Quantization of evanescent electromagnetic waves: Momentum of the electromagnetic field very close to a dielectric medium

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The problem of the momentum of the electromagnetic field near a homogeneous dielectric plane surface is studied. In the particular case of evanescent waves, our calculations are consistent with the experimental result of S. Huard and C. Imbert which points out that, during an interaction between a moving atom and a surface wave of pulsation ω, the exchanged momentum is greater than $\hbar\omega/c$. This work leads at first to a straightforward procedure for quantizing the momentum parallel to the diopter of the field. The authors proceed to investigate the form of the component perpendicular to the interface which is transmitted by the field to an atom during an interaction. (In the case of an evanescent wave, this problem involves the interpretation of the imaginary part of the wave vector.) A general theory shows that the presence of a medium widens and shifts the $\vec{k}$ levels of the field. This result agrees with the uncertainty principle.

INTRODUCTION

The recent experimental results obtained by Huard and Imbert[1] show that the momentum parallel to the interface transmitted to a moving atom by an evanescent state of the electromagnetic field is $\hbar\vec{K}_\parallel$ (where $\vec{K}_\parallel$ is the real part of the complex wave vector of the evanescent wave). This result had been predicted by Costa de Beauregard *et al.*[2] and has been used by one of us in a theory of the Čerenkov effect[3] and in a theory of absorption and emission near a dielectric interface.[4] Although it agrees with the de Broglie formula $\vec{P}=\hbar\vec{K}$, this result is nevertheless surprising, since it means that, when there is an interaction between an electromagnetic field of pulsation ω and matter, the modulus of the exchanged momentum can be greater than $\hbar\omega/c$.

In order to explain this experimental result, we have examined the problem of quantization of the electromagnetic field momentum in a space which is filled with a homogeneous dielectric on the left of the plane $z=0$ and is empty on the right of the plane. In the quantum field theory the expressions of dynamic variables are obtained from Noether's theorem and from the principle of stationary action. The momentum thus corresponds to the invariant under a space translation. In the case we are now considering, the field is invariant under a translation parallel to the interface. We can therefore define the momentum $\vec{P}_\parallel$ of the field in the xy plane.

In Secs. II and III of this paper we study $\vec{P}_\parallel$. For this we use the triplet modes previously introduced by Carniglia and Mandel[5] for quantizing the energy in such a space. By expanding the field in terms of these modes it is possible to demonstrate that the field momentum parallel to the interface reduces to the sum of the momenta of independent harmonic oscillators. The quantization is therefore straightforward and proceeds as it does for a free field. In the case of an evanescent mode the momentum quantum is seen to be greater than $\hbar\omega/c$. This is consistent with the experimental results obtain by Huard and Imbert.[1] When the wave incident on the diopter is a circularly polarized wave, our result shows, moreover, that the momentum always lies in the plane of incidence, even in the case of total internal reflection, where a transverse energy flux is observed. This energy flux in a direction which is not collinear with the momentum was foreseen theoretically by Costa de Beauregard[6] and demonstrated experimentally by Imbert.[7] We also calculate the total contribution of evanescent modes to the momentum of the field. We can therefore draw the following conclusion: the momentum density of the transmitted evanescent part of the modes does not intervene in the calculation. Therefore our result does not depend on the choice of the electromagnetic tensor in the evanescent wave. As a final conclusion of Sec. III we discuss the importance of the formalism of triplet modes.

In Sec. IV we investigate the form of the momentum $\vec{P}_\perp$ perpendicular to the interface which is transmitted by the field to an atom during an interaction close to the diopter. In the particular case of an evanescent wave this problem involves the interpretation of the imaginary part of the wave vector of the transmitted field. The field is not invariant under a translation normal to the interface. We can therefore state that it will not be possible to reduce $\vec{P}_\perp$ to the sum of the momenta of independent harmonic oscillators. In fact, in addition to the terms containing the distribution $\delta(\vec{k}-\vec{k}')$, which express the independence of the different modes in the calculation of $\vec{P}_{\parallel}$, we have now to consider in the general expression of $\vec{P}_{\perp}$ the principal-part terms $\mathcal{P}(1/(k_z-k_z'))$. Because of the definition of the principal parts these terms clearly show, when the momentum of a mode $\vec{k}$ is calculated, the influence of all the other modes $\vec{k}'$. This result has the fundamental consequence of making it impossible to consider formally the interface-field system as a free field. This system thus appears as an interacting one. Therefore, by using the usual expansion in field theory for the determination of the dynamic variables in an interacting field, it is possible to define the momentum perpendicular to the interface $\vec{P}_{\perp}$. After calculations, $\vec{P}_{\perp}$ appears as the sum of a self-momentum $\vec{P}_0^z$ and of an interaction momentum $\vec{P}_{\text{int}}^z$. As a consequence of this result we can understand the influence of the dielectric: the presence of a spatial discontinuity of the field (physically described by the interface) widens and shifts its momentum states, just as a time perturbation of the field widens and shifts its energy states. This result occurs, of course, not only in an evanescent wave but also in a homogeneous one transmitted very close to the interface. It agrees with the uncertainty principle: the partial localization of the field (more exactly its unequal repartition in the two half-spaces $z \geq 0$ and $z \leq 0$) makes it impossible to know with certainty its momentum along $\hat{z}$.

I. NOTATIONS

Let us consider a space which is filled with an isotropic medium of refractive index n_0 on the left of the plane $z=0$ and is empty everywhere on the right of this plane (see Fig. 1). We use the triplet modes previously introduced by Carniglia and Mandel[5] and are keeping their notation. When

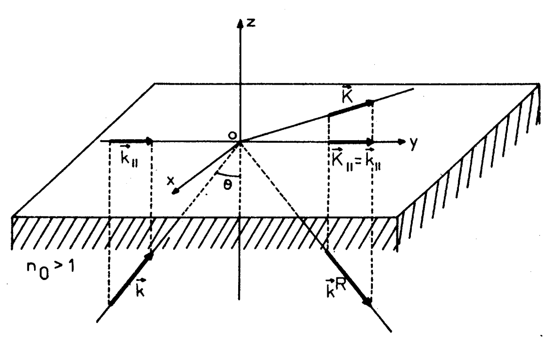

FIG. 1. Space filled with isotropic medium of refractive index n_0, where $\vec{k}$ represents the wave vector of the incident wave. In the case in this picture the plane of incidence is yz.

the incident wave on the diopter is coming from the $z<0$ or from the $z>0$ half space, the corresponding modes of the field are, respectively, called left modes (subscript L) or right modes (subscript R). These modes are characterized by the wave vector of their incident part ($\vec{k}$ for a left mode and $\vec{K}$ for a right one) and by their polarization s [$s=1$ for a transverse electric mode (TE) and $s=2$ for a transverse magnetic mode (TM)]. Each mode can be expressed as the sum of its incident, reflected, and transmitted parts (superscripts I, R, T):

$$\vec{\mathcal{E}}_L(\vec{k},s,\vec{r}) = \vec{\mathcal{E}}_L^I(\vec{k},s,\vec{r}) + \vec{\mathcal{E}}_L^R(\vec{k},s,\vec{r}) + \vec{\mathcal{E}}_L^T(\vec{k},s,\vec{r}), \quad (1)$$

$$\vec{\mathcal{E}}_R(\vec{K},s,\vec{r}) = \vec{\mathcal{E}}_R^I(\vec{K},s,\vec{r}) + \vec{\mathcal{E}}_R^R(\vec{K},s,\vec{r}) + \vec{\mathcal{E}}_R^T(\vec{K},s,\vec{r}). \quad (2)$$

The wave vectors of these components are $\vec{k}$, $\vec{k}^R$, and $\vec{K}$ for a left mode and $\vec{K}$, $\vec{K}^R$, and $\vec{k}$ for a right one (Figs. 2 and 3). Their modulus and their components parallel to the diopter verify

$$\left.\begin{array}{l} |\vec{k}| = |\vec{k}^R| = n_0(\vec{K}\cdot\vec{K})^{1/2} = n_0\omega/c \\ \vec{k}_{\parallel} = \vec{k}_{\parallel}^R = \vec{K}_{\parallel} \end{array}\right\} \text{left mode}, \quad (3)$$
$$\left.\begin{array}{l} |\vec{K}| = |\vec{K}^R| = |\vec{k}|/n_0 = \omega/c \\ \vec{K}_{\parallel} = \vec{K}_{\parallel}^R = \vec{k}_{\parallel} \end{array}\right\} \text{right mode}.$$

We write $\vec{\epsilon}_L^I(s)$, $\vec{\epsilon}_L^R(s)$, $\vec{\epsilon}_L^T(s)$ and $\vec{\epsilon}_R^I(s)$, $\vec{\epsilon}_R^R(s)$, and $\vec{\epsilon}_R^T(s)$ as the polarization of the incident, reflected, and transmitted waves of the modes. In the TE case ($s=1$) all these vectors are equal to a real unit vector $\hat{\epsilon}$ lying in the plane $z=0$ and perpendicular to the plane of incidence. In the TM case ($s=2$) they are defined by $(\hat{\epsilon}\times\vec{\kappa})/(\vec{\kappa}\cdot\vec{\kappa})^{1/2}$, where $\vec{\kappa}$ is the possibly complex wave vector of the corresponding component of the mode. The normalized complex amplitudes a_L (a_R) of each component of $\vec{\mathcal{E}}_L(\vec{k},s,\vec{r})$ [$\vec{\mathcal{E}}_R(\vec{K},s,\vec{r})$] are given by

$$a_R^I(1) = a_R^R(2) = n_0 a_L^I(1) = n_0 a_L^I(2) = 1/\sqrt{2}, \quad (4)$$

$$a_R^R(1) = -n_0 a_L^R(1) = (1/\sqrt{2})(K_z - k_z)/(K_z + k_z),$$
$$a_R^R(2) = -n_0 a_L^R(2) \quad (5)$$
$$= (1/\sqrt{2})(n_0^2 K_z - k_z)/(n_0^2 K_z + k_z),$$

$$a_R^T(1) = n_0(K_z/k_z)a_L^T(1) = (1/\sqrt{2})2K_z/(K_z + k_z),$$
$$a_R^T(2) = n_0(K_z/k_z)a_L^T(2) \quad (6)$$
$$= (1/\sqrt{2})2n_0 K_z/(n_0^2 K_z + k_z).$$

Noting that $n(\vec{r})=1$ for $z>0$ and $n(\vec{r})=n_0$ for $z<0$, we introduce

$$\vec{d}_{L,R}^I(s) = n^2(\vec{r})a_{L,R}^I(s)\vec{\epsilon}_{L,R}^I(s). \quad (7)$$

With this notation we can write for a left and a right mode

$$n^2(\vec{r})\vec{\mathcal{E}}_L^I(\vec{k}, s, \vec{r}) = \vec{d}_L^I(s)e^{j\vec{k}\cdot\vec{r}}, \tag{8}$$

$$n^2(\vec{r})\vec{\mathcal{E}}_R^I(\vec{K}, s, \vec{r}) = \vec{d}_R^I(s)e^{j\vec{k}\cdot\vec{r}}. \tag{9}$$

Similar relations can be obtained for other parts of the modes by changing superscript I into R or T. For clarity, it must be realized that $\vec{\mathcal{E}}_L^I$, $\vec{\mathcal{E}}_L^R$, $\vec{\mathcal{E}}_R^T$ and $\vec{\mathcal{E}}_R^I$, $\vec{\mathcal{E}}_R^R$, and $\vec{\mathcal{E}}_L^T$ are zero for $z>0$ and $z<0$, respectively.

By combining all possible modes with arbitrary complex amplitudes $u(\vec{k}, s)$ and $v(\vec{K}, s)$, we can form a representation of any arbitrary electric field (see Carniglia and Mandel[5]):

$$\vec{E}(\vec{r}, t) = \frac{1}{(2\pi)^3}\int_{k_z>0} d\vec{k}\sum_{s=1}^{2}\left(\frac{\hbar\omega}{\epsilon_0}\right)^{1/2}[u(\vec{k}, s)\vec{\mathcal{E}}_L(\vec{k}, s, \vec{r})e^{-j\omega t} + \text{c. c.}] + \frac{1}{(2\pi)^3}\int_{K_z<0} d\vec{K}\sum_{s=1}^{2}\left(\frac{\hbar\omega}{\epsilon_0}\right)^{1/2}[v(\vec{K}, s)\vec{\mathcal{E}}_R(\vec{K}, s, \vec{r})e^{-j\omega t} + \text{c. c.}]. \tag{10}$$

In the same way the magnetic field can be written

$$\begin{aligned}\vec{\mathcal{B}}_L(\vec{k}, s, \vec{r}) &= \vec{\mathcal{B}}_L^I(\vec{k}, s, \vec{r}) + \vec{\mathcal{B}}_L^R(\vec{k}, s, \vec{r}) + \vec{\mathcal{B}}_L^T(\vec{k}, s, \vec{r}),\\ \vec{\mathcal{B}}_R(\vec{K}, s, \vec{r}) &= \vec{\mathcal{B}}_R^I(\vec{K}, s, \vec{r}) + \vec{\mathcal{B}}_R^R(\vec{K}, s, \vec{r}) + \vec{\mathcal{B}}_R^T(\vec{K}, s, \vec{r}).\end{aligned} \tag{11}$$

Each component of $\vec{\mathcal{B}}_L$ and $\vec{\mathcal{B}}_R$ is deduced from the corresponding one of $\vec{\mathcal{E}}_L$ and $\vec{\mathcal{E}}_R$ by the help of Maxwell's equation $\overrightarrow{\text{curl}}\ \vec{E} = -\partial\vec{B}/\partial t$.

By introducing

$$\vec{b}_{L,R}^I(s) = (c/\omega)a_{L,R}^I(s)\vec{\kappa}_{L,R}^I \times \vec{\epsilon}_{L,R}^I(s), \tag{12}$$

we can write

$$\vec{\mathcal{B}}_L^I(\vec{k}, s, \vec{r}) = \vec{b}_L^I(s)e^{j\vec{k}\cdot\vec{r}}, \tag{13}$$

$$\vec{\mathcal{B}}_R^I(\vec{K}, s, \vec{r}) = \vec{b}_R^I(s)e^{j\vec{k}\cdot\vec{r}}, \tag{14}$$

and similar relations for the other parts of the modes. As above, $\vec{\mathcal{B}}_L^I$, $\vec{\mathcal{B}}_L^R$, and $\vec{\mathcal{B}}_R^T$ and $\vec{\mathcal{B}}_R^I$, $\vec{\mathcal{B}}_R^R$, and $\vec{\mathcal{B}}_L^T$ are zero for $z>0$ and $z<0$, respectively.

The more general magnetic field can then be written

$$\vec{B}(\vec{r}, t) = \frac{1}{(2\pi)^3}\int_{k_z>0} d\vec{k}\sum_{s=1}^{2}\left(\frac{\hbar\omega}{\epsilon_0 c^2}\right)^{1/2}[u(\vec{k}, s)\vec{\mathcal{B}}_L(\vec{k}, s, \vec{r})e^{-j\omega t} + \text{c. c.}] + \frac{1}{(2\pi)^3}\int_{K_z<0} d\vec{K}\sum_{s=1}^{2}\left(\frac{\hbar\omega}{\epsilon_0 c^2}\right)^{1/2}[v(\vec{K}, s)\vec{\mathcal{B}}_R(\vec{K}, s, \vec{r})e^{-j\omega t} + \text{c. c.}]. \tag{15}$$

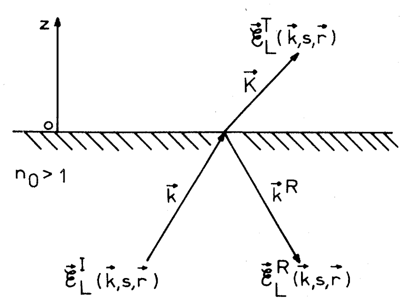

FIG. 2. Definition of a left mode. $\mathcal{E}_L(\vec{k}, s, \vec{r})$ represents the electric field. Superscripts attached to $\mathcal{E}_L(\vec{k}, s, \vec{r})$ correspond respectively to the incident, reflected, and transmitted parts of the field. The wave vector is $\vec{k}$ in the dielectric, and $\vec{K}$ in the vacuum.

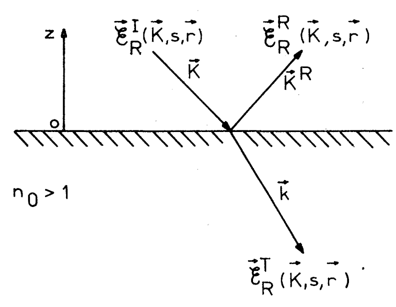

FIG. 3. Definition of a right mode. Notations are as in Fig. 2.

II. COMPONENT PARALLEL TO INTERFACE OF FIELD MOMENTUM

According to field theory, the expressions for dynamic variables are obtained by using Noether's theorem and the principle of stationary action. Thus the momentum corresponds to the invariant in a space translation. In the case we are studying, the field is invariant only under a translation parallel to the plane interface. This situation enables us to define and to quantize the momentum $\vec{P}_\parallel$ parallel to the interface. By using the Maxwell-Minkowski energy-momentum density, we have

$$\vec{P}_\parallel = \int d\vec{r}\,[\vec{D}(\vec{r},t)\times\vec{B}(\vec{r},t)]_\parallel\,. \qquad (16)$$

Using $\vec{D}(\vec{r},t)=n^2(\vec{r})\epsilon_0\vec{E}(\vec{r},t)$, substituting (10) and (15) into (16), and noting (for clarity) $u(\vec{k},s)\equiv u$, $u(\vec{k}',s')\equiv u'$, $\vec{\mathcal{E}}_L(\vec{k},s,\vec{r})\equiv\vec{\mathcal{E}}_L$, etc., we obtain

$$\begin{aligned}
\vec{P}_\parallel = {} & \frac{1}{(2\pi)^6}\iint_{\substack{k_z>0,\\ k'_z>0}} d\vec{k}\,d\vec{k}'\sum_{ss'=1}^{2}\frac{\hbar}{c}(\omega\omega')^{1/2}\Big\{\Big[u^*u'e^{j(\omega-\omega')t}\Big(\int d\vec{r}\,n^2(\vec{r})\vec{\mathcal{E}}_L^*\times\vec{\mathcal{B}}'_L\Big)_\parallel+\text{c. c.}\Big] \\
& \qquad + \Big[uu'e^{-j(\omega+\omega')t}\Big(\int d\vec{r}\,n^2(\vec{r})\vec{\mathcal{E}}_L\times\vec{\mathcal{B}}'_L\Big)_\parallel+\text{c. c.}\Big]\Big\} \\
& +\frac{1}{(2\pi)^6}\iint_{\substack{k_z>0,\\ K'_z<0}} d\vec{k}\,d\vec{K}'\sum_{ss'=1}^{2}\frac{\hbar}{c}(\omega\omega')^{1/2}\Big\{\Big[u^*v'e^{j(\omega-\omega')t}\Big(\int d\vec{r}\,n^2(\vec{r})\vec{\mathcal{E}}_L^*\times\vec{\mathcal{B}}'_R\Big)_\parallel+\text{c. c.}\Big] \\
& \qquad + \Big[uv'e^{-j(\omega+\omega')t}\Big(\int d\vec{r}\,n^2(\vec{r})\vec{\mathcal{E}}_L\times\vec{\mathcal{B}}'_R\Big)_\parallel+\text{c. c.}\Big]\Big\} \\
& +\frac{1}{(2\pi)^6}\iint_{\substack{K_z<0,\\ k'_z>0}} d\vec{K}\,d\vec{k}'\sum_{ss'=1}^{2}\frac{\hbar}{c}(\omega\omega')^{1/2}\Big\{\Big[v^*u'e^{j(\omega-\omega')t}\Big(\int d\vec{r}\,n^2(\vec{r})\vec{\mathcal{E}}_R^*\times\vec{\mathcal{B}}'_L\Big)_\parallel+\text{c. c.}\Big] \\
& \qquad + \Big[vu'e^{-j(\omega+\omega')t}\Big(\int d\vec{r}\,n^2(\vec{r})\vec{\mathcal{E}}_R\times\vec{\mathcal{B}}'_L\Big)_\parallel+\text{c. c.}\Big]\Big\} \\
& +\frac{1}{(2\pi)^6}\iint_{\substack{K_z<0,\\ K'_z<0}} d\vec{K}\,d\vec{K}'\sum_{ss'=1}^{2}\frac{\hbar}{c}(\omega\omega')^{1/2}\Big\{\Big[v^*v'e^{j(\omega-\omega')t}\Big(\int d\vec{r}\,n^2(\vec{r})\vec{\mathcal{E}}_R^*\times\vec{\mathcal{B}}'_R\Big)_\parallel+\text{c. c.}\Big] \\
& \qquad + \Big[vv'e^{-j(\omega+\omega')t}\Big(\int d\vec{r}\,n^2(\vec{r})\vec{\mathcal{E}}_R\times\vec{\mathcal{B}}'_R\Big)_\parallel+\text{c. c.}\Big]\Big\}\,.
\end{aligned} \qquad (17)$$

To study (17) we must calculate eight spatial integrals, which we denote $\vec{I}_1, \vec{I}_2, \ldots, \vec{I}_8$, according to their position. For the sake of space, we present a calculation only of the first one:

$$\vec{I}_1 = \int d\vec{r}\,n^2(\vec{r})(\vec{\mathcal{E}}_L^*\times\vec{\mathcal{B}}'_L)_\parallel\,. \qquad (18)$$

According to Eqs. (1), (8), (11), and (13), this expression can be written

$$\begin{aligned}
\vec{I}_1 = {} & \int d\vec{r}(\vec{d}_L^{I*}\times\vec{b}_L^{I'})_\parallel\exp[-j(\vec{k}-\vec{k}')\vec{r}] + \int d\vec{r}(\vec{d}_L^{R*}\times\vec{b}_L^{R'})_\parallel\exp[-j(\vec{k}^R-\vec{k}^{R'})\vec{r}] + \int d\vec{r}(\vec{d}_L^{T*}\times\vec{b}_L^{T'})_\parallel\exp[-j(\vec{K}^*-\vec{K}')\vec{r}] \\
& + \int d\vec{r}(\vec{d}_L^{I*}\times\vec{b}_L^{R'})_\parallel\exp[-j(\vec{k}-\vec{k}^{R'})\vec{r}] + \int d\vec{r}(\vec{d}_L^{R*}\times\vec{b}_L^{I'})_\parallel\exp[-j(\vec{k}^R-\vec{k}')\vec{r}]\,.
\end{aligned} \qquad (19)$$

Making use of (3), we can bring out of the above integration the term

$$\iint dx\,dy\,\exp[-j(k_x-k'_x)x]\exp[-j(k_y-k'_y)y] = (2\pi)^2\delta(k_x-k'_x)\delta(k_y-k'_y)\,, \qquad (20)$$

which shows that only those modes which have the same plane of incidence contribute to $\vec{I}_1$. When integrating the equation on variable z, we obtain because of the discontinuity at $z=0$ ($\mathcal{P}$ denoting the principal part)

$$\begin{aligned}
\vec{I}_1 = {} & \tfrac{1}{2}(2\pi)^3\delta(\vec{k}-\vec{k}')\Big((\vec{d}_L^{I*}\times\vec{b}_L^{I})_\parallel + (\vec{d}_L^{R*}\times\vec{b}_L^{R})_\parallel + n_0^2\frac{K_z+K_z^*}{2k_z}(\vec{d}_L^{T*}\times\vec{b}_L^{T})_\parallel\Big) + (2\pi)^2\delta(k_x-k'_x) \\
& \times\delta(k_y-k'_y)j\mathcal{P}\Big(\frac{(\vec{d}_L^{I*}\times\vec{b}_L^{I'})_\parallel-(\vec{d}_L^{R*}\times\vec{b}_L^{R'})_\parallel}{k_z-k'_z} + \frac{(\vec{d}_L^{I*}\times\vec{b}_L^{R'})_\parallel-(\vec{d}_L^{R*}\times\vec{b}_L^{I'})_\parallel}{k_z+k'_z} - n_0^2\frac{K_z^*+K'_z}{k_z^2-k_z'^2}(\vec{d}_L^{T*}\times\vec{b}_L^{T'})_\parallel\Big)\,.
\end{aligned} \qquad (21)$$

In order to calculate this expression in the simplest possible way, we must consider separately the different possible values of s and s' and calculate the different vector products $\vec{d}\times\vec{b}'$. When $s\neq s'$, (21) is seen to be zero. Finally, we obtain

$$\vec{I}_1=\int d\vec{r}\, n^2(\vec{r})(\vec{\mathcal{E}}_L^*\times\vec{\mathcal{B}}_L')_{\parallel}$$
$$=\tfrac{1}{2}(2\pi)^3(c\vec{K}_{\parallel}/\omega)\delta_{ss'}\delta(\vec{k}-\vec{k}')\,. \tag{22}$$

Defining $\vec{\alpha}(s)=\pm(2\pi)^3 n_0^2(c/\omega)\vec{K}_{\parallel}\delta_{ss'}$ (sign is + when $s=1$, is − when $s=2$), we can evaluate the seven other integrals in the same way:

$$\vec{I}_2=\vec{\alpha}(s)\delta(\vec{k}_{\parallel}+\vec{k}_{\parallel}')\delta(k_z-k_z')a_L^R a_L^I\,, \tag{23}$$

$$\vec{I}_3=\vec{0}\,, \tag{24}$$

$$\vec{I}_4=\vec{\alpha}(s)\delta(\vec{k}_{\parallel}+\vec{K}_{\parallel}')\delta(k_z+k_z')a_R'^T a_L^I\,, \tag{25}$$

$$\vec{I}_5=\vec{0}\,, \tag{26}$$

$$\vec{I}_6=\vec{\alpha}(s)\delta(\vec{K}_{\parallel}+\vec{k}_{\parallel}')\delta(k_z+k_z')a_R^T a_L'^I\,, \tag{27}$$

$$\vec{I}_7=\tfrac{1}{2}(2\pi)^3(c/\omega)\vec{K}_{\parallel}\delta_{ss'}\delta(\vec{K}-\vec{K}')\,, \tag{28}$$

$$\vec{I}_8=\vec{\alpha}(s)\delta(\vec{K}_{\parallel}+\vec{K}_{\parallel}')\delta(K_z-K_z')a_R^R a_R^I\,. \tag{29}$$

Using these results, we can now calculate the eight terms in Eq. (17). For the sake of clarity, they will be denoted $\vec{P}_{\parallel}^1, \vec{P}_{\parallel}^2, \ldots, \vec{P}_{\parallel}^8$, according to their order in (17). Using (22) and (28), we obtain

$$\vec{P}_{\parallel}^1=\frac{1}{2(2\pi)^3}\int_{k_z>0}d\vec{k}\sum_{s=1}^{2}\hbar\vec{K}_{\parallel}(u^*u+uu^*)\,, \tag{30}$$

$$\vec{P}_{\parallel}^7=\frac{1}{2(2\pi)^3}\int_{K_z<0}d\vec{K}\sum_{s=1}^{2}\hbar\vec{K}_{\parallel}(v^*v+vv^*)\,. \tag{31}$$

We now want to state that all the other terms vanish. Using (23) and (29), respectively, we see that the expressions to be integrated in $\vec{P}_{\parallel}^2$ and $\vec{P}_{\parallel}^8$ appear to be odd functions in k_x and k_y; $\vec{P}_{\parallel}^2$ and $\vec{P}_{\parallel}^8$ are then identically equal to zero. Because of (24) and (26), $\vec{P}_{\parallel}^3$ and $\vec{P}_{\parallel}^5$ must also vanish. We have then only $\vec{P}_{\parallel}^4$ and $\vec{P}_{\parallel}^6$ to consider. Using (25) and (27) and after integrating on $\vec{K}'$ in $\vec{P}_{\parallel}^4$ and on $\vec{K}$ in $\vec{P}_{\parallel}^6$, we obtain [where $\beta(s)=+1$ when $s=1$ and $\beta(s)=-1$ when $s=2$]

$$\vec{P}_{\parallel}^4=\frac{1}{(2\pi)^3}\sum_{s=1}^{2}\int_{k_z>0}d\vec{k}\,\hbar\vec{K}_{\parallel}u(k_x,k_y,k_z)\times v(-k_x,-k_y,-K_z)\times e^{-2j\omega t}a_R^T a_L^I\beta(s)\,, \tag{32}$$

$$\vec{P}_{\parallel}^6=-\frac{1}{(2\pi)^3}\sum_{s=1}^{2}\int_{k_z'>0}d\vec{k}'\,\hbar\vec{K}_{\parallel}'u(k_x',k_y',k_z')\times v(-k_x',-k_y',-K_z')\times e^{-2j\omega' t}a_R'^T a_L'^I\beta(s)\,. \tag{33}$$

The sum of these last two expressions vanishes. Consequently we have

$$\vec{P}_{\parallel}=\vec{P}_{\parallel}^1+\vec{P}_{\parallel}^7=\frac{1}{(2\pi)^3}\left(\int_{k_z>0}d\vec{k}\sum_{s=1}^{2}\hbar\vec{K}_{\parallel}u^*(\vec{k},s)u(\vec{k},s)+\int_{K_z<0}d\vec{K}\sum_{s=1}^{2}\hbar\vec{K}_{\parallel}v^*(\vec{K},s)v(\vec{K},s)\right). \tag{34}$$

Since each mode satisfies the Helmholtz equation,[5] this expression shows that $\vec{P}_{\parallel}$ reduces to the sum of the momenta of independent harmonic oscillators. The quantization is therefore straightforward and proceeds as in a free field. The complex amplitudes $u(\vec{k},s)$, $u^*(\vec{k},s)$ and $v(\vec{K},s)$, $v^*(\vec{K},s)$ are replaced by Hilbert-space operators $\hat{u}(\vec{k},s)$, $\hat{u}^\dagger(\vec{k},s)$ and $\hat{v}(\vec{K},s)$, $\hat{v}^\dagger(\vec{K},s)$, which can be given the usual interpretation of annihilation and creation operators for quantum modes $(\vec{k},s)$ and $(\vec{K},s)$:

$$\hat{\vec{P}}_{\parallel}=\frac{1}{(2\pi)^3}\int_{k_z>0}d\vec{k}\sum_{s=1}^{2}\hbar\vec{K}_{\parallel}\hat{u}^\dagger(\vec{k},s)\hat{u}(\vec{k},s)+\frac{1}{(2\pi)^3}\int_{K_z<0}d\vec{K}\sum_{s=1}^{2}\hbar\vec{K}_{\parallel}\hat{v}^\dagger(\vec{K},s)\hat{v}(\vec{K},s)\,. \tag{35}$$

The similarity between Eq. (35) and the expression

$$\hat{H}=\frac{1}{(2\pi)^3}\int_{k_z>0}d\vec{k}\sum_{s=1}^{2}\hbar\omega\hat{u}^\dagger(\vec{k},s)\hat{u}(\vec{k},s)+\frac{1}{(2\pi)^3}\int_{K_z<0}d\vec{K}\sum_{s=1}^{2}\hbar\omega\hat{v}^\dagger(\vec{K},s)\hat{v}(\vec{K},s) \tag{36}$$

obtained by Carniglia and Mandel[5] for the Hamiltonian of the field allows us to attribute to the modes whose energy and polarization are $\hbar\omega$ and s a momentum parallel to the interface $\vec{P}_{\parallel}=\hbar\vec{K}_{\parallel}$.

III. DISCUSSION

Equation (35) shows that the quantum of the momentum of the electromagnetic field is $\hbar\vec{K}_{\parallel}$ ($\hbar\vec{K}_{\parallel}=\hbar\vec{k}_{\parallel}$). This result agrees with the de Broglie relation $\vec{P}=\hbar\vec{K}$ when it is restricted by invariance considerations to its component parallel to the interface. In the case of a left mode, by expressing it in terms of n_0 and θ (Fig. 1), the modulus of the momentum parallel to the diopter absorbed or emitted close to the interface can be written

$$P_{\parallel} = \hbar K_{\parallel} = (\hbar\omega/c) n_0 \sin\theta \, . \tag{37}$$

For an evanescent mode ($n_0 \sin\theta > 1$) we have

$$P_{\parallel} = (\hbar\omega/c) n_0 \sin\theta > \hbar\omega/c \, . \tag{38}$$

This value, greater than that we always have in a free field,

$$P = \hbar\omega/c \, , \tag{39}$$

is characteristic of the evanescent field. It is the cause of some specific interactions between the field and the matter very close to an interface. To illustrate this, let us consider the case of a free electron traveling at constant velocity in a medium or in the vacuum parallel and very close to a plane dielectric. Its possible dynamic states can be represented by the $E = (c^2P^2 + m_0^2c^4)^{1/2}$ hyperbola, and because of (39) the absorption or the emission of a photon can be represented by a parallel to the light-cone vector. Therefore when looking at Fig. 4 we can understand that such an interaction cannot exist. In fact, if the electron had to absorb or to emit a photon (39), its final state F would no longer be on the hyperbola and would thus not be an allowed dynamic state for it. With an evanescent field things are quite different: the momentum quantum being greater than $\hbar\omega/c$, the evanescent mode can be formally represented by a vector which points out from the light cone. Thus the transition is possible. As we have seen in Refs. 2, 3, and 9, it corresponds to the Čerenkov effect. Inequality (38) is *a priori* surprising, since it formally implies a spacelike energy-momentum transition. It is nevertheless consistent with the experimental results of Huard and Imbert.[1]

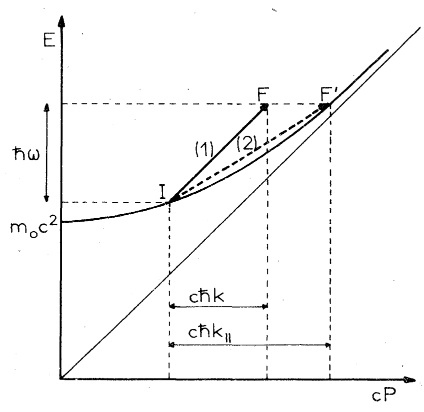

FIG. 4. Example of specific interaction near an interface: the Čerenkov effect. The possible dynamic states of a free electron are represented by hyperbola $E = (c^2P^2 + m_0^2c^4)^{1/2}$. Because of the relation $c\hbar k > \hbar\omega$ a free electron can emit or absorb an evanescent mode (2), whereas the interaction was impossible in a free field (1) ($c\hbar k = \hbar\omega$).

All the above results have been obtained by using the Maxwell-Minkowski's momentum density as expressed in (16). As is well known,[2] the Maxwell-Minkowski and de Broglie momentum densities are yet quite different in the evanescent wave. It should then be interesting to know whether the de Broglie tensor leads to the same result (35). Of course it does. Briefly, the reason for this is that the momentum density of the evanescent transmitted part of the mode never intervenes in the calculation of $\vec{P}_{\parallel}$. To make this result more precise, let us calculate separately the total contribution of the evanescent modes to the momentum $\vec{P}_{\parallel}$ of the electromagnetic field: in accordance with (34) only the $\vec{P}_{\parallel}^1$ (left modes) and the $\vec{P}_{\parallel}^7$ (right modes) terms do not vanish. As evanescent modes are left modes, their contribution to $\vec{P}_{\parallel}$ will appear only in the left term $\vec{P}_{\parallel}^1$. In order to perform the discussion in the simplest possible way, we consider, instead of (30), the expression for $\vec{P}_{\parallel}^1$ as written in the first term on the right-hand side of (17). Then, using (18) and the first term on the right-hand side of (21) [recall that we have demonstrated that the principal-part term in (21) is zero], and after integrating on $\vec{k}'$ and adding on s', we obtain

$$\vec{P}_{\parallel}^1 = \frac{1}{2(2\pi)^3} \int_{k_z > 0} d\vec{k} \sum_{s=1}^{2} \frac{\hbar\omega}{c} \Big[u^* u \Big((\vec{d}_L^{I*} \times \vec{b}_L^{I})_{\parallel} + (\vec{d}_L^{R*} \times \vec{b}_L^{R})_{\parallel} + n_0^2 \frac{K_z + K_z^*}{2k_z} (\vec{d}_L^{T*} \times \vec{b}_L^{T})_{\parallel} \Big) + \text{c. c.} \Big] . \tag{40}$$

In this expression, which represents the total contribution of the left modes to $\vec{P}_{\parallel}$, the vector products $\vec{d} \times \vec{b}$ are directly connected with the momentum density $\vec{D}(\vec{r}, t) \times \vec{B}(\vec{r}, t)$ as written in (16). The total contribution of the evanescent modes may be simply obtained by restricting the integral (40) to the domain $k_z^2 < k_x^2 + k_y^2$. Since in this case of total internal reflection K_z is imaginary, the multiplicative factor $K_z + K_z^*$ of the momentum density $\vec{D}^T \times \vec{B}^T$ of the transmitted part of the field is zero. Thus we have (using the subscript "ev" to denote evanescence)

$$\vec{P}_{\parallel \text{ev}} = \frac{1}{2(2\pi)^3} \int_{\substack{k_z > 0, \\ k_z^2 < k_x^2 + k_y^2}} d\vec{k} \times \sum_{s=1}^{2} \frac{\hbar\omega}{c} \{ u^* u [(\vec{d}_L^{I*} \times \vec{b}_L^{I}) + (\vec{d}_L^{R*} \times \vec{b}_L^{R})_{\parallel}] + \text{c. c.} \} . \tag{41}$$

As explained above, (41) clearly shows that the momentum density of the evanescent part of the modes does not contribute to $\vec{P}_{\parallel \text{ev}}$, nor consequently to $\vec{P}_\parallel$. If we use the de Broglie tensor, we obtain a quite similar result. It is possible simply to realize that, by calculating directly the momentum of the evanescent transmitted part of the field. In such a calculation, whatever the tensor may be, we must consider the integral

$$\int_0^\infty \exp[-(K_z^* - K_z')z]\,dz = \left(\frac{K_z+K_z^*}{2K_z}\right)\left(\frac{K_z'+K_z'^*}{2K_z'}\right)\pi\delta(K_z-K_z') - j\mathcal{P}\left(\frac{1}{K_z^*-K_z'}\right). \quad (42)$$

It is then possible to see that the $\delta(\vec{k}-\vec{k}')$ term always vanishes in the evanescent case ($K_z+K_z^*=0$). Of course, this presentation is not sufficient, since it does not show how the principal-part term vanishes. For this we must integrate the momentum density over the entire space and then take into account the momentum of the incident and of the reflected part of the field, as in the above calculations. When we proceed in such a way, the principal-part term vanishes, and we obtain by using the de Broglie tensor the same result (35) as with the Maxwell-Minkowski tensor. In conclusion, we can see that, although they lead to quite different momentum densities in the evanescent wave, the Maxwell and de Broglie energy-momentum tensors are, as expected, quite equivalent when calculating $\vec{P}_\parallel$ (as they obviously are for the Hamiltonian H).

As a third remark, let us emphasize that, the contribution $\hbar\vec{K}_\parallel$ of each oscillator being in the plane of incidence of the corresponding mode, $\vec{P}_\parallel$ will always lie in the plane of incidence, regardless of the polarization of the incident part of the wave. In the evanescent case this result must be emphasized when the incident wave is circularly polarized, by referring to the Imbert transversal shift.[7] In fact, as Costa de Beauregard has previously pointed out,[6] this means that in here the momentum is not collinear with the energy transfer.

As a final conclusion, we would like to add the following remarks on the importance of the use of triplet modes. Because the interface is included in the definition of them, they permit the performance of calculations which automatically take into account the combinatorial aspects of the different parts of the field arising from the presence of the diopter. Besides, we can see from Secs. II and III that these modes are at the same time eigenmodes of the Hamiltonian $\hat{H}$ and of the momentum $\hat{\vec{P}}_\parallel$ of the field. All of this allows us to proceed in a formal way as in a free field and therefore to present calculations in the simplest way possible when performing calculations on the interaction between matter and the electromagnetic field very close to a dielectric. We have mentioned this fact on several occasions and particularly in our study of absorption[4] and of Raman scattering[8] in the vicinity of an interface.

IV. COMPONENT PERPENDICULAR TO SURFACE OF FIELD MOMENTUM

When calculating the total momentum of the field, the discontinuity introduced by the surface at $z=0$ causes the expressions obtained in the two half spaces $z \geq 0$ and $z<0$ to be integrated separately. Then, as we saw in Secs. II and III, we have principal-part terms $\mathcal{P}(1/(k_z \pm k_z'))$ coming from integrals as

$$\int_0^\infty e^{-j(k_z-k_z')z}\,dz = \pi\delta(k_z-k_z') - j\mathcal{P}(1/(k_z-k_z')). \quad (43)$$

Owing to the continuity relations of the field through the interface, these principal-part terms vanish in the calculation of $\vec{P}_\parallel$. Thus only the distributions $\delta(\vec{k}-\vec{k}')$ remain, meaning that in the calculation of $\vec{P}_\parallel$ each mode has an independent contribution $\hbar\vec{K}_\parallel$ to the total momentum of the field. In the calculation of $\vec{P}_\perp$, the principal-part terms no longer vanish. Because of the definition these terms $\mathcal{P}(1/(k_z-k_z'))$ clearly show, when the momentum of a mode $(\vec{k}, s)$ is calculated, the influence of all the other modes $(\vec{k}', s')$. This result has the fundamental consequence of making it impossible to write $\vec{P}_\perp$ as the sum of the momenta of independent harmonic oscillators. Thus in consideration of $\vec{P}_\perp$ the interface-field system now appears as an interacting one. Therefore it is no longer possible to obtain $\vec{P}_\perp$ from Noether's theorem and principle of stationary action. In that case we must use, instead of action, the unitary operator which transforms the state vector ϕ of the system under an infinitesimal perpendicular to the diopter translation. We use for that purpose the presentation and notations of Bogoliubov and Schirkov[10] (with units such that $\hbar=c=1$).

Let ϕ be the state vector of the system. As this state is an interacting one, we call it $\phi(g)$, where g is a function characterizing the interaction and whose values are to be taken in $[0,1]$. To make discussion clearer, we consider the most general case of a space-time translation. We denote the space-time coordinates by x^μ with the understanding that $x^0=t$, $x^1=x$, $x^2=y$, and $x^3=z$. We use a metric tensor $g_{\mu\nu}$ with components $g^{ll}=-1$ ($l=1,2,3$), $g^{00}=+1$. Without any interaction, under the transformation

$$x \rightarrow x' + Lx = x + \delta x \quad (44)$$

the state vector ϕ of the system will become (where u_L is a unitary operator that keeps invariant the norm $|\phi^*\phi|^2$)

$$\phi \rightarrow \phi' = u_L \phi . \quad (45)$$

Since in this case $\phi = \phi(g)$, we must take into account that $g(x)$ also undergoes the same transformation:

$$g(x) \rightarrow g'(x) = Lg(x) = g(L^{-1}x) . \quad (46)$$

Then we can write

$$\phi(g) \rightarrow \phi'(g) = u_L \phi(L^{-1}g) \quad (47)$$

and, when we consider an infinitesimal translation,

$$\phi(g) \rightarrow \phi'(g) = (1 + \delta u_L)\phi(L^{-1}g) ; \quad (48)$$

u_L being Hermitian, δu_L must be anti-Hermitian. It must also be written as a linear combination of the infinitesimal values δx^k. So by introducing P_0^k, an Hermitian operator the physical meaning of which will appear below, we have

$$\delta u_L = j \sum_k g^{kk} P_0^k \delta x^k . \quad (49)$$

Therefore it follows from (48) that

$$\phi'(g) - \phi(g) = \phi(L^{-1}g) - \phi(g) + \delta u_L \phi(g) + \delta u_L[\phi(L^{-1}g) - \phi(g)]. \quad (50)$$

Defining $\delta g(x)$ as $g(Lx) - g(x)$ and using (46), we can write

$$\phi'(g(x)) - \phi(g(x)) = \phi(g(Lx)) - \phi(g(x)) + \delta u_L \phi(g(x)) , \quad (51)$$

$$\phi'(g) - \phi(g) = \phi(g + \delta g) - \phi(g) + \delta u_L \phi(g) .$$

Using then the definition of the functional derivative of $\phi(g)$ vs g at point x, we have

$$\phi'(g) - \phi(g) = \int \frac{\delta\phi(g)}{\delta g(x)} \delta g(x) + \delta u_L \phi(g) . \quad (52)$$

Equation (52) may be transformed by writing

$$\delta g(x) = g(Lx) - g(x) = \sum_k \frac{\partial g}{\partial x^k} \delta x^k . \quad (53)$$

We then obtain, using the variational form of the Schrödinger equation,

$$j\delta\phi(g) = \int H(x,g)\phi(g)\delta g(x) dx \quad (54)$$

and substituting into (52), we obtain

$$\phi'(g) - \phi(g) = j[P(g)A]\phi(g) , \quad (55)$$

where $P(g)A$ denotes the scalar product of the four-vector $A \equiv \delta x$ with P, whose space-time coordinates are

$$P^k(g) = P_0^k - g^{kk} \int H(x,g) \frac{\partial g(x)}{\partial x^k} dx . \quad (56)$$

It can be demonstrated [see Ref. (10)] that the eigenvalues and the average values of the $P^k(g)$ operators so-defined do not depend on g and therefore really correspond to physical observables. Equation (56) then defines the energy momentum of the interacting system. Making $g = 0$ in (56), we can see that P^k reduces to P_0^k, which can then be identified as the energy momentum of a free field.

Very close to the interface $z = 0$ and for symmetry reasons the functions $g(x)$, which represents in our case the interaction between the field and the dielectric, cannot vary vs x, y, or t. Therefore its only nonzero derivative is $\partial g/\partial z$ and (56) yields

$$\begin{aligned} H &= H , \\ P^x &= P_0^x , \\ P^y &= P_0^y , \\ P^z &= P_0^z + \int H(x,g) \frac{\partial g}{\partial z} dx = P_0^z + P_{\text{int}}^z . \end{aligned} \quad (57)$$

The first three equations show that the energy and the momentum parallel to the diopter of the dielectric-field system are the same as in the free field (this agrees with the results presented in Secs. II and III). The last means that the momentum perpendicular to the diopter ($P_\perp \equiv P^z$) now appears as the sum of a "self-momentum" P_0^z and of an "interaction momentum" P_{int}^z.

A good way to make this result somewhat clearer is to compare the present case to the one of a time perturbation. When a time-dependent interaction is switched on at time τ in free space, the only nonzero derivative of g is $\partial g/\partial t$. Hence instead of (57) we obtain from (56)

$$H = H_0 + H_{\text{int}} , \quad P^k = P_0^k \ (k = x, y, z) . \quad (58)$$

As is well known, this result shows that the total momentum is the same as in the absence of interaction, whereas the Hamiltonian is the sum of a nonperturbated Hamiltonian and an interaction one. With the help of (58) it becomes possible to understand the physical consequences of (57): in just the same way as energy states of a time-dependent interacting system would be enlarged and shifted, we can predict near an interface a widening and a shift of the momentum state $P_\perp$ of the field. This result agrees with the uncertainty principle: the partial localization of the field (more exactly, its unequal repartition in the two half spaces $z \geq 0$ and $z < 0$) makes it impossible to know precisely its momentum along z. It is important to note that these results occur of course not only in the evanescent wave but also in a homogeneous one very close to an interface. Generally, when considering the left case, the explicit calculation of

$$\vec{P}_\perp = \int d\vec{r}[\vec{D}(\vec{r},t)\times\vec{B}(\vec{r},t)]_\perp \quad (59)$$

leads us to the intermediate result

$$\int d\vec{r}\, n^2(r)(\vec{\mathcal{E}}_L^* \times \vec{\mathcal{B}}_L')$$

$$= \frac{(2\pi)^3}{2}\frac{n_0^2 c}{\omega}\frac{K_z + K_z^*}{2k_z}(k_z + K_z)a_L^T a_L^{T*}\delta(\vec{k}-\vec{k}') + \mathcal{P}, \quad (60)$$

whatever the evanescent or homogeneous character of the mode may be. In the particular case of an evanescent mode where $K_z + K_z^* = 0$ the shift and the widening of the momentum state is then made from the value $P_\perp = 0$.

V. CONCLUSION

In conclusion, this work, which is a continuation of Ref. 11, has the following results: In Secs. II and III we have seen that triplet modes are eigenmodes not only of $\hat{H}$ but also of $\hat{\vec{P}}_\parallel$. This allows us to consider the interface-field system as a free field when we are concerned with $\hat{H}$ or $\hat{\vec{P}}_\parallel$. Therefore the use of triplet modes, previously introduced by Carniglia and Mandel[5] and then developed by one of us,[8] makes it possible to study interactions between matter and the radiation field very close to an interface with the same method as in a free field. Our results are consistent with the experimental results of Huard and Imbert,[1] who have pointed out the possibility of absorbing near a diopter a momentum $\vec{P}_\parallel$ whose modulus is greater than $\hbar\omega/c$.

By using the usual expansion in field theory for the determination of the dynamic variables in an interacting field, we have defined the momentum perpendicular to the diopter $\vec{P}_\perp$ of the interface-field system and shown how the presence of a spatial discontinuity of the field (physically described by the medium) widens and shifts its momentum states. The form of P_{int}^z could be studied by looking at the form of $g(x)$. This would permit a new approach to such problems as, for example, that of the momentum of the photon in a dielectric medium. The method we have used may also be useful in studying the angular momentum exchanged between the field and an atom during an interaction very close to an interface, particularly in an evanescent wave. Concerning this problem, it can be noted that our theoretical results[4] have now received a first experimental proof.[12] At this point we must moreover mention the recent paper of Huard on the spin-angular momentum of a field interacting with a plane interface.[13]

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