Intensity fluctuations and spatial coherence of a single polarized component of a saturating amplified-spontaneous-emission source

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Intensity fluctuations of the amplified-spontaneous-emission (ASE) output of a saturated 3.51μ m xenon-helium cw gas-laser amplifier have been characterized by measurements of the first seven normalized cumulants of the intensity probability distribution function (IPDF) for a single linear polarization. ASE output at all amplifier lengths measured (60 to 200 cm) displays some degree of gain-saturation distortion of the negative exponential IPDF expected for linearly amplified spontaneous emission. At long lengths (heavy saturation) the higher cumulants [initially (n - 1)! for the *n*th normalized cumulant] are substantially reduced to order unity. However, the normalized variance (second cumulant) is only slightly reduced and appears to be approaching a long-length limiting value of approximately 0.45. The output shows a high degree of spatial coherence at both long and short lengths studied.

I. INTRODUCTION AND BACKGROUND

Long cylindrical volumes filled with excited atoms produce intense highly directional beams of amplified spontaneous emission (ASE).¹ When the amplification is linear, one expects each polarization component of the output to be the sum of many independent, randomly phased, spontaneous emissions variously amplified depending on the length traveled through the medium. As has been demonstrated,² this results in a negative-exponential intensity probability distribution function (IPDF)

$$P_{1}(I) = \langle I \rangle^{-1} \exp(-I/\langle I \rangle) . \tag{1}$$

Since the two polarization components of a traveling wave are independent in the linear amplifier, and are noninterfering, the IPDF of the total intensity is given by

$$P_{2}(I) = (I/\langle I \rangle^{2}) \exp(-I/\langle I \rangle).$$
⁽²⁾

When the intensity is sufficiently high, the amplifier gain becomes nonlinear because the number of upper-state atoms is reduced by induced transitions. In a saturating homogeneously broadened medium, the evolution of the intensity in the direction of propagation is given by³

$$\frac{dI(z)}{dz} = \frac{gI(z)}{1 + sI_{\tau}(z)},$$
(3)

where I_T is the total intensity illuminating the atoms. Restricting our study to the special case where $I_T = I$, the first incremental distortion of the IPDF can be determined. A particular way to characterize an IPDF and thus to measure the distortion is to calculate the normalized cumulants of the distribution defined by

$$K_{n} = \langle I \rangle^{-n} \left(\frac{\partial^{n}}{\partial \lambda^{n}} \ln \langle \exp(\lambda I) \rangle \right)_{\lambda = 0} .$$
 (4)

The values of K_n for the special cases of the negative-exponential distribution characteristic of the thermal light and the constant intensity distribution characteristic of perfectly coherent light are given, respectively, by

$$K_{nT} = (n-1)!, \quad n > 1$$
 (5)

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and

$$K_{nc} = 0, \quad n > 1$$
 . (6)

The effect of saturation on the cumulants is given by

$$\frac{dK_n}{dz} = \frac{d}{dz} \left[\langle I \rangle^{-n} \left(\frac{\partial^n}{\partial \lambda^n} \ln \langle \exp \lambda I \rangle \right)_{\lambda = 0} \right], \qquad (7)$$

which can be written as

$$\frac{dK_n}{dz} = \langle I \rangle^{-n} \left[\frac{\partial^n}{\partial \lambda^n} \ln \left(\sum_{m=0}^{\infty} \frac{\lambda^m}{m!} \frac{d\langle I^m \rangle}{dz} \right) \right]_{\lambda=0} - n \langle I \rangle^{-1} \frac{d\langle I \rangle}{dz} K_n.$$
(8)

Using Eq. (3), we find that the lowest-order effect of saturation is given by

$$\frac{dI}{dz} = gI - gsI^2 , \qquad (9)$$

so that, again to lowest order,

$$\frac{d\langle I^n \rangle}{dz} = \left\langle nI^{n-1} \frac{dI}{dz} \right\rangle$$
$$= ng \langle I^n \rangle - ngs \langle I^{n+1} \rangle . \tag{10}$$

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To calculate the effect on an initially negativeexponential distribution, we can use the initial characteristic relation

$$\langle I^n \rangle = n! \langle I \rangle^n , \qquad (11)$$

in which case Eq. (8) becomes

$$\frac{dK_n(0)}{dz} = \langle l \rangle^{-n} \frac{\partial^n}{\partial \lambda^n} \left[\ln \sum_{m=0}^{\infty} \left(\frac{gm(m1)\lambda^m \langle l \rangle^m}{m!} - \frac{gs m\lambda^m (m+1)! \langle l \rangle^{m+1}}{m!} \right) \right]_{\lambda=0}$$
$$- (ng - 2nsg\langle l \rangle) K_n. \qquad (12)$$

It follows that

$$\frac{dK_n(0)}{dz} = ngK_n - gsn(n+1)\langle t \rangle K_n - ngK_n + 2nsg\langle t \rangle K_n.$$
(13)

So finally,

$$\frac{dK_n(0)}{dz} = -gsn(n-1)\langle t \rangle K_n, \qquad (14)$$

where the linear gain terms have disappeared as expected since linear gain amounts only to a scale change in the variable. Thus, at least initially, the higher cumulants decline more rapidly. Since, as Eq. (8) demonstrates, the modes are coupled in their evolution, it is clear that the decoupling using Eq. (11) is only valid as an initial approximation. A more detailed approach is required for exact solution.

It is easily shown⁴ that the limiting distribution for a very long (heavily saturated) amplifier governed by Eq. (3) is a constant intensity distribution if there is only a single beam present. Recently, however, models which couple the copropagating, yet orthogonally polarized, components of the field indicate that competition for the energy stored in the atoms results in substantial fluctuations in each component even when the amplifier is heavily saturated.^{5,6} When consideration is given to the counterpropagating fields as well, the coupling causes even greater fluctuations in the limiting case. In these simple models, the signals are coupled through their mutual interaction with incoherently excited atoms. The details of these analyses are presented elsewhere.⁷ It is sufficient to note here that we have established a formalism for any number of coupled signals in which the competition is shown to be equivalent to the mode competition in multimode laser theories recently reported.^{8,9} In particular, the normalized variance (second cumulant) of the IPDF of each signal approaches a limiting value of $\frac{1}{3}$ when two signals are present and $\frac{3}{5}$ for four signals.

Preliminary measurements of the IPDF of a polarized component of an ASE source were recently reported.¹⁰ We have improved the measurement system and report here measurements on a different (longer) discharge tube permitting us to observe more heavily saturated conditions in the amplifier.

II. EXPERIMENT

A Brewster-angle quartz window was used to select a single linearly polarized component from the output of a 2-m long ASE source. The source was a 4-mm-diameter (i.d.) discharge tube filled with 190 mTorr of xenon (natural isotopic abundance) and 4 Torr of helium. This resulted in a pressure-broadened homogeneous linewidth of about 80 MHz in addition to the Doppler and isotopic inhomogeneously-broadened linewidth of approximately 150 MHz. The cold cathode discharge was maintained using a 6 kv dc power supply at a discharge current of 4.5 mA. A nonexcited return path guaranteed homogeneity along the discharge. The length of the discharge could be changed by selecting among various anodes.

As shown in Fig. 1, the light passed through an initial 2.5 mm aperture, was selected by the Brewster-angle window, passed through a chopping wheel, and a second aperture (105 cm from the first), and was focused on a high-speed reversebiased InAs detector by a quartz lens. The beam was chopped only in making lock-in measurements of the mean power. Extreme caution was taken to avoid back reflections of any light into the amplifier at either end. A second detector was used to monitor possible back reflections and to confirm their elimination.

The detector bandwidth exceeded that of the homodyne signal so the full fluctuating photocurrent corresponded to the slowly varying portion of the intensity. The photocurrent power spectrum varied between 35 and 50 MHz FWHM as the discharge length was changed.

The photocurrent was amplified and then sampled at a 20-kHz rate by a Tektronix 3S1 Sampling Oscilloscope plug-in used as a fast sample-and-hold. The output signal was gated by an Ortec Model 226



FIG. 1. Experimental setup: A, anodes; C, cold aluminum cathode; S_1 , S_2 , S_3 , blackened pinhole apertures; F, quartz flat; W, chopping wheel; Q, quartz lens; D, indium-arsenide reverse-biased detectors.

Linear Gate and the resulting pulses were fed to a Canberra Series 30 Multichannel Analyzer operating in a pulse-height-analysis mode. The resulting histograms and the corresponding histograms of the electronic noise (each containing approximately three million samples) were fed to an IBM 5100 minicomputer for analysis of the cumulants. On the assumption that the noise from all sources was an independent, zero-mean, random variable added to the photocurrent, the signal cumulants were calculated by subtracting the noise cumulants from the signal-plus-noise cumulants. The validity of this step was verified and a simultaneous check of the detector linearity was accomplished by measurements of the same ASE signal adjusted to different intensity levels by the insertion of passive attenuators between the source and detector.

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The results for the normalized cumulants versus discharge length are indicated in Fig. 2. Among the more obvious features is the more rapid fractional reduction of the higher cumulants in comparison with the lower cumulants as expected. We also notice that the second cumulant seems to have leveled out (rather than approaching zero) at a value approximately that predicted by the simple four-signal model. Quantitative agreement is not particularly expected since the experimental parameters are not exactly those of full homogeneous broadening. Other complications such as propagation effects, atomic memory effects, and distributed spontaneous emission have also not been considered in the theoretical model.



FIG. 2. Normalized cumulants as a fraction of their values for thermal light $[K_n/\bar{I}^n(n-1)!]$ versus discharge length denoted by $\Box, n=2; \Delta, n=3; \forall, n=4; \exists, n=5; \Delta, n=6; \nabla, n=7$. Typical error bars are shown for n=2.

Spatial coherence. The diameter of the second aperture was varied to investigate the spatial coherence of the output beams from the 200-cm source and the 70-cm source. The resulting cumulants for aperture diameters from 3 to 0.5 mm are shown in Table I. Because of the poor accuracy in determining the mean at low signal levels, we have tabulated the cumulants normalized to the appropriate power of K_2 which could be measured more accurately. Of particular note is the almost complete lack of variation. Even assuming that the effective source is at the far end of the discharge, the spatial coherence parameter¹¹ κ varied from 0.064 to 0.38 at the 200-cm length and from 0.11 to 0.65 at the 70-cm length without any evidence of the variation in the cumulants expected for a thermal source.¹² Here $\kappa = kr_1r_2/z_1R$, where $k = 2\pi/\lambda$, $2z_1$ is the first zero of the Bessel Function $J_1(z)$, r_1 and r_2 are source and receiver aperture radii, and R is their separation. The power output measurements indicate a e^{-1} beam diameter of 3.4 mm for the 200-cm case and 2.8 mm for the 70-cm case at the location of the second aperture.

III. CONCLUSION

These measurements demonstrate that saturation in a partially homogeneously broadened ASE source leads to partial reduction in the fluctuations characteristic of the initial negative exponential IPDF. Relatively large fluctuations are main-

TABLE I. Cumulants are normalized to the $\frac{1}{2}n$ power of the variance. Uncertainties are least count unless otherwise indicated.

I Second pinhole diameter (mm)	Discharge 1 Relative intensity (mV)	length 200 Norm <i>i</i> = 3	cm alized cun i=4	nulants $i=5$
3.0	3 44	1 94	2.06	9.59
2.5	0.11	1.24	2.00	0.00
2.5	2.03	1.24	2.04	3.50
2.0	1.90	1.24	2.06	3.56
1.4	1.06	1.24	2.04	3.44
1.0	0.49	1.24	2.03	3.43
0.8	0.34	1.24	2.04	3.51
0.5	0.12	1.28	1.7 ± 0.6	3.2 ± 0.8
Discharge length 70 cm				
(mm)	(μV)			
3.0	450	1.58	3.80	11.2
2.5	360	1 61	3 91	12.0
· 20	250	1 61	2.06	10 5
2.0	200	1.01	3.90	12.0
1.4	156	1.62	4.03	12.9
1.0	81	1.62	4.04	12.6
0.8	54	1.8 ± 0.3	3.2 ± 1.5	

tained, however, even for strong saturation. The beam displays a high degree of spatial coherence which, together with the directionality resulting from the narrow-diameter cylindrical geometry, may indicate that cw ASE will be useful as a signal carrier. It may also be worthy of study in the context of the speckle reduction sought in the use of quasihomogeneous sources.

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