# Optical second-harmonic generation in $n$-InSb

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The phenomenon of nonlinear second-harmonic generation of laser radiation in $n$-InSb contained inside a rectangular waveguide is investigated. The nonlinearity arises through the ponderomotive force on electrons. The power conversion efficiency is resonantly enhanced at some optimum values of waveguide density and size such that $2\beta_1 = \beta_2$, i.e., $n^2\pi^2/b^2 + (m^2-4)\pi^2/a^2 = 3\omega p^2/c^2$; $\beta_1$ and $\beta_2$ are the propagation constants of fundamental and second-harmonic modes, $a$ and $b$ are the $x$ and $y$ dimensions of the waveguide, and $n$ and $m$ are integers. For a typical 1-kW $CO_2$ laser the second-harmonic efficiency takes on values as high as 1%.

## I. INTRODUCTION

The phenomena of harmonic generation and nonlinear mixing of laser radiation in $n$-InSb have attained considerable importance in the past.[1-10] The nonlinearity responsible for these phenomena arises through the energy-dependent relaxation time of electrons and the nonparabolicity of the conduction band. This nonlinearity, in the absence of any dc electric field, gives rise to the generation of third-harmonic and second-order sum and difference frequency ($2\omega_1 \pm \omega_2$) waves, whose yield is substantially low. The application of a suitable dc electric field ($\gtrsim 300$ V/cm) gives rise to the generation of second harmonic; however, again the yield is poor.

In this paper, we have investigated a different mechanism of nonlinearity, viz., the ponderomotive force and employed dimensional resonance effects to enhance the efficiency of second-harmonic generation of laser radiation in $n$-InSb. The ponderomotive force on electrons due to a plane uniform beam is purely longitudinal (i.e., aligned in the direction of propagation of the laser beam) and hence, gives rise to purely electrostatic oscillations. However, a beam having a nonuniform distribution of intensity along its wavefront gives rise to a transverse ponderomotive force driving a transverse second-harmonic current and thus producing a second-harmonic laser beam. The nonuniformity in the intensity distribution of the laser pump (and hence the large values of the transverse ponderomotive force) could be easily obtained by propagating the laser through a waveguide configuration. For waveguide dimensions of the order of a few laser wavelengths, the ponderomotive nonlinearity is much higher than that due to an energy-dependent relaxation time and nonparabolic energy bands.

Furthermore, one would expect resonant enhancement in the efficiency of second-harmonic generation when the propagation constants $\beta_1 = \beta(\omega)$ and $\beta_2 = \beta(2\omega)$ of the fundamental and the second-harmonic mode satisfy the resonance condition $2\beta_1 = \beta_2$, i.e., when the phase velocities of the fundamental and second-harmonic waves are equal. For a laser pump propagating in the $TE_{10}$ mode $\beta_1$ is given by[11]

$$\beta_1 = (\omega^2\epsilon_L/c^2 - \omega_p^2/c^2 - \pi^2/a^2)^{1/2}.$$

The propagation constant of the second harmonic (for the general $TE_{mn}$ or $TM_{mn}$ modes of propagation) may be written

$$\beta_2 = (4\omega^2\epsilon_L/c^2 - \omega_p^2/c^2 - m^2\pi^2/a^2 - n^2\pi^2/b^2)^{1/2}.$$

Whenever, the condition $2\beta_1 = \beta_2$ is satisfied for a given set of $m$ and $n$, that particular mode is resonantly excited, while others remain of low magnitude. Thus there are many possible combinations of $m$, $n$, and $\omega_p/\omega$ values for which resonance condition is satisfied.

In Sec. II we have obtained the expression for second-harmonic current density due to a laser pump propagating in the $TE_{10}$ mode in a rectangular waveguide filled with $n$-InSb. Indium antimonide is specifically chosen here, because the effective mass of electrons is very low (0.01 times the free space mass) and hence high nonlinearities are expected at relatively much lower powers. In Sec. III we have obtained the power conversion efficiency of second harmonic by using appropriate boundary conditions on the field components. Numerical estimates of the power conversion efficiency have been made in the same section. A brief discussion of results is given in Sec. IV.

## II. SECOND-HARMONIC CURRENT DENSITY

We consider the propagation of an electromagnetic wave in the $TE_{10}$ mode in a rectangular waveguide, filled with $n$-InSb,

$$\vec{E} = \hat{y}A\sin(\pi x/a)\exp i(\omega t - \beta_1 z), \qquad (1)$$

 

where

$$\beta_1 = [(\omega^2/c^2)\epsilon(\omega) - \pi^2/a^2]^{1/2},$$

$$\epsilon(\omega) = \epsilon_L - \omega_p^2/\omega^2,$$

$\epsilon_L$ is the dielectric constant, $\omega_p = (4\pi Ne^2/m_0)^{1/2}$, and $-e$, $m_0$, and $N$ are the electronic charge, mass, and equilibrium electron density, respectively.

In the presence of the pump wave the response of electrons is governed by the Vlasov equation (for $\nu \ll \omega$, $\nu$ is the momentum-transfer collision frequency of electrons)

$$\frac{\partial f}{\partial t} + \vec{v}\cdot\vec{\nabla} f - \frac{e}{m_0}\left(\vec{E} + \vec{E}_2 + \frac{\vec{v}\times\vec{B}}{c}\right)\cdot\frac{\partial f}{\partial \vec{v}} = 0, \qquad (2)$$

where $\vec{B} = -c\vec{\nabla}\times\vec{E}/i\omega$ is the magnetic field of the pump and $\vec{E}_2$ is the self-consistent electric field of the second harmonic.

The distribution function can be expanded as[3,10]

$$f = f_0 + f_1 \exp(i\omega t) + f_2 \exp(i2\omega t), \qquad (3)$$

where

$$f_0 = N(m_0/2\pi T)^{3/2} \exp - m_0(v^2/2T)$$

is the unperturbed part of the distribution function and $T$ is the electron temperature in energy units. Assuming $f_2 < f_1 < f_0$, the response of electrons at the fundamental and second-harmonic frequencies could be obtained from Eq. (2) as

$$f_1 = \frac{e\vec{E}}{m_0 i\omega}\cdot\frac{\partial f_0}{\partial\vec{v}} + \frac{2ef_0}{m_0\omega^2 v_e^2}\left(\vec{v}\cdot\vec{\nabla}(\vec{E}\cdot\vec{v})\right), \qquad (4)$$

and

$$f_2 = \left(\frac{e}{2m_0 i\omega}\right)\left(\vec{E}_2\cdot\frac{\partial f_0}{\partial\vec{v}} + \vec{E}\cdot\frac{\partial f_1}{\partial\vec{v}} + \frac{\vec{v}\times\vec{B}}{c}\cdot\partial f_1/\partial\vec{v}\right), \qquad (5)$$

where $v_e = (2T/m_0)^{1/2}$. If we use Eqs. (3)–(5), second-harmonic current density can be written

$$\vec{J}_2 = -e\int \vec{v} f_2 d^3\vec{v}$$

$$= \frac{Ne^2\vec{E}_2}{2m_0 i\omega} - \frac{Ne^2}{2m_0^2\omega^3}\left(E\frac{\partial E}{\partial x}\hat{x} + i\beta_1 E^2\hat{z}\right). \qquad (6)$$

## III. POWER CONVERSION EFFICIENCY

Substituting Eq. (6) in the wave equation, we obtain

$$\vec{\nabla}^2\vec{E}_2 - \vec{\nabla}(\vec{\nabla}\cdot\vec{E}_2) + \frac{4\omega^2}{c^2}\epsilon(2\omega)\vec{E}_2$$

$$= -i\frac{\omega_p^2}{\omega^2}\frac{e}{m_0 c^2}\left(E\frac{\partial E}{\partial x}\hat{x} + i\beta_1 E^2\hat{z}\right). \qquad (7)$$

The second term is Eq. (7) can be simplified by using the Poisson's equation and the equation of continuity,

$$\vec{\nabla}\cdot\vec{E}_2 = -(2\pi i/\epsilon(2\omega)\omega)\vec{\nabla}\cdot\vec{J}_2^{NL}. \qquad (8)$$

Using Eq. (8) in Eq. (7), we obtain the following equations for the three components of the second-harmonic electric field:

$$\nabla^2 E_{2x} + \left(\frac{4\omega^2}{c^2}\right)\epsilon(2\omega)E_{2x}$$

$$= -i\left(\frac{\omega_p}{\omega}\right)^2 A\frac{|v_d|}{c}\delta_1 \sin\left(\frac{2\pi x}{a}\right)\exp i(2\omega t - 2\beta_1 z), \qquad (9)$$

$$\nabla^2 E_{2y} + \left(\frac{4\omega^2}{c^2}\right)\epsilon(2\omega)E_{2y} = 0, \qquad (10)$$

$$\nabla^2 E_{2z} + \left(\frac{4\omega^2}{c^2}\right)\epsilon(2\omega)E_{2z}$$

$$= \left(\frac{\omega_p^2}{\omega_2}\right)A\frac{|v_d|}{c}\left[\delta_2 + \delta_3\cos\left(\frac{2\pi x}{a}\right)\right]\exp i(2\omega t - 2\beta_1 z), \qquad (11)$$

where

$$\delta_1 = [\omega^2/c^2 + (\pi^2/a^2 + \beta_1^2)/\epsilon(2\omega)](\pi c/2a\omega),$$

$$\delta_2 = [\omega^2/c^2 + \beta_1^2/\epsilon(2\omega)](c\beta_1/2\omega),$$

$$\delta_3 = [\omega^2/c^2 + (\pi^2/a^2 + \beta_1^2)/\epsilon(2\omega)](c\beta_1/2\omega),$$

$$|v_d| = eA/m_0\omega.$$

The solution of Eq. (10) can be taken to be $E_{2y} = 0$ as it is independent of the pump wave. The $z$ dependence of $E_{2x}$ and $E_{2z}$, in compliance with Eqs. (9) and (11), can be taken as $\exp[-(i2\beta_1 z)]$. Subsequently Eqs. (9) and (11) simplify to give

$$\frac{\partial^2 E_{2x}}{\partial x^2} + \frac{\partial^2 E_{2x}}{\partial y^2} + k_2^2 E_{2x} = -i\left(\frac{\omega_p^2}{\omega^2}\right)A\left(\frac{|v_d|}{c}\right)$$

$$\delta_1 \sin\left(\frac{2\pi x}{a}\right)\exp i(2\omega t - 2\beta_1 z), \qquad (12)$$

$$\frac{\partial^2 E_{2z}}{\partial x^2} + \frac{\partial^2 E_{2z}}{\partial y^2} + k_2^2 E_{2z}$$

$$= \left(\frac{\omega_p^2}{\omega^2}\right)A\left(\frac{|v_d|}{c}\right)\left[\delta_2 + \delta_3\cos\left(\frac{2\pi x}{a}\right)\right]$$

$$\times \exp i(2\omega t - 2\beta_1 z), \qquad (13)$$

where

$$k_2^2 = [(4\omega^2/c^2)\epsilon(2\omega) - 4\beta_1^2]; \quad \epsilon(2\omega) = \epsilon_L - \omega_p^2/4\omega^2.$$

The solutions of Eqs. (12) and (13) with appropriate boundary conditions

$E_{2x} = 0$ at $y = 0, b$

$E_{2z} = 0$ at $x = 0, a$ and $y = 0, b$

could be written

$$E_{2x} = i(\omega_p/\omega)^2 A(|v_d|/c) \times [\cos(\beta_1^1 y) + g_1 \sin(\beta_1^1 y) - 1] \times \sin(2\pi x/a)\exp i(2\omega t - 2\beta_1 z)\,, \qquad (14)$$

and

$$E_{2z} = (\omega_p^2/\omega^2)A(|v_d|/c)\phi(x) \times (-\cos\beta_3 y + g_2 \sin\beta_3 y + 1) \times \exp i(2\omega t - 2\beta_1 z)\,, \qquad (15)$$

where

$$\beta_1^1 = (k_2^2 - 4\pi^2/a^2)^{1/2}\,,$$

$$g_1 = (1 - \cos\beta_1^1 b)/\sin\beta_1^1 b\,,$$

$$\phi(x) = \alpha_1 \cos k_2 x + \alpha_2 \sin k_2 x + [\delta_2/k_2^2 + \delta_3/k_2^2(1 + 4\pi^2/a^2k_2^2)\cos(2\pi x/a)]\,,$$

$$\alpha_1 = -[\delta_2/k_2^2 + \delta_3/k_2^2(1 + 4\pi^2/a^2k_2^2)]\,,$$

$$\alpha_2 = \alpha_1(1 - \cos k_2 a)/\sin k_2 a\,,$$

$$\beta_3 = (4\pi^2 m^2/a^2 - k_2^2)^{1/2}\,; \quad m = 1, 2, 3, \ldots\,,$$

$$g_2 = (\cos\beta_3 b - 1)/\sin\beta_3 b\,.$$

The power flow through the waveguide can be obtained by integrating the power density over the cross section of the waveguide as

$$P_2 = \left(\frac{c}{16\pi\omega}\right)\left(\frac{\omega_p^2 A|v_d|}{\omega^2 c}\right)^2 [A_1A_2 + A_3A_4]\,, \qquad (16)$$

where

$$A_1 = \frac{\delta_1 a\beta_1}{\beta_1^{12}}\,;$$

$$A_2 = \frac{\sin 2\beta_1^1 b}{4\beta_1^1} + \frac{3b}{2} - \frac{2\sin\beta_1^1 b}{\beta_1^1} + \frac{g_1^2}{2}\left(b - \frac{\sin 2\beta_1^1 b}{2\beta_1^1}\right) + \left(\frac{2g_1}{\beta_1^1}\right)(\cos\beta_1^1 b - 1) - \left(\frac{g_1}{2\beta_1^1}\right)(\cos 2\beta_1^1 b - 1)\,,$$

$$A_3 = \frac{2\pi\delta_1}{ak_2^2}\left\{\frac{\delta_2}{k_2^2}\left(1 + \frac{4\pi^2}{a^2k_2^2}\right) + \frac{2\pi\alpha_1 a}{4\pi^2 - a^2k_2^2}[\sin k_2 a(1 + \alpha_2 k_2^2 a^2) - \alpha_2 k_2^2 a^2]\right\}\,,$$

$$A_4 = \frac{\sin\beta_3 b}{\beta_3} + \frac{\sin\beta_1^1 b}{\beta_1^1} - b - \frac{g_1(\cos\beta_1^1 b - 1)}{\beta_1^1} - \frac{\sin(\beta_1^1 + \beta_3)b}{2(\beta_1^1 + \beta_3)}(1 + g_1 g_2) - \frac{\sin(\beta_3 - \beta_1^1)b}{2(\beta_3 - \beta_1^1)}(1 - g_1 g_2) + (g_1 - g_2) \times \frac{\cos(\beta_3 + \beta_1^1)b - 1}{2(\beta_3 + \beta_1^1)} - (g_1 + g_2)\frac{\cos(\beta_3 - \beta_1^1)b - 1}{2(\beta_3 - \beta_1^1)}\,.$$

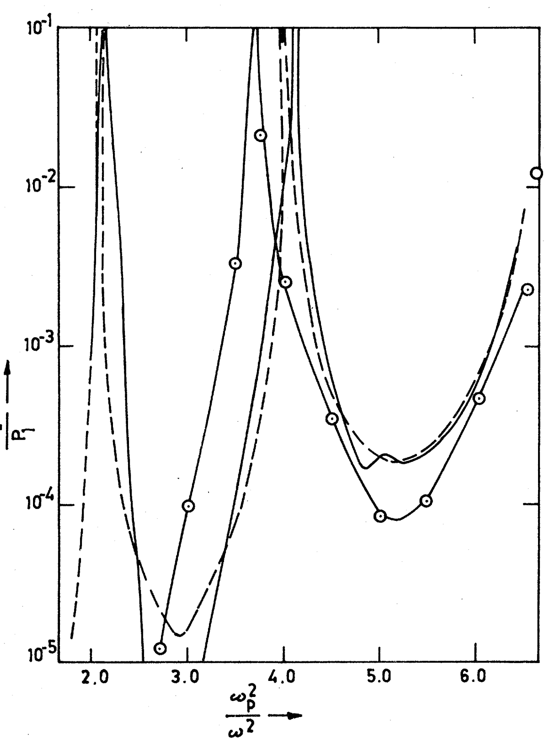


FIG. 1. Variation of $P_2/P_1$ with $\omega_p^2/\omega^2$ for $|v_d|/c \simeq 10^{-2}$, $m=4$ ———, O—O—O and - - - - curves corresponds to $a\omega/c = 3\pi/2$, $b\omega/c = 2\pi$; $a\omega/c = \pi$, $b\omega/c = 3\pi/2$, and $a\omega/c = 7\pi/4$, $b\omega/c = 2\pi$, respectively.

The power carried by the fundamental beam is given by

$$P_1 = c^2 abA^2\beta_1/16\pi\omega\,. \qquad (17)$$

It must be noted here that the second-harmonic power shows resonant enhancement at (i) $\beta_1^1 b = n\pi$, $n = 1, 2, 3, \ldots$, (ii) $k_2 a = n\pi$, $n > 2$, and (iii) $\beta_3 b = n\pi$, $n = 1, 2, 3, \ldots$, where

$$\beta_1^{12} = 3\omega_p^2/c^2\,, \quad k_2^2 = 4\pi^2/a^2 + \frac{3\omega_p^2}{c^2} \text{ and } \beta_3^2 = \frac{4\pi^2(m^2-1)}{a^2} - 3\omega_p^2/c^2\,,$$

and $n$ is an integer. All these conditions are particular cases of the general resonance condition $2\beta_1 = \beta_2$, i.e.,

$$\frac{n^2\pi^2}{b^2} + \frac{(m^2-4)\pi^2}{a^2} = 3\omega_p^2/c^2\,.$$

In order to have numerical estimate of power conversion efficiency ($P_2/P_1$), we have carried out the calculations for the following set of typical parameters:

$$\omega(CO_2 \text{laser}) \simeq 2.0\times 10^{14}\ \text{rad sec}^{-1}\,,$$

$$\omega_p^2/\omega^2 = 1.0\text{–}7.0\,, \quad |v_d|/c = 10^{-2}\,, \quad a\omega/c = 2.0\text{–}7.0\,,$$

$$\epsilon_L = 17.5\,, \quad b\omega/c = 2.0\text{–}9.0\,, \quad m = 4\,.$$

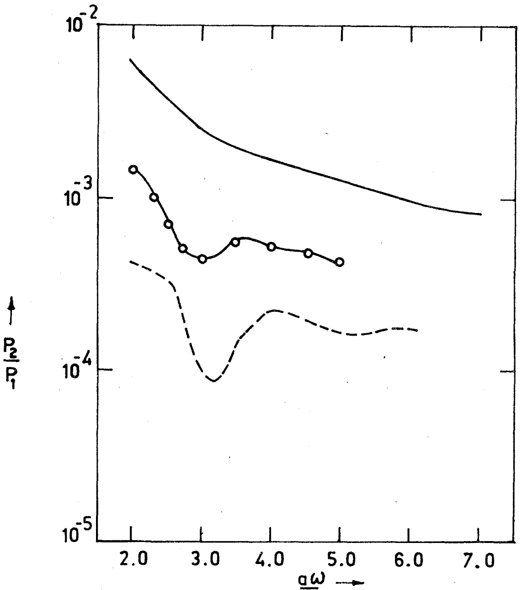


FIG. 2. Variation of $P_2/P_1$ with the $x$ dimension of the waveguide for $|v_d|/c \simeq 10^{-2}$, $m=4$, $b\omega/c=3\pi/2$. The value of $\omega_p^2/\omega^2$ for ———, O—O—O and - - - - curve is 4, 6, and 5, respectively.

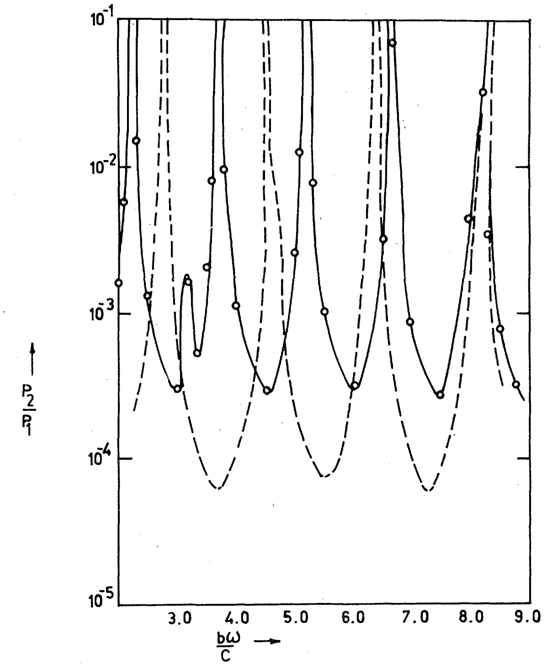


FIG. 3. Variation of $P_2/P_1$ with the $y$ dimension of the waveguide for $|v_d|/c \simeq 10^{-2}$, $m=4$, $a\omega/c=3\pi/2$. The values of $\omega_p^2/\omega^2$ for O—O—O and - - - - curve is 6 and 4, respectively.

These values correspond to laser powers ~1 kW. Figure 1 shows the variation of $P_2/P_1$ with $\omega_p^2/\omega^2$ for a fixed dimension of the waveguide. It is found that $P_2/P_1$ is resonantly enhanced for some optimum values of $\omega_p^2/\omega^2$.

Figure 2 shows the variation of $P_2/P_1$ with the $x$ dimension of the waveguide. $P_2/P_1$ decreases with the increasing $x$ dimension of the waveguide for low values of $\omega_p^2/\omega^2 \lesssim 4.0$. For higher values of $\omega_p^2/\omega^2$ the conversion efficiency shows minimum at some optimum values of $(a\omega/c)$.

Figure 3 shows the variation of $P_2/P_1$ with the $y$ dimension of the waveguide $(b\omega/c)$. The conversion efficiency is resonantly enhanced at some particular values of the $y$ dimension. As the density of the plasma increases resonance become narrower.

## IV. DISCUSSION

A high-power laser beam propagating through a rectangular waveguide gives rise to large yields of second-harmonic generation due to ponderomotive effects. The power conversion efficiency is greatly enhanced by the dimensional resonances. For a typical 1-kW $CO_2$ laser the power conversion efficiency in an $n$-InSb sample at room temperature takes on values as high as $10^{-2}$ (for $|v_d|/c \simeq 10^{-2}$). The optimum size of the waveguide for harmonic generation is of the order of a few wavelengths, and $\omega_p^2/\omega^2$ lies between 3 and 8.

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