Electron-atom scattering in the field of a low-frequency laser

Marvin H. Mittleman

Physics Department, The City College of the City University of New York, New York, New York 10031

(Received 29 January 1979)

Previous work on potential scattering in the field of a low-frequency laser and a model multichannel problem are combined to present a theory of electron-atom scattering in the field of a low-frequency laser. The atomic distortion by the laser is treated in perturbation theory, which is consistent with the other approximations of the theory. The only additional assumption necessary is that the scattering in the absence of the laser has no sharp resonances which participate in the scattering in the presence of the laser. A result is obtained which is a simple multichannel generalization of that of Kroll and Watson. It is used to calculate the average number of photons absorbed or emitted during scattering.

I. INTRODUCTION

A brief discussion of the history of the problem of the scattering of a charged particle in the field of a low-frequency laser is given. The cross section for electron-atom scattering in the field of a low-frequency laser is derived with the approximation that only nonresonant scattering occurs. There is a cancellation of two terms of order ω which yields a simple generalization of the potential-scattering cross section obtained by Kroll and Watson.[1] The result is used to derive an expression for the average number of photons transferred between the projectile and the laser, and it is found that it can be either positive or negative, depending upon the details of the momentum-transfer cross section.

II. NONRESONANT ELECTRON-ATOM SCATTERING

It has long been known[2] that the interaction of low-frequency electromagnetic waves with matter is much simpler than a similar problem at higher frequencies. Recently Kroll and Watson[1] carried this result beyond previous work by eliminating the need for the expansion in powers of the field. More precisely, for the problem of scattering of a charged particle by a local potential in the field of a low-frequency (classical) laser, they assumed that the laser frequency ($\hbar = 1$) was small compared to the particle energy. More recently Krüger and Jung[3] have pointed out that it is also necessary to assume that the T matrix in the absence of the laser has no resonance structure which is comparable in width to ω. The field is allowed to be intense in the sense that the parameter $\alpha_0 = eE/m\omega^2$ is not assumed small compared to the range of the potential. We shall see that this nevertheless allows for a perturbation treatment of the field when dealing with bound states.

The low-frequency result of Kroll and Watson can then be written

$$\frac{d\sigma}{d\Omega}(\vec{p}_f(l), \vec{p}_i; l)$$

$$= \frac{|p_f(l)|}{|p_i|} J_l^2([\vec{p}_f(l) - \vec{p}_i] \cdot \vec{\alpha}_0) \frac{d\sigma}{d\Omega}(\vec{P}_f(l), \vec{P}_i(l)),$$

$$(2.1)$$

where the integer l describes the number of photons transferred between projectile and field during the scattering. It enters into the energy-conservation condition

$$p_f^2(l)/2m = (p_i^2/2m) + l\omega \qquad (2.2)$$

which relates the magnitudes of the initial and final momenta of the projectile. The cross section appearing on the right-hand side of (2.1) is that in the absence of the field, but the momenta appearing are shifted from the initial and final momenta by a vector of order ω,

$$\vec{P}_i(l) = \vec{p}_i - \omega l m \vec{\alpha}_0 / x(l),$$

$$\vec{P}_f(l) = \vec{p}_f(l) - \omega l m \vec{\alpha}_0 / x(l), \qquad (2.3)$$

where

$$x(l) = [\vec{p}_f(l) - \vec{p}_i] \cdot \vec{\alpha}_0$$

and, since

$$P_i^2(l) = P_f^2(l), \qquad (2.4)$$

this cross section is evaluated on shell.

An interesting application of (2.1) can be made to evaluate the average number of photons, $\bar{l}$, transferred during a scattering event. It is defined

$$\bar{l}\sigma_T = \sum_l l \int \frac{d\sigma}{d\Omega}(\vec{p}_f(l), \vec{p}_i; l) d\Omega_f, \qquad (2.5)$$

where σ_T is the total cross section (for all l and

directions of $\vec{p}_f$). To evaluate $\bar{I}$ it is useful to rewrite the cross section on the right-hand side of (2.1) as

$$\frac{d\sigma}{d\Omega}(\vec{P}_f(l),\vec{P}_i(l)) = \frac{d\sigma}{d\Omega}\left(\frac{P_i^2(l)}{2m},\hat{P}_f(l)\cdot\hat{P}_i(l)\right), \quad (2.6)$$

where the scattering potential has been assumed to be central. The evaluation of (2.5) is further facilitated by the expansion of (2.1) in powers of $l\omega$. That is, we write

$$|p_f(l)/p_i| = 1 + l\omega m/p_i^2 + \cdots,$$

$$J_i^2(x(l)) = J_i^2(x(0)) + 2J_i(x(0))J_i'(x(0))$$

$$\times \frac{l\omega m}{p_i^2}\vec{p}_f(0)\cdot\vec{\alpha}_0 + \cdots,$$

$$\frac{d\sigma}{d\Omega}\left(\frac{P_i^2(l)}{2m},\hat{P}_f(l)\cdot\hat{P}_i(l)\right) \quad (2.7)$$

$$= \frac{d\sigma}{d\Omega}(\epsilon_i,\mu) - \frac{l\omega\vec{p}_i\cdot\alpha_0}{x(0)}\frac{\partial}{\partial\epsilon_i}\frac{d\sigma}{d\Omega}(\epsilon_i,\mu)$$

$$+ \frac{l\omega m}{p_i^2}(\mu-1)\frac{\vec{\alpha}_0\cdot(\vec{p}_i+\vec{p}_f(0))}{x(0)}\frac{\partial}{\partial\mu}\frac{d\sigma}{d\Omega}(\epsilon_i,\mu) + \cdots$$

where quantities such as $\vec{p}_f(0)$ and $x(0)$ are evaluated at $l = 0$ and we use

$$\epsilon_i = P_i^2/2m, \quad \mu = \hat{P}_i\cdot\hat{P}_f. \quad (2.8)$$

These can be substituted back into (2.1) and (2.5), and the l sums can be done with the aid of

$$\sum_l J_l^2(x) = 1, \quad \sum_l l^2 J_l^2(x) = \frac{x^2}{2}, \quad \sum_l l J_l^2(x) = 0. \quad (2.9)$$

The integral over the final direction of the scat-

tering can then be done with the result

$$\bar{I}\sigma_T(\epsilon_i) = \tfrac{1}{2}\omega m\alpha_0^2[1 + (\hat{p}_i\cdot\alpha_0)^2]\sigma_6(\epsilon_i)$$

$$+ \omega m\alpha_0^2(\hat{p}_i\cdot\hat{\alpha}_0)^2\epsilon_i\frac{\partial\sigma_6}{\partial\epsilon_i}(\epsilon_i), \quad (2.10)$$

where $\sigma_T(\epsilon_i)$ and $\sigma_6(\epsilon_i)$ are the total cross section and the momentum-transfer cross section, respectively, both evaluated in the absence of the field. The first term of (2.10) is positive definite and represents heating in the sense that the particle, on the average, absorbs energy from the laser. The sign of the second term depends upon the energy derivative of the momentum-transfer cross section and so can cause a net transfer of energy to or from the laser. That is, it is in principle possible to amplify the laser with the energy of an electron beam if the momentum-transfer cross section is decreasing with energy. On the other hand, if the projectiles have an initial isotropic Maxwellian distribution, it can be shown that $\bar{I}$ is positive so that particle heating by the laser will always occur.

The derivation of Kroll and Watson[1] has been repeated[4] in a more systematic way confirming their results and giving the first off-shell contributions to the T matrix in the ω^2 terms. The procedure used was an expression for the exact T matrix as a power series in the scattering potential

$$T_{\vec{p}_f,\vec{p}_i}(l) = \sum_{n=0}^{\infty} T_{\vec{p}_f,\vec{p}_i}^{n+1}(l), \quad (2.11)$$

where

$$T_{\vec{p}_f,\vec{p}_i}^{n+1}(l) = \sum_{L_1\cdots L_n}\int\left(\frac{d^3k}{(2\pi)^3}\right)^n U(p_f - k_1; l - l_1)\frac{1}{\Delta_1 + \omega L_1}U(\vec{k}_1 - \vec{k}_2; l_1 - l_2)\frac{1}{\Delta_2 + \omega L_2}\cdots\frac{1}{\Delta_n + \omega L_n}U(\vec{k}_n - \vec{p}_i; l_n), \quad (2.12)$$

where

$$U(\vec{K}; a) = \vec{V}(\vec{K})J_a(\vec{K}\cdot\vec{\alpha}_0) \quad (2.13)$$

and

$$\Delta_j = \epsilon_i - k_j^2/2m + i\eta. \quad (2.14)$$

Here V is the (local) scattering potential in momentum space. Equations (2.11)–(2.14) lend themselves to an interesting interpretation. The laser modifies the scattering problem by adding another discrete degree of freedom denoted by the integer L. The scattering potential in this augmented space is U and the diagonal propagator is $(\Delta + \omega L)^{-1}$. The propagators can then be expanded in powers of ω (assuming no rapid variation of T with respect to energy) and the L sums can then be done[4] yielding the Kroll and Watson result and higher-order terms in ω. It must be pointed out here that this result was obtained by this method

and originally by Kroll and Watson by the explicit use of the fact that the original scattering potential was local in configuration space so that

$$\langle\vec{k}|V|\vec{k}'\rangle = \vec{V}(\vec{k} - \vec{k}'),$$

which means that momentum translations of the form (2.3) will leave $\vec{V}$ unchanged. This result will be modified by a nonlocal potential which we expect to enter in the electron-atom scattering potential through exchange scattering.

A subsequent paper[5] allowed for a multichannel scattering from a two-state model atom by describing the atom in an adiabatic approximation. That is, with the assumption

$$\omega \ll W, \quad (2.15)$$

where W is the separation of the two dressed

states of the atom, the atomic states could be obtained analytically. They could then be used to calculate the 2×2 scattering-potential matrix. In fact the only property of these states that was used was that they could be written

$$e^{-i\overline{W}_n t}\Phi_n(\omega t),\qquad (2.16)$$

where $\overline{W}_n$ is the dressed energy level and Φ_n is a periodic function of ωt. It has been pointed out by Shirley[6] that this is a general property of states in a periodic field and has nothing to do with the two-state nature of the model. An expansion technique similar to (2.11) and (2.12) was used with (2.12) replaced by (neglecting overall phases)

$$T_{f,i}^{n+1}(l)=\sum_{\substack{L_1\cdots L_n\\ S_0\cdots S_n}}(-1)^{S_0}\int\left(\frac{d^3k}{(2\pi)^3}\right)^n\left(u_f,U(\vec{p}_f-\vec{k}_1;lS_0L_1S_1)\frac{1}{D_1}U(\vec{k}_1-\vec{k}_2;L_1S_1L_2S_2)\frac{1}{D_2}\cdots U(k_n-p_i;L_nS_n00)u_i\right),\qquad (2.17)$$

where

$$U(\vec{K};aS_0bS_1)=V_{S_0-S_1}(\vec{K})J_{a-b}(\vec{K}\cdot\vec{\alpha}_0)\qquad (2.18)$$

and

$$D_j=\epsilon_{p_i}+\overline{W}_i-\frac{k_j^2}{2m}-h+\omega(S_j+L_j)+i\eta.\qquad (2.19)$$

The scattering potential is similar to that occurring in the potential-scattering problem (2.13), except that now the indices S_0 and S_1 on $V(\vec{K})$ describe the periodic behavior of the scattering potential induced by the periodic behavior of the bound-state wave functions. More precisely, if these are expanded

$$\Phi_n(\omega t)=\sum_S\Phi_n^{(S)}e^{-iS\omega t},\qquad (2.20)$$

then

$$V_{S-S'}(\vec{K})=\int(\Phi_n^{(S)},V\Phi_{n'}^{(S')})d^3r\,e^{-i\vec{K}\cdot\vec{r}},\qquad (2.21)$$

where V is the projectile-atom interaction. The scattering potential on the left-hand side of (2.21)

is clearly a matrix in the space of the atomic states, but these indices (n,n') have been suppressed. The energy denominators (2.19) are similar to Δ, (2.14), except that the additional degree of freedom, the atomic state, also appears in (2.19) via the atomic Hamiltonian h which in this simple model is a diagonal 2×2 matrix of dressed energies. In addition the integer S_j describes the energy transferred to the projectile by the periodic variation of the scattering potential, (2.21). In effect, L describes the number of photons transferred directly between the projectile and the laser, and S describes the indirect photon transfer with the atomic target acting at the intermediary.

Again (2.17) can be expanded in powers of ω, and all the sums can be performed with the result that the energy-conservation condition is now

$$p_f^2(l)/2m+\overline{W}_f=p_i^2/2m+\overline{W}_i+\omega l\qquad (2.22)$$

and the T matrix is given by

$$T_{fi}(l)=\int_0^{2\pi}\frac{d\theta}{2\pi}\exp[il\theta+ix(l)\sin\theta]\left(\vec{P}_f(l),f\left|T(\theta)+\frac{i\omega}{x(l)}\frac{\partial}{\partial\theta}\left(T(\theta)\frac{\alpha_0\cdot\vec{P}}{(E_i-H)^2}T(\theta)\right)+\frac{i\omega}{2}\left(\frac{\partial T(\theta)}{\partial\theta}\frac{1}{(E_2-H)^2}T(\theta)\right)\right.\right.$$
$$\left.\left.-\frac{i\omega}{2}\left(T(\theta)\frac{1}{(E_i-H)^2}\frac{\partial T(\theta)}{\partial\theta}\right)\right|\vec{P}_i(l),i\right),\qquad (2.23)$$

where

$$E_i=(p_i^2/2m)+\overline{W}_i+i\eta,$$

$\vec{P}$ is the momentum operator, and H is the unperturbed Hamiltonian

$$H=h+P^2/2m.\qquad (2.24)$$

The T matrix is constructed from a scattering potential $V(\theta)$ in the usual way,

$$T(\theta)=V(\theta)+V(\theta)[1/(E_i-H)]T(\theta),\qquad (2.25)$$

and $V(\theta)$ is the periodic matrix given by

$$V(\theta)=(\Phi_n(\theta),V\Phi_{n'}(\theta)).\qquad (2.26)$$

That is, it is the time-dependent potential matrix where the time dependence arises from the perio-

dic variation of the atomic states induced by the laser. Then time is frozen during the scattering process $(\theta=\omega t)$, the particle scatters from this potential as described by $T(\theta)$, and then the *scattering amplitude* is *coherently* averaged over the phase of the laser. The last three terms of (2.23) are corrections to this process which we shall see are actually of order ω^2 and so will be of no interest here. Note that the matrix element is formed between atomic states f and i and between $\vec{P}_f(l)$ and $\vec{P}_i(l)$ [Eq. (2.23)]; so the first-order result of Kroll and Watson is contained here.

This model is unrealistic in that the atom is treated as a two-state model but, as has been indicated above, the generalization needed to include the multistate atom is straightforward. h

must be extended to a dressed diagonal matrix of infinite dimensionality, and V must be similarly extended. The final modification necessary to make the model realistic is the inclusion of exchange. It is well known[7] that exchange can be inserted at the cost of making the scattering potential nonlocal in configuration space. Then the translation of momenta, which is necessary to obtain the first-order Kroll-Watson result, no longer leaves the scattering potential unchanged. When this translation, by a term of order ω, is performed it can be expected to induce additional terms in the transition matrix of order ω because of this nonlocality.

Our first task in extending (2.23) to a real atom will be to find the realistic V matrix to order ω. This requires a knowledge of the target states to order ω. Fortunately these can be obtained by straightforward perturbation theory since the coupling between atoms and the (classical, single-mode) laser field in dipole approximation is

$$\frac{e}{m}\sum_{j=1}^{z}\vec{p}_j\cdot\vec{A}(t)=\omega\vec{\alpha}_0\cdot\sum_{j=1}^{z}\vec{p}_j\cos\omega t, \qquad (2.27)$$

which is of order ω. If the states are written as in (2.16), then the dressed energy $\overline{W}_n$ is just the unperturbed energy since the shift, the dynamical Stark effect, is of order ω^2. The shifted states can be obtained by using first-order time-dependent perturbation theory treating the laser-atom coupling [Eq. (2.27)] as the perturbing term. They can be written

$$\Phi_n(1\cdots z;\omega t)$$
$$= \left(1-im\omega\vec{\alpha}_0\cdot\sum_{j=1}^{z}\vec{r}_j\cos\omega t\right)u_n(1\cdots z), \qquad (2.28)$$

where the u_n are the unperturbed states. These will enter in density matrices

$$\rho_{nn'}(rx;r'x';\omega t)$$
$$= \int d^3x_3\cdots d^3x_z\Phi_n^*(rxx_3\cdots x_z;\omega t)$$
$$\times \Phi_{n'}(r'x'x_3\cdots x_z;\omega t) \qquad (2.29)$$

which we expand as

$$\rho_{nn'}(rx;r'x';\omega t)$$
$$= \rho_{nn'}^{(0)}(rx;r'x')+\rho_{nn'}^{(1)}(rx;r'x';\omega t)+\cdots. \qquad (2.30)$$

Insertion of (2.28) into (2.29) and (2.30) yields

$$\rho_{nn'}^{(1)}(rx;r'x';\omega t)$$
$$= im\omega\vec{\alpha}_0\cdot(\vec{r}+\vec{x}-\vec{r}'-\vec{x}')$$
$$\times \cos\omega t\rho_{nn'}^{(0)}(rx;r'x'). \qquad (2.31)$$

We now expand the total wave function in a close-coupling form:

$$\Psi=A_0\sum_{n'}\Phi_{n'}(1\cdots z;\omega t)F_{n'}(r_0,\omega t)e^{-iW_{n'}t}, \qquad (2.32)$$

where A_0 is the antisymmetrization operator

$$A_0=1-\sum_{j=1}^{z}X_{j0} \qquad (2.33)$$

and X_{j0} is the electron exchange operator. The full Hamiltonian can be written

$$H=\sum_{j=0}^{z}\left(\frac{p_j^2}{2m}+\frac{e}{m}\vec{p}_j\cdot A(\vec{t})+V_j\right)+\sum_{i>j=0}^{z}\mathcal{v}_{ij} \qquad (2.34)$$

and then the operation

$$\int d^3x_1\cdots d^3x_z\Phi_n^*(1\cdots z;\omega t)\left(\frac{i\partial}{\partial t}-H\right)\Psi=0 \qquad (2.35)$$

results in a set of coupled equations for the F. These are

$$\left[\frac{i\partial}{\partial t}-\left(\frac{p_0^2}{2m}+\omega\vec{\alpha}_0\cdot\vec{p}_0\cos\omega t+V_0+W_n\right)\right]F_n(r_0,t)-\sum_{n'}\int d^3x\rho_{nn'}(x,x;\omega t)\mathcal{v}(r_0-x)F_{n'}(r_0,t)$$

$$-\sum_{n'}\int V_{nn'}^X(r_0,r';\omega t)F_{n'}(r',t)d^3r'=0, \qquad (2.36)$$

where

$$\rho_{nn'}(r,r';\omega t)=\int d^3x\rho_{nn'}(rx;r'x;\omega t). \qquad (2.37)$$

The exchange potential arises from the X_{j0} in (2.33) and is given by

$$V_{nn'}^X(r_0,r';\omega t)F_{n'}(r',\omega t)=zi\frac{\partial}{\partial t}[\rho_{nn'}(r',r_0;\omega t)F_{n'}(r',t)]$$

$$-z\left[W_n+\frac{p_0^2}{2m}+\omega\vec{\alpha}_0\cdot\vec{p}_0\cos\omega t+V_0+\mathcal{v}(r_0-r')\right]\rho_{nn'}(r',r_0;\omega t)F_{n'}(r',t)$$

$$-z(z-1)\int d^3x\rho_{nn'}(r'x;r_0x;\omega t)\mathcal{v}(r_0-x)F_{n'}(r',t). \qquad (2.38)$$

The exchange potential can now be expanded in ω with the aid of (2.30) and (2.31), with the result

$$V_{nn'}^{X\,(0)}(r_0,r') = z\rho_{nn'}^{(0)}(r',r_0)\, i\frac{\partial}{\partial t} - z\left[W_n + \frac{p_0^2}{2m} + V_0 + \mho(r_0 - r')\right]\rho_{nn'}^{(0)}(r',r_0)$$

$$- z(z-1)\int d^3x\,\rho_{nn'}^{(0)}(r'x,r_0x)\mho(r_0-x) \tag{2.39}$$

and

$$V_{nn'}^{X\,(1)}(r_0,r';\omega t) = z\rho_{nn'}^{(1)}(r',r_0;\omega t)\, i\frac{\partial}{\partial t} - z\left[W_n + \frac{p_0^2}{2m} + V_0 + \mho(r_0-r')\right]\rho_{nn'}^{(1)}(r',r_0;\omega t)$$

$$- z(z-1)\int d^3x\,\rho_{nn'}^{(1)}(r'x,r_0x;\omega t)\mho(r_0-x) - z\,\omega\vec{\alpha}_0\cdot\vec{p}_0\cos\omega t\,\rho_{nn'}^{(0)}(r,r_0). \tag{2.40}$$

Insertion of (2.31) into (2.40) with a little algebra results in

$$V_{nn'}^{X\,(1)}(r,r';\theta)$$

$$= -im\omega\vec{\alpha}_0\cdot(\vec{r}-\vec{r}')\cos\theta V_{nn'}^{X\,(0)}(r,r'). \tag{2.41}$$

As a brief digression it should be pointed out that this result as well as Eq. (2.31) are, in the $\omega \to 0$ limit, simply consequences of the gauge invariance of the theory.

The direct potential in (2.36) can be written

$$V_{nn'}^{D}(r,\omega t)$$

$$= \delta_{nn'}V(r) + \int d^3x\,\rho_{nn'}(x,x;\omega t)\mho(r-x). \tag{2.42}$$

If this is expanded in powers of ω, it is easily shown that the first-order terms vanish because of (2.31) and (2.37).

Returning to (2.23), it is seen that the last three terms are really order ω^2. This arises because of the $\theta = \omega t$ derivative. From (2.39) and (2.42), the zero-order potentials are independent of t so that the θ derivative of T (proportional to the θ deriva-

tive of V) is of order ω. The first term of (2.23) then contains the entire result to order ω.

There are two sources of corrections of order ω that arise in T. The first simply comes from the correction to the exchange potential of order ω and there is no correction to the direct potential of this order. The second comes from the translation of momenta that was necessary to get the Kroll-Watson result, that is, to get the capitalized momenta (2.3) occurring in the initial and final states of (2.23). As was pointed out above, the locality of the potential was used, but, for a nonlocal potential, corrections will arise. For nonlocal potentials, (2.18) must be replaced by

$$(\bar{k}aS_0|U|\bar{k}'bS_1)$$

$$= (\bar{k}|V_{S_0-S_1}|\bar{k}')J_{a-b}(\bar{k}-\bar{k}'\cdot\vec{\alpha}_0) \tag{2.43}$$

and the translation (2.3) replaces this by

$$(\bar{k}+\vec{\lambda}|V_{S_0-S_1}|\bar{k}'+\vec{\lambda})J_{a-b}(\bar{k}-\bar{k}'\cdot\vec{\alpha}_0),$$

where $\vec{\lambda} = m\omega l\vec{\alpha}_0/x$. This can be expanded in powers of ω yielding

$$[(\bar{k}|V_{S_0-S_1}|\bar{k}') + (\omega ml/x)\vec{\alpha}_0\cdot(\vec{\nabla}_k+\vec{\nabla}_{k'})(\bar{k}|V_{S_0-S_1}|\bar{k}')]J_{a-b}(\bar{k}-\bar{k}'\cdot\vec{\alpha}_0). \tag{2.44}$$

The second term of (2.44) is the second source of ω corrections in $T(\theta)$. It can be transformed to configuration space and to the θ representation instead of the S representation with the result that it is

$$- (i\omega ml/x)\vec{\alpha}_0\cdot(\vec{r}-\vec{r}')V^{X\,(0)}(r,r'), \tag{2.45}$$

where only the zero-order part of the exchange (nonlocal) potential enters. It is θ independent.

The two first-order potentials are now included in the T matrix in (2.23). When they are treated to first order they generate a new term in (2.23) which is

$$\int_0^{2\pi}\frac{d\theta}{2\pi}\exp(il\theta + ix\sin\theta)(\vec{P}_f(l),f|T^{(0)}(V^{(0)})^{-1}\delta V(\theta)$$

$$\times (V^{(0)})^{-1}T^{(0)}|\vec{P}_i(l),i), \tag{2.46}$$

where $V^{(0)}$ is the potential in the $\omega \to 0$ limit and $T^{(0)}$ is the T matrix which it generates. Both are independent of θ. $\delta V(\theta)$ is the sum of (2.41) and (2.45), and since the $\cos\theta$ factor in (2.41) is the only remaining θ dependence in the bracket in (2.46), it can be removed by an integration by parts such that it is replaced by $-l/x$.

Then (2.41) and (2.45) exactly cancel each other, and the T matrix for nonresonant electron-atom scattering in the field of a low-frequency laser is

given by

$$T_{fi}(l) = J_l(x)(\vec{P}_f(l), f \, | T^{(0)}(E) | \, \vec{P}_i(l), i)$$
$$+ O(\omega^2). \tag{2.47}$$

As in the result of Kroll and Watson the T matrix on the right-hand side is evaluated on-shell so there is no difficulty in interpreting it in terms of the T matrix in the absence of the laser. That is, the energy-conservation equation

$$p_i^2/2m + W_i = p_f^2(l)/2m + W_f - l\omega, \tag{2.48}$$

when combined with (2.3), yields

$$E = P_i^2(l)/2m + W_i = P_f^2(l)/2m + W_f. \tag{2.49}$$

The cross section can then be written

$$\frac{d\sigma}{d\Omega}(\vec{p}_f f; \vec{p}_i i; l)$$

$$= J_l^2(x(l)) \left| \frac{p_f(l)}{P_f(l)} \right| \left| \frac{P_i(l)}{p_i} \right| \frac{d\sigma}{d\Omega}$$

$$\times (\vec{P}_f(l), f; \vec{P}_i(l), i). \tag{2.50}$$

If we again rewrite the cross section in the absence of the laser as

$$\frac{d\sigma_{fi}}{d\Omega}\left(\frac{P_i^2(l)}{2m}, \hat{P}_i(l) \cdot \hat{P}_f(l) \right),$$

then a result similar to (2.10) can be obtained:

$$\bar{l}\sigma_T = \omega m \alpha_0^2 (\hat{p}_i \cdot \hat{\alpha}_0)^2 \frac{p_i^2}{2m} \sum_f \frac{\partial}{\partial E} \sigma_{fi,6}(E)$$

$$+ \frac{m\omega\alpha_0^2}{2} [1 + (\hat{p}_i \cdot \hat{\alpha}_0)^2] \sum_f \sigma_{fi,6}(E), \tag{2.51}$$

where we define a generalized momentum-transfer cross section

$$\sigma_{fi,6}(E) = \int d\Omega \left(1 - \frac{p_f}{p_i} \mu \right) \frac{d\sigma_{fi}}{d\Omega}(E). \tag{2.52}$$

It is difficult to make statements about the sign of $\bar{l}$ in this case. One reason is the fact that (2.52) can be negative for superelastic scattering. However, if all the important $\sigma_{fi,6}$ in (2.51) are positive, and if the distribution of the free electrons is Maxwellian, then again $\bar{l}$ is positive definite.

ACKNOWLEDGMENT

---

[1]N. K. Kroll and K. M. Watson, Phys. Rev. A **8**, 804 (1973); and see M. Gavrilla and M. J. van der Weil, Comments At. Mol. Phys. **8**, 1 (1978) for a comprehensive review and bibliography.

[2]See, for example, F. Low, Phys. Rev. **110**, 974 (1958).

[3]H. Krüger and C. Jung, Phys. Rev. A **17**, 1706 (1978).

[4]M. H. Mittleman, Phys. Rev. A **19**, 99 (1979).

[5]M. H. Mittleman, Phys. Rev. A **19**, 134 (1979).

[6]J. H. Shirley, Phys. Rev. B **138**, 979 (1965).

[7]H. Feshbach, Ann. Phys. (N.Y.) **19**, 287 (1962).