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Screening effects in pair production near threshold

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Pair production cross sections near threshold have been calculated numerically in partial waves for a screened central potential, permitting a comparison of theory with recent experiments. Exchange effects are found to remain small. The level of agreement with experiment is variable and does not suggest any systematic trends. A more approximate theory of screening, energy-shift screening theory (EST), identifying screened cross sections with Coulomb cross sections of shifted spectrum energy V_0, is in agreement with the full calculation if the proper choice is made for the energy shift V_0; however, the procedure becomes more delicate as threshold is approached. A previous attempt to apply EST to threshold experiments made an incorrect choice for V_0.

In the absence of detailed numerical calculations of atomic electron screening effects in threshold pair production (within 100 keV), recent experiments[1-3] have been interpreted using point-Coulomb (unscreened) predictions together with approximate treatments of screening.[4,5] Here we wish to report direct screened calculations which can be compared with these experiments. These calculations also demonstrate that the approximate treatment of screening through energy-shift screening theory[5] can be used *if* the energy shift V_0 is appropriately chosen (which was not done in the attempt to interpret these experiments).

Our full numerical calculations are based on an exact partial-wave formulation, which has been described previously.[4] For low photon energies only a few, but more than one, partial waves in the positron, electron, and photon series contribute. Our results for the pair production cross sections[6] are shown in Table I and Figs. 1 and 2.

Earlier experimental measurements of the pair production cross section, and comparisons with theory, have been discussed by Motz, Olsen, and Koch,[7] by Øverbø,[8] and by Tseng and Pratt.[5] Here we examine recent experimental measurements of near-threshold pair production cross sections. The data[9] are shown in Table II. Agreement with the experimental results of Coquette[3,10] for $Z=32$ is good for the cases $k=2.08$ and $2.178m_ec^2$, but there is about a 12% discrepancy for the intermediate energy $k=2.127m_ec^2$. For the case $Z=82$, $k=2.190m_ec^2$, our result lies almost in the middle of the five experimental values.[1,11-14] For the case $Z=92$, $k=2.190m_ec^2$, there is about a 33% difference from the experimental data of Girard, Avignone, and Huntsburger.[2,15]

One may ask how sensitively our results depend on the choice of realistic screened central potential. Previously[4,5] we had used the Kohn-Sham (KS) potential,[16] which includes an approximate exchange term which is actually not appropriate for positrons. To see whether such features are of serious concern, we have made numerical calculations with a Hartree-Fock-Slater potential with the ex-

TABLE I. Pair production cross sections $\sigma(E_+) \equiv Z^{-2} d\sigma/dE_+$ computed with the partial-wave method for atomic numbers $Z=32, 82, 92$ and photon energies $k=2.08, 2.1, 2.127, 2.178, 2.19m_ec^2$, by using the Hartree-Fock-Slater potential with the exchange term omitted (HFN). Here $y\equiv(E_+-1)/(k-2)$ and σ_{tot} is the total pair production cross section.

y \ k (m_ec^2), Z	2.08, 32	2.10, 92	2.127, 32	2.178, 32	2.19, 82	2.19, 92
0.95	3.746	4.476	7.445	11.457	24.263	24.352
0.9	3.504	3.817	7.141	11.197	21.917	21.679
0.7	2.469	1.706	5.615	9.564	12.859	11.888
0.5	1.402	0.5179	3.664	6.903	5.489	4.636
0.3	0.4907	0.06957	1.558	3.380	1.157	0.8520
0.1	0.02706	0.000544	0.1199	0.3395	0.01796	0.01006
σ_{tot} (mb/atom)	0.130	1.06	0.472	1.15	10.7	12.7

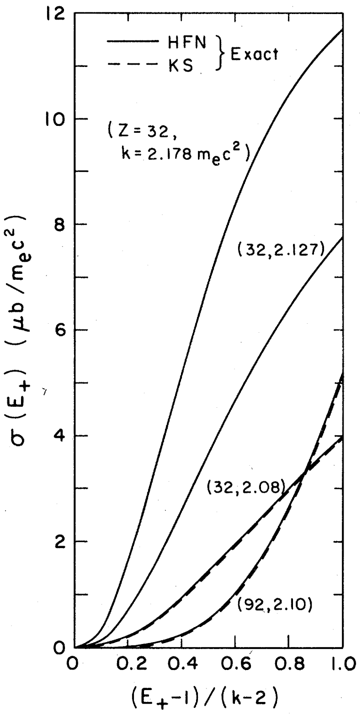

FIG. 1. Pair production differential cross sections $\sigma \equiv Z^{-2} d\sigma/dE_+(E_+)$ of the exact numerical calculation in partial waves for the HFN potential (solid lines) and the KS potential (broken lines) for $Z=32$, $k=2.08$, 2.127, and $2.178 m_e c^2$; and $Z=92$, $k=2.10 m_e c^2$.

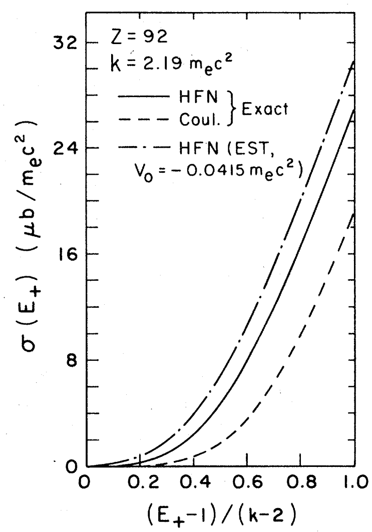

FIG. 2. Pair production differential cross section $\sigma(E_+)$ of the exact numerical calculation in partial waves for the HFN potential (solid line) and the point-Coulomb potential (Coul, broken line) for $Z=92$, $k=2.13 m_e c^2$. The dotted line is the result with the energy-shift theory, and $V_0 \approx -0.0415 m_e c^2$.

change term entirely omitted (HFN). Comparisons between the results obtained with the KS potential and with the HFN potential are shown in Fig. 1. The exchange effect on the total cross section and on the dominant part of the energy spectrum is less than 3% for $Z=92$ and $k=2.1 m_e c^2$, and less than 1.5% for $Z=32$ and $k=2.08 m_e c^2$, becoming less important as energy increases or Z decreases. This can be understood qualitatively from Fig. 3, where we show the potential difference $V - V_c$ between the screened (V_s) and the unscreened ($V_c = -Z\alpha/r$) potentials in these two models. For photon energies from threshold to about 5 MeV, the pair production matrix element is determined at small distances[4,5] (about the order of the electron Compton wavelength), and at such distances wave functions are point Coulomb in shape and independent of screening. However, for low energies, wave-function normalizations are determined at larger distances, where the two screened potentials are similar, and hence similar screening effects are predicted in both potentials. (At higher energies, where the normalizations are de-

TABLE II. Comparisons between theory and experiment for the total pair production cross section σ near threshold. Symbols "B" and "expt" refer to the point-Coulomb Born approximation and experiments, respectively.

Z	k (MeV)	k ($m_e c^2$)	Reference	$\sigma_{\rm expt}/\sigma_{\rm B}$	$\sigma_{\rm expt}$ (mb/atom)	$\sigma_{\rm HFN}/\sigma_{\rm B}$	$\sigma_{\rm HFN}$ (mb/atom)	$\sigma_{\rm expt}/\sigma_{\rm HFN}$
	1.063	2.080		2.03 ± 0.19		1.83	0.130	1.1 ± 0.10
32	1.087	2.127	3	2.24 ± 0.26		1.77	0.472	1.27 ± 0.15
	1.113	2.178		1.68 ± 0.13		1.67	1.15	1.01 ± 0.08
			1		14.5 ± 0.8			1.36 ± 0.07
			11		14.8 ± 0.7			1.38 ± 0.07
82	1.119	2.190	12		13.4 ± 1.0	1.97	10.7	1.25 ± 0.09
			13	1.8 ± 0.2				0.91 ± 0.10
			14	1.59 ± 0.16				0.81 ± 0.08
92	1.119	2.190	2		17.4 ± 0.5	1.86	12.7	1.37 ± 0.04

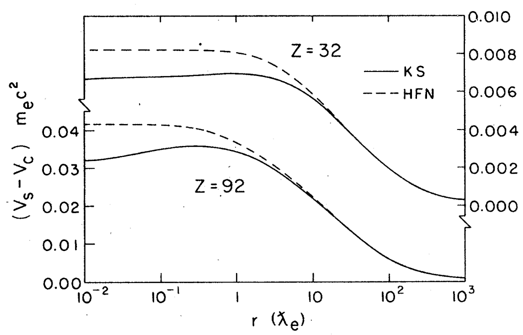

FIG. 3. Difference $V_s - V_c$ between the screened (KS or HFN) and the point-Coulomb potential as a function of distance r, in units of the Compton electron wavelength λ_e. The right-hand scale is for $Z=32$, and the left-hand scale is for $Z=92$.

termined at smaller distances, the entire effect of screening becomes unimportant.)

In our previous work[5,17] we found that at reasonably small distances a screened continuum electron or positron wave function of shifted energy $\delta E_\pm = \pm V_0$ (plus sign for positrons and minus sign for electrons)[18] is closer in shape to a point-Coulomb wave function than is a screened wave function of the same energy. Here $\delta E = E_s - E_c$ (the subscripts s and c stand for screened and point-Coulomb potentials, respectively). We proposed a simple relation between screened and point-Coulomb pair production energy distributions for a given photon energy k:

$$\sigma_s(E_+ + V_0, E_- - V_0) = \sigma_c(E_+, E_-)\,. \qquad (1)$$

The validity of the relationship for cross sections depended on the fact (which follows from the energy shift in comparing shapes) that a screened normalization and a Coulomb normalization of shifted energy are related as

$$\tilde{N}_s(E_\pm \pm V_0) = \tilde{N}_c(E_\pm)\,,$$

with $\tilde{N} \equiv (pE)^{1/2} N$, where N is a continuum-state normalization. This approach [called the energy-shift screening theory (EST)] was verified numerically[5] for photon energies k down to $2.5 m_e c^2$, and we used the method to convert all the point-Coulomb results of Øverbø, Mork, and Olsen[19] and of Øverbø[8] into screened predictions for photon energies $k = 3$ to $10 m_e c^2$. Can this approach be used for lower photon energies, and how should V_0 be determined? From Fig. 3 we see that, for the KS potential which we had used previously, there is little difference between the value of V_0 which characterizes very small distances (near the nuclear surface) and the value which characterizes the region of several Compton wavelengths. However, for the HFN potential the difference is significant, and so this case can be used to test the assertion made above that for these energies V_0 should characterize the range of several λ_e. (For still lower energies larger distances and a smaller effective V_0 would be expected.) In Fig. 2, we show for $Z=92$, $k=2.19 m_e c^2$ the pair production cross section $\sigma(E_+)$ calculated numerically in partial waves both with the HFN potential (solid line) and with the point-Coulomb potential (broken line). We also show the result with the EST for $V_0 = -0.0415 m_e c^2$ (the broken line), as would be determined at very small distances. It is evident

TABLE III. Values of $\tilde{N}_s(E_\pm = V_0)/\tilde{N}_c(E_\pm)$ for electrons and positrons, for the case shown in Fig. 2, with energy shift $\delta E_\pm = \pm V_0$ (plus sign for positrons and minus sign for electrons) for two possible choices of the V_0 which should characterize the HFN potential. The symbol E_c refers to energies of electrons or positrons (negative value for positrons, positive value for electrons). We define $\tilde{N} \equiv (pE)^{1/2} N$, where N is the continuum-state normalization and κ the angular momentum.

y	V_0 ($m_e c^2$)	E_c ($m_e c^2$)	κ: −1	+1	−2	+2	−5	+5
0.9	−0.0415	−1.171	0.862	0.890	0.812	0.856	0.643	0.693
		1.019	1.001	1.002	1.01	1.08	1.70	2.29
	−0.0279	−1.171	1.01	1.03	0.992	1.01	0.903	0.934
		1.019	0.994	0.975	0.972	0.960	1.14	1.34
0.7	−0.0415	−1.133	0.781	0.816	0.720	0.775	0.522	0.581
		1.057	1.000	1.002	1.01	1.06	1.35	1.55
	−0.0279	−1.133	0.992	1.02	0.964	0.996	0.838	0.881
		1.057	0.994	0.978	0.974	0.964	1.04	1.11
0.3	−0.0415	−1.057	0.164	0.187	0.126	0.162	0.039	0.060
		1.133	1.000	1.001	1.01	1.03	1.17	1.24
	−0.0279	−1.057	0.683	0.738	0.607	0.685	0.358	0.433
		1.133	0.994	0.981	0.978	0.970	0.997	1.02

that the EST with this choice of V_0 is incorrect. Clearly, also, there is a value (or small range of values) for V_0 (which we have taken as $\cong -0.0279 m_e c^2$) for which the EST prediction is correct,[20] and this value for V_0 does characterize the potential at several Compton wavelengths. To demonstrate that this is the proper choice of energy shift at these energies, we also, in Table III, show values of $\tilde{N}_s(E_\pm \pm V_0)/\tilde{N}_c(E_\pm)$ for electrons and positrons for this case. With $V_0 = -0.0415 m_e c^2$, which is determined near the nuclear radius for the HFN potential, $\tilde{N}_s/\tilde{N}_c \neq 1$ for the important positron partial waves. However, for $V_0 = -0.0279 m_e c^2$, we find that $\tilde{N}_s/\tilde{N}_c \cong 1$ for the important partial waves of both electron and positrons for the portion of the energy spectrum which contributes most to the total cross section. Even for $y \equiv (E_+ - 1)/(k-2) = 0.3$ (which does not contribute much to the total cross section) the choice with $V_0 = -0.0279 m_e c^2$ is better than that with $V_0 = -0.0415 m_e c^2$ (the positron energy is low, and we may anticipate that a smaller effective V_0 would be better). Note that for a given energy, electron waves are much less sensitive to screening than positron waves, which become very small near enough to the nucleus for it to appear unscreened.

Thus we conclude that the energy-shift normalization theory can be used to describe screening effects in atomic-field pair production near threshold, with a suitable choice for the energy shift. However, use of an energy shift characterizing very small (rather than Compton wavelength) distances in a potential is incorrect and will misestimate the actual screened cross sections in that potential. Also, as very low energies (particularly for the positron) are needed, larger distances become important and a constant V_0 characterizing several Compton wavelengths is not appropriate. For the current threshold experiments, the EST is adequate.

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[9]Note that, in Table II, the photon energies k given in MeV are for experimental results, while those given in $m_e c^2$ are for the theory.

[10]Note that in Fig. 1 of Ref. 3 the value of σ_{TP}/σ_B is the interpolated value from Table IV of Ref. 5 for $k \geqslant 2.10 m_e c^2$, while for $k < 2.10 m_e c^2$ the value of σ_{TP}/σ_B is the extrapolated value estimated by the author of Ref. 3.

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[15]The values of σ_{TP} given in Refs. 1 and 2 are the interpolated values from Table IV of Ref. 5. For the photon energy $k = 1.119$ MeV, the values we had been asked to provide were for $k = 1.1205$ MeV ($\approx 2.193 m_e c^2$). As a result the discrepancy is due partly to the energy of the source, since near threshold the total cross section σ varies quite rapidly with photon energy k. The theoretical values σ_Φ for the screened central potential (the Hartree-Fock-Slater potential with the exchange term omitted) quoted in Refs. 1 and 2 were obtained with the energy-shift screening theory, using $V_0 = -0.0160$, -0.0344, and $-0.0419 m_e c^2$ for $Z = 50$, 82, and 92, respectively. These values are incorrect when applied to threshold pair production, as will be discussed later in this work.

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[18]In our previous work using the KS potential we calculated the constant term $-V_0$ of the power-series expansion in r of the screened central potential near the nuclear radius. This is in fact not really proper, as will be discussed in this work, but for the KS (but not HFN potential) the value so determined is fairly good and adequate where screening effects are not very large. Hence, the values given in Table IV of Ref. 5 may still be used, since the shift was applied only in the region $k = 3$ to $10 m_e c^2$, where the screening effect is less than 5%.

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[20]This was also realized by J. Øverbø [Arkiv for Det Fysiske Seminar i Trondheim No. 9, 1978 (unpublished)]. In this paper Øverbø determined V_0 near the nuclear radius, so that his results for $k \lesssim 2.5 m_e c^2$ must be recalculated.