kim and J. P. Desclaux (Refs. 1 and 41).
[b] L. Armstrong Jr., W. R. Fielder, and D. L. Lin (Ref. 2); and L Armstrong, Jr. (private communication).
[c] A. W. Weiss (Ref. 8).
[d] R. D. Cowan (Ref. 12).
[e] Extrapolated from B. Edlén (Ref. 15).
[f] K. T. Cheng and W. R. Johnson (Ref. 3).
[g] A. W. Weiss (Ref. 9).
[h] Extrapolated from B. Edlén (Ref. 16).
[i] D. H. Sampson, R. E. H. Clark, and L. B. Golden (Ref. 42).
[j] H. Nussbaumer and P. J. Storey (Ref. 10).

TABLE II. Mean lives (nanoseconds) of the $2p\ ^2P_{3/2,1/2}$ states of lithiumlike Kr^{33+} and the $2s2p\ ^3P_1$ state of beryllium-like Kr^{32+}.

State	Experiment(*ns*)	Theory (corrected for hydrogenic Lamb shift) Relativistic Hartree-Fock		Hartree-Fock with relativistic corrections		Perturbation expansion	Relativistic random-phase approximation
		KD[a]	AFL[b]	Weiss[c]	Cowan[d]	This work	
$Kr^{33+}\ 2p\ ^2P_{3/2}$	0.052±0.003	0.050	0.050	0.054	0.053	0.050	
$2p\ ^2P_{1/2}$	0.32 ±0.03	0.357	0.357	0.418	0.413	0.354	
		KD[a]	AFL[b]/CJ[e]	Weiss[f]	Cowan[d]	This work	LJ[g]
$Kr^{32+}\ 2s2p\ ^3P_1$	2.3 ±0.3	2.46	2.45	2.68	2.58	2.42	2.54(5.0)
				SCG[h]	NS[i]	VS[j]	
				2.6	2.73	2.07	

[a] Y.-K. Kim and J. P. Desclaux (Refs. 1 and 41).
[b] L. Armstrong Jr., W. R. Fielder, and D. L. Lin (Ref. 2); and L. Armstrong, Jr. (private communication).
[c] A. W. Weiss (Ref. 8).
[d] R. D. Cowan (Ref. 12).
[e] K. T. Cheng and W. R. Johnson (Ref. 3).
[f] A. W. Weiss (Ref. 9).
[g] C. D. Lin and W. R. Johnson (Ref. 18) and W. R. Johnson (private communication).
[h] D. H. Sampson, R. E. H. Clark, and L. B. Golden (Ref. 42).
[i] H. Nussbaumer and P. J. Storey (Ref. 10).
[j] Extrapolated from D. S. Victorov and U. I. Safronova (Ref. 21).

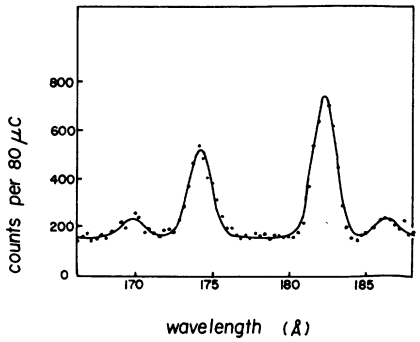

FIG. 1. Spectra from decays in flight of krypton ions traveling at 4×10^9 cm/sec after passage through a 600-$\mu g/cm^2$ carbon foil. The peak at 170 Å is the first-order line from the $2s2p\ ^3P_1 \rightarrow 2s^2\ ^1S_0$ transition in berylliumlike Kr^{32+}. The peaks near 174 and 182 Å are the first-order line from the $2p\ ^2P_{1/2} \rightarrow 2s\ ^2S_{1/2}$ and second-order line from the $2p\ ^2P_{3/2} \rightarrow 2s\ ^2S_{1/2}$ transitions in lithiumlike Kr^{33+}. The monochromator slit widths were 200 μm, and 80 μC of charge was collected at each point.

170-Å $2s2p\ ^3P_1 - 2s^2\ ^1S_0$ transition in berylliumlike Kr^{32+} was made after the second Fe calibration. HILAC source changes accompanied by several hours interruption were mandated in switching from Kr to Fe and by the need for periodic replacement of worn out sources (every 20–30 hours at high intensity). The wavelength of the He^+ 304-Å line from our static source was measured frequently during both Fe and Kr runs to check for apparatus instabilities such as those caused by temperature variation or mechanical shock. Instabilities in the apparatus were found to be small compared to other sources of error.

Prior to the Kr measurements, extensive wavelength measurements were made on the lithiumlike Fe spectra to establish a procedure by which the 192- and 255-Å wavelengths could be accurately obtained from the measurements. We made eight scans over the 192-Å line ($^2P_{3/2} \rightarrow {}^2S_{1/2}$) and six scans over the 255-Å ($^2P_{1/2} \rightarrow {}^2S_{1/2}$) line. The second Fe calibration after the Kr wavelength measurements consisted of seven scans over the 192-Å line. The monochromator slits were 100 μm wide. With these slits the expected instrumental linewidth is 0.75 Å and the Doppler width 0.3 Å. The observed linewidths were (0.9 ± 0.1) Å. All spectral data were fitted to Gaussian line shapes. Measured wavelengths were corrected for time dilation (transverse Doppler shift). Remaining differences between our measured Fe^{23+} wavelengths and the known Fe^{23+} wavelengths are attributed to linear Doppler shifts caused by angular misalignment of the spectrometer axis from perpendicularity to the beam axis. The average linear Doppler shift thus obtained was $\Delta\lambda/\lambda = (1.3\pm0.4)\times10^{-3}$. The quoted uncertainty, sufficient to include the values of $\Delta\lambda/\lambda$ for all Fe scans, is at least in part due to the lack of tighter beam collimation. As two Fe^{33+} lines overdetermine the linear Doppler-shift correction, the ability to reproduce both of the known Fe^{23+} lines using the linear Doppler shift correction, and the fixed-source standards is a useful measure of experimental accuracy. The Fe^{23+} wavelengths produced by this procedure are (192.02 ± 0.08) and (255.12 ± 0.10)-Å.

The accurate wavelength measurements of the 91-Å lithiumlike Kr line were made in second order at 182 Å after determining that the second-order line had the same lifetime as the first-order line. Four scans using 100-μm slits over the 182-Å line were taken. Gaussian fitting of the data yielded a standard deviation of the mean of the individual wavelength measurements of 0.02 Å. The measured wavelength is corrected for time dilation, and for a linear Doppler shift of $(+0.24\pm0.07)$ Å. A correction of $(+0.03\pm0.03)$ Å is applied to the measurement of this wavelength because the short lifetime of the $2\ ^2P_{3/2}$ state results in a larger contribution from photons Doppler shifted towards shorter wavelengths. Combining this value with our "measured" wavelength of the 192-Å Fe^{23+} calibration yields (182.15 ± 0.20) Å in second order and (91.08 ± 0.10) Å for the wavelength of the $^2P_{1/2} \rightarrow {}^2S_{1/2}$ transition in lithiumlike Kr^{33+}.

An uncertainty in the wavelength and the lifetime of the lithiumlike $Kr^{33+}\ ^2P_{1/2} \rightarrow {}^2S_{1/2}$ transition arises from the presence of a short-lived component seen at small distances downstream from the foil (Fig. 2). We do not currently understand the origin of this component. Anomalies which may be of a similar nature have been observed in the decays of beam-foil-produced heliumlike atoms.[37] One possible mechanism is discussed by Lin and Armstrong.[38] Measurement of the $Kr^{33+}\ ^2P_{1/2} \rightarrow {}^2S_{1/2}$ transition wavelength (174 Å) as a function of distance downstream from the foil indicates that the wavelength of the short-lived component is within less than 0.25 Å of the long-lived component. We know of no other transition near this wavelength among the few electron ions produced in our beam-foil source. Our accurate wavelength measurements of the 174-Å line were performed with the foil positioned such that the short-lived component contributed about 15% of the total intensity. We account for the presence of the short-lived component by applying a correction to the measured wavelength of (-0.04 ± 0.07) Å. Seven scans over the 174-Å Kr^{33+} line were taken using a monochromator slit width of 100 μm. The standard deviation of the mean was 0.04 Å. Correction for time dilation, the short-lived component, and a linear Doppler shift of $(+0.23\pm0.07)$ Å and com-

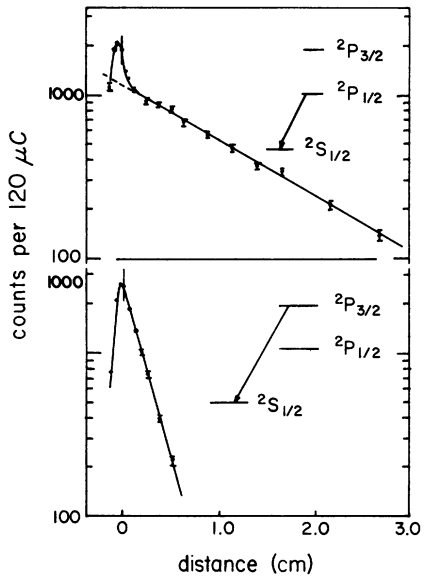

FIG. 2. Decay curves in second order of the $^2P_{3/2} \rightarrow {}^2S_{1/2}$ transition (lower curve) and the $^2P_{1/2} \rightarrow {}^2S_{1/2}$ transition (upper curve) in lithiumlike Kr^{33+}. The error bars are counting statistics, and the vertical line at zero distance is the foil location. 120 μC of charge was collected at each point.

parison with the 192-Å Fe^{23+} calibration line yield a value of (174.15 ± 0.26) Å for the lithiumlike Kr^{33+} $^2P_{1/2} \rightarrow {}^2S_{1/2}$ transition wavelength.

Only two scans, with 400-μm slits, over the 170-Å berylliumlike Kr^{32+} line were taken. An uncertainty of 0.32 Å, twice the average deviation from the mean of the two runs is assigned. Correction for time dilation, a linear Doppler shift of (+0.22 ± 0.07) Å and comparison with 192-Å Fe^{23+} calibration line yields a wavelength of (169.9 ± 0.5) Å for the berylliumlike Kr^{32+} $2s2p\ ^3P_1 \rightarrow 2s^2\ ^1S_0$ transition.

Decay curves for the three transitions, shown in Figs. 2 and 3, and the associated mean lives were obtained by techniques similar to those described in Ref. 33. Decays of the lithiumlike Kr^{33+} $^2P_{3/2,1/2}$ states were measured using 400-μm monochromator slits, which allowed a view of 1.9 mm of ion path length, and the berylliumlike Kr^{32+} $2s2p\ ^3P_1$ decay with 2-mm slits which provided a view of 4.2-mm ion path length. The mean life of the lithiumlike Kr^{33+} $^2P_{3/2}$ state was determined from three decay curves, one taken in first order at 91 Å and two taken in second order. The $^2P_{1/2}$ state lifetime was determined from two decay curves with the initial portion of the decay curve containing the short-lived component omitted from the fit. The berylliumlike Kr^{32+} $2s2p\ ^3P_1$ lifetime was obtained from a single decay curve. In all cases the decays are fit by single exponentials.

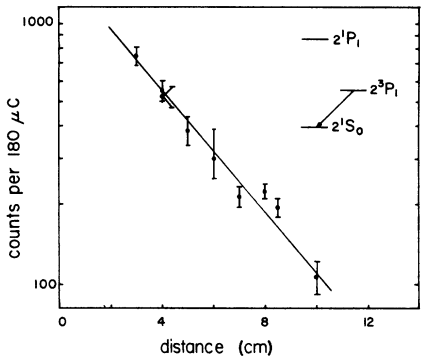

FIG. 3. Decay curve of the $2s2p\ ^3P_1 \rightarrow 2s^2\ ^1S_0$ transition in berylliumlike Kr^{32+}. 180 μC of charge was collected at each point.

The effect of cascading from higher states is expected to be negligible at this level of accuracy because the mean lives of the higher states that have significant initial populations are much shorter than the mean lives of the $^2P_{3/2,1/2}$ and $2s2p\ ^3P_1$ states. The mean lives (Table I) include corrections for time dilation (0.9%) and a slowing of the ions by the foil.[39] The uncertainties include ion velocity uncertainty of 1% and in the case of the $^2P_{3/2,1/2}$ state mean lives, the uncertainties are large enough to include all measured values. For the $2s2p\ ^3P_1$ lifetime the quoted uncertainty is twice that produced by the exponential fit to the single decay curve. (Larger than statistical errors are chosen to partially account for quasisystematic effects such as changes in the background counting rate, foil deterioration, and drifts in the beam position which would smooth or show up as scatter if many decay curves were measured.)

IV. RESULTS

In Tables I and II measured values of wavelengths and lifetimes are compared with the various theoretical calculations. All of the wavelengths calculated by *ab initio* methods have been adjusted by decreasing the binding energy of the 2s state by 13 940 cm^{-1} to account for the one electron Lamb shift. This Lamb shift value is obtained by interpolation of values calculated by Mohr for hydrogenlike atoms.[40] Theoretical lifetimes are corrected for the change in transition energies due to the Lamb shift by employing the E^3 dependence (E being the transition energy) of the electric dipole decay rate. We neglect the effect of core electrons upon the Lamb shift (leading term of relative order Z^{-1}) and radiative corrections to the electric dipole matrix elements. In berylliumlike Kr we also neglect the effect of configuration mixing upon the Lamb shift. Without the Lamb shift included, no calculated wavelength in Table I would agree with

our experiment.

Among the *ab initio* methods, only the RHF calculations[1–3,41] show consistent agreement with measured values of wavelengths and lifetimes. Deviations from experiment with lifetimes calculated by other methods are largely the result of error in the calculated wavelengths. The small differences in wavelengths among the different RHF calculations apparently arise predominantly from the way in which the Breit interaction is treated.[41] The lifetime in Table I calculated by the relativistic random-phase approximation (RRPA) uses the experimental wavelength. The value obtained using the RRPA-generated wavelength is shown in parentheses.

The perturbation-expansion calculation of transition energies and mean lives of the lithium isoelectronic sequence yields mean lives of the $^2P_{3/2,1/2}$ states and $^2P_{3/2} \rightarrow {}^2S_{1/2}$ transition energy [Eqs. (1) and (2)] for lithiumlike Kr which are in agreement with experiment. There is some disagreement between the perturbation-expansion calculation and experiment for the $^2P_{1/2} \rightarrow {}^2S_{1/2}$ transition energy in lithiumlike Kr. At lower Z the perturbation expansion wavelengths are comparable to the RHF values. At higher Z, however, large discrepancies result. This may be due to the absence in the perturbation expansion of higher-order relativistic corrections which are contained in RHF theory via the Breit interaction. With the terms from Refs. 19, 20 and 29 the Z-expansion calculation is less satisfactory in predicting the berylliumlike wavelengths.

The semiempirical wavelength values for lithiumlike Kr^{33+} were obtained by an extrapolation from Edlén's tables using the formulas in Ref. 15. The agreement of the semiempirical theory with all measurements in the Li isoelectronic sequences speaks for itself. However, further extrapolation to higher Z will require a formula different from the series expansion for the Lamb shift. The semiempirical wavelength for berylliumlike Kr is obtained by a very long extrapolation of Fig. 2 in Ref. 16.

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