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Theory of Compton scattering by interfering electromagnetic fields produced by two independent sources

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The interference effect produced by two independent sources can be understood in terms of the detection process, owing to the inability of the detector to know from which of the two sources the photons come. Detection based on the Compton effect seems to be a powerful method for recording the interference between high-frequency coherent fields. It is shown that by taking advantage of the uncertainty over the photon momenta, the scattering probability for an electron in an interference fringe is very high when the electrons are not relativistic. When the fields have the same frequency in the laboratory system, relativistic electrons can "see" the interference only when the angles between the fields are very small. In general, relativistic electrons see interference when the fields are suitably shifted in frequency in the laboratory system.

I. INTRODUCTION

The conditions under which interference effects occur when the interfering beams come from independent sources have been considered in the past.[1-5] The classical explanation is quite simple: each beam is a classical field and the intensity pattern is proportional to the square of the sum of the amplitudes of the two fields. In the quantum picture, i.e., in terms of photons, the explanation is not so obvious.[6] In a conventional interference experiment, like the two-hole Young experiment, each photon is assumed to interfere only with itself[7]; we may in fact suppose that each photon goes partly into each beam so that we are unable to tell which of the possible paths is taken by the photon. In interference effects between independent sources this point of view is not really true, because the beams come from two separate sources. In this case the interference effect could be associated with an indetermination in the detection process.[1,4] The existence of the interference effect is linked with our inability to know the origin, then the momentum, of the photon; the detection process involves a localization of a photon in a space-time point. Thus, the detection process rules out the possibility of knowing the photon's momentum and our ability to tell from which of the two sources it comes.

In a quantum-mechanical description the interference is described from a knowledge of the expectation values of the intensity in space-time points. If the expectation values are made on a set of pure states which are not an orthogonal set of the radiation field, interference terms appear; moreover, for "coherent" states the result is the same as the classical treatment. A pure coherent state describes the field of an ideal laser source. For two independent beams a more realistic description of the field involves an ensemble of mixed states,[1,4,5] for which the random-phase assumption is necessary to describe a succession of independent trails. This assumption gives an expectation value of intensity which indicates no interference effects. Interference effects are recovered if intensity correlations at two space-time points are considered; they are, however, random. Jordan and Ghiemetti[3] show that in a very short time interval interference phenomena can be described by interference terms in the average intensity, since the average of the field operator is not zero for laser radiation (i.e., coherent states).

The radiation from two independent lasers would be considered coherent in the same way as the radiation from a single source, but this coherence implies no relation between the density matrices for the two beams. This is a property possessed separately by each beam and depends on the magnitude of the expectation value of each of the annihilation operators of the field in the interference term. This point of view describes the existence of stable interference fringes, produced by two independent lasers when they operate continuously at a single well stabilized frequency.

Let us now consider the detection process. Generally it is photoelectric and Mandel[4] has calculated that the probability of photon absorption by a detector contains interference terms which are proportional to the photon number. We are interested in considering the problem of the detection of the interference effect via the Compton effect,

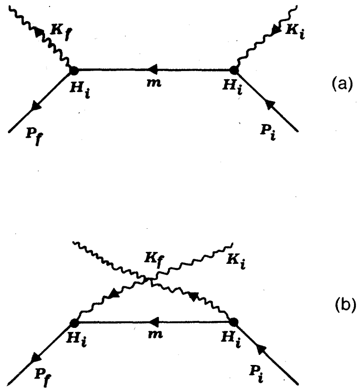

FIG. 1. Diagrammatic representation of the type of interaction which contributes to the second-order scattering process.

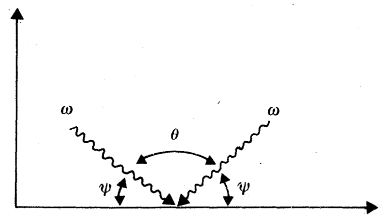

FIG. 2. Interfering fields which come from two separate laser sources.

a necessary technique when the interfering fields are at high frequency (x or γ rays). Therefore, we generalize the photocounting probability to a scattering probability for a Compton process by free electrons with the hypothesis that the process is detected in a time short compared with the coherence time $\tau = 1/\Delta\omega$ of the laser sources, i.e., in a time in which the phase of the beams remains fairly fixed. Thus, we can consider our source an ideal laser, and we can average the field operators on a pure coherent state, using the same method as Jordan and Ghielmetti.[3] It must be remembered that in a detection process it is possible to consider the scattering by free electrons only for radiation fields with wavelength much smaller than the atomic dimensions.

We first perform the scattering calculation in general and then in two particular cases: scattering from nonrelativistic free electrons, corresponding to a scattering between an electron beam and low-frequency interfering fields, and scattering from relativistic free electrons (Compton scattering), relative to inner atomic electrons of a detector, or to an accelerated beam. We will show that, the Compton scattering being a second-order process (see Fig. 1), new interference terms not present in first-order processes may appear in some cases.

II. GENERAL CASE

We wish to define in general the scattering probability for an electron in a space-time point $(\vec{r}, t)$, when the interfering fields come from two separate equal sources having the same frequency and polarization and meeting at an angle of θ (see Fig. 2). We are interested in the way in which the scattering depends upon the radiation field. For this reason, we derive the scattering probability as function of the expectation value of the field operators first in a generic state $|R_0\rangle$, then in a pure coherent state. We use the general form of the transition probability for a second-order scattering:

$$P_{\text{tot}} = \Sigma_f \left| \frac{1}{2}\left(\frac{1}{i\hbar}\right)^2 \int_{t_0}^{t} \int_{t}^{t_1} dt_1\, dt_2 \Sigma_m [\langle f|H_I(t_1)|m\rangle\langle m|H_T'(t_2)|i\rangle + \langle f|H_I'(t_1)|m\rangle\langle m|H_I(t_2)|i\rangle] \right|^2, \tag{1}$$

where $|i\rangle = |P_0, R_0\rangle = |P_0\rangle|R_0\rangle$ is the initial state of the electron plus the field, $|P_0\rangle$ is the initial state of the electron, $|R_0\rangle$ is the initial state of the radiation field, $|f\rangle = |P\rangle|R\rangle$ is the final state of the electron plus the field, $|P\rangle$ is the final state of the electron, $|R\rangle$ is final state of the radiation field, $|m\rangle = |P'\rangle|R'\rangle$ are the intermediate states of the electron plus the field, and $H_I(t)$ is the interaction Hamiltonian. For the moment, we do not specify the kind of the radiation field.

The time integration is made on a time interval shorter than the coherence time of the sources. In Eq. (1) each matrix element contains also a spatial integration over the detector volume. By writing explicitly this spatial integration and factorizing the matrix elements relative to the electron and the radiation field, we obtain:

$$P_{\text{tot}} = \frac{1}{4}\frac{1}{\hbar^4}\Sigma_P \Sigma_R \left| \int_V d\vec{r}_1 \int_V d\vec{r}_2 \int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2 \Sigma \{ M_{Pi} [\langle R|\hat{A}^{(-)}(\vec{r}_1, t_1)|R'\rangle\langle R'|\hat{A}^{(+)}(\vec{r}_2, t_2)|R_0\rangle + \langle R|\hat{A}^{(+)}(\vec{r}_1, t_1)|R''\rangle\langle R''|\hat{A}^{(-)}(\vec{r}_2, t_2)|R_0\rangle] \} \right|^2, \tag{2}$$

where M_{Pi} is the matrix elements relative to the electron interaction, $\hat{A}(\vec{r},t)=\hat{A}^{(-)}(\vec{r},t)+\hat{A}^{(+)}(\vec{r},t)$ are the field operators, $\hat{A}^{(-)}(\vec{r},t)\sim\hat{a}^{\dagger}$ is the emission operator, and $\hat{A}^{(+)}(\vec{r},t)\sim\hat{a}$ is the absorption operator of the radiation field.

Using the completeness property of the radiation fields, which is valid for each state of the field, squaring the matrix elements relative to the radiation field, and using the commutation rules we have

$$P_{\mathrm{tot}}=\frac{1}{4}\frac{1}{\hbar^4}\Sigma_P\int_{V'}\int_{V'}\int_{V}\int_{V}d\vec{r}_2'\,d\vec{r}_1'\,d\vec{r}_1 d\vec{r}_2\int_{t_0}^{t'}dt_2'\int_{t_0}^{t_2'}dt_1'\int_{t_0}^{t}dt_1\int_{t_0}^{t_1}dt_2\,|\Sigma(M_{Pi})|^2$$
$$\times(\langle R_0|\hat{A}^{(-)}(\vec{r}_2',t_2')\hat{A}^{(+)}(\vec{r}_1',t_1')\hat{A}^{(-)}(\vec{r}_1,t_1)\hat{A}^{(+)}(\vec{r}_2,t_2)|R_0\rangle$$
$$+\langle R_0|\hat{A}^{(-)}(\vec{r}_1',t_1')\hat{A}^{(+)}(\vec{r}_2',t_2')\hat{A}^{(-)}(\vec{r}_2,t_2)\hat{A}^{(+)}(\vec{r}_1,t_1)|R_0\rangle$$
$$+\langle R_0|\hat{A}^{(-)}(\vec{r}_2',t_2')\hat{A}^{(+)}(\vec{r}_1',t_1')\hat{A}^{(-)}(\vec{r}_2,t_2)\hat{A}^{(+)}(\vec{r}_1,t_1)|R_0\rangle$$
$$+\langle R_0|\hat{A}^{(-)}(\vec{r}_1',t_1')\hat{A}^{(+)}(\vec{r}_2',t_2')\hat{A}^{(-)}(\vec{r}_1,t_1)\hat{A}^{(+)}(\vec{r}_2,t_2)|R_0\rangle).\qquad(3)$$

For fixed polarization,

$$\hat{A}(\vec{r},t)=\hat{A}^{(-)}(\vec{r},t)+\hat{A}^{(+)}(\vec{r},t)$$
$$=\Sigma_K[A_K\exp(-i\omega_Kt+i\vec{K}\vec{r})\hat{a}_K+\hat{A}_K^*\exp(i\omega_Kt-i\vec{K}\vec{r})\hat{a}_K^{\dagger}],\qquad(4)$$

where

$$\hat{A}^{(-)}(\vec{r},t)\sim\hat{a}_K^{\dagger},\quad \hat{A}^{(+)}(\vec{r},t)\sim\hat{a}_K.$$

The initial state of the field is the superposition of two states corresponding to the fields, which come from the two sources:

$$|R_0\rangle=|R_{01},R_{02}\rangle=|R_{01}|R_{02}\rangle.\qquad(5)$$

We can separate the field operators which act on the $|R_{01}\rangle$ state and $|R_{02}\rangle$ state:

$$\hat{A}(\vec{r},t)=\hat{A}_1(\vec{r},t)+\hat{A}_2(\vec{r},t)$$
$$=\Sigma_{K1}[\hat{a}_{K1}f_{K1}(\vec{r},t)+\hat{a}_{K1}^{\dagger}f_{K1}^*(\vec{r},t)]+\Sigma_{K2}[\hat{a}_{K2}f_{K2}(\vec{r},t)+\hat{a}_{K2}^{\dagger}f_{K2}^*(\vec{r},t)].\qquad(6)$$

Equation (3) then becomes

$$P_{\mathrm{tot}}=\frac{1}{4}\frac{1}{\hbar^4}\Sigma_P\int_{v'}dt_2'\int_{v'}d\vec{r}_1'\int_{v}d\vec{r}_1\int_{v}d\vec{r}_2\int_{t_0}^{t'}dt_2'\int_{t_0}^{t_2'}dt_1'\int_{t_0}^{t}dt_2\int_{t_0}^{t_1}dt_2\,|\Sigma(M_{Pi})|^2$$
$$\times\{\langle R_0|[\hat{A}_1^{(-)}(\vec{r}_2',t_2')+\hat{A}_2^{(-)}(\vec{r}_2',t_2')][\hat{A}_1^{(+)}(\vec{r}_1',t_1')+\hat{A}_2^{(+)}(\vec{r}_1',t_1')][\hat{A}_1^{(-)}(\vec{r}_1,t_1)+\hat{A}_2^{(-)}(\vec{r}_1,t_1)]$$
$$\times[\hat{A}_1^{(+)}(\vec{r}_2,t_2)+\hat{A}_2^{(+)}(\vec{r}_2,t_2)]|R_0\rangle+\langle R_0|[\hat{A}_1^{(-)}(\vec{r}_1',t_1')+\hat{A}_2^{(-)}(\vec{r}_1',t_1')][\hat{A}_1^{(+)}(\vec{r}_2',t_2')+\hat{A}_2^{(+)}(\vec{r}_2',t_2')]$$
$$\times[\hat{A}_1^{(-)}(\vec{r}_2,t_2)+\hat{A}_2^{(-)}(\vec{r}_2,t_2)][\hat{A}_1^{(+)}(\vec{r}_1,t_1)+\hat{A}_2^{(+)}(\vec{r}_1,t_1)]|R_0\rangle+\langle R_0|[\hat{A}_1^{(-)}(\vec{r}_2',t_2')+\hat{A}_2^{(-)}(\vec{r}_2',t_2')]$$
$$\times[\hat{A}_1^{(+)}(\vec{r}_1',t_1')+\hat{A}_2^{(+)}(\vec{r}_1',t_1')][\hat{A}_1^{(-)}(\vec{r}_2,t_2)+\hat{A}_2^{(-)}(\vec{r}_2,t_2)][\hat{A}_1^{(+)}(\vec{r}_1,t_1)+\hat{A}_2^{(+)}(\vec{r}_1,t_1)]|R_0\rangle$$
$$+\langle R_0|[\hat{A}_1^{(-)}(\vec{r}_1',t_1')+\hat{A}_2^{(-)}(\vec{r}_1',t_1')][\hat{A}_1^{(+)}(\vec{r}_2',t_2')+\hat{A}_2^{(+)}(\vec{r}_2',t_2')]$$
$$\times[\hat{A}_1^{(-)}(\vec{r}_1,t_1)+\hat{A}_2^{(-)}(\vec{r}_1,t_1)][\hat{A}_1^{(+)}(\vec{r}_2,t_2)+\hat{A}_2^{(+)}(\vec{r}_2,t_2)]|R_0\rangle\}.\qquad(7)$$

Equation (7) is the general scattering probability between a free electron and a radiation field, which is a superposition of two fields.

If the initial set of radiation states is an orthogonal set, the expectation value of the mixed terms of the field operators (interference terms), relative to the two sources and obtained by developing the products in the Eq. (7), vanishes. It does not vanish if the initial state is a set of coherent states (nonorthogonal set) and if the time integration is made on a time interval shorter than the coherence time of the sources, because with this condition the phase of the fields can be considered fixed. We will come back to this point at the end of this section. Now if in Eq. (7) we perform the space and time integrations for the terms relative to the radiation field over a time much shorter than the coherence time of the beams and over a volume much smaller than the coherence volume of the beams we note that the four average sets of field operators coincide. Then, developing the products and assuming that the initial state of the radiation field is a coherent pure state (i.e., ideal laser source), we obtain for the total electron scattering probability in a space-time point $(\vec{r},t)$ which belongs to an interference fringe:

$$
\begin{aligned}
P_{\text{tot}} = \frac{1}{\hbar^4}\Big\{ & \langle I_{1i}(\vec{r},t)\rangle\langle I_{1s}(\vec{r},t)+\hbar\omega_{1s}^2/VK_{1s}\rangle+\langle I_{2i}(\vec{r},t)\rangle\langle I_{2s}(\vec{r},t)+\hbar\omega_{2s}^2/VK_{2s}\rangle\\
&+\langle\hat{A}_{1i}^{(-)}(\vec{r},t)\rangle\langle\hat{A}_{2i}^{(+)}(\vec{r},t)\rangle\langle\hat{A}_{1s}^{(+)}(\vec{r},t)\rangle\langle\hat{A}_{2s}^{(-)}(\vec{r},t)\rangle\langle\hat{A}_{2i}^{(-)}(\vec{r},t)\rangle\langle\hat{A}_{1i}^{(+)}(\vec{r},t)\rangle\langle\hat{A}_{2s}^{(+)}(\vec{r},t)\rangle\langle\hat{A}_{1s}^{(-)}(\vec{r},t)\rangle\\
&+\langle\hat{A}_{1i}^{(-)}(\vec{r},t)\rangle\langle\hat{A}_{2i}^{(+)}(\vec{r},t)\rangle\langle\hat{A}_{2s}^{(+)}(\vec{r},t)\rangle\langle\hat{A}_{1s}^{(-)}(\vec{r},t)\rangle+\langle\hat{A}_{2i}^{(-)}(\vec{r},t)\rangle\langle\hat{A}_{1i}^{(+)}(\vec{r},t)\rangle\langle\hat{A}_{1s}^{(+)}(\vec{r},t)\rangle\langle\hat{A}_{2s}^{(-)}(\vec{r},t)\rangle\\
&+\langle I_{2i}(\vec{r},t)\rangle\langle I_{1s}(\vec{r},t)+\hbar\omega_{1s}^2/VK_{1s}\rangle+\langle I_{1i}(\vec{r},t)\rangle\cdot\left\langle I_{2s}(\vec{r},t)+\frac{\hbar\omega_{2s}^2}{VK_{2s}}\right\rangle\\
&+[\langle\hat{A}_{1i}^{(-)}(\vec{r},t)\rangle\langle\hat{A}_{2i}^{(+)}(\vec{r},t)+\langle\hat{A}_{2i}^{(-)}(\vec{r},t)\rangle\langle\hat{A}_{1i}^{(+)}(\vec{r},t)\rangle]\left\langle I_{1s}(\vec{r},t)+\frac{\hbar\omega_{1s}^1}{VK_{1s}}\right\rangle\\
&+[\langle\hat{A}_{1i}^{(-)}(\vec{r},t)\rangle\langle\hat{A}_{2i}^{(+)}(\vec{r},t)\rangle+\langle\hat{A}_{2i}^{(-)}(\vec{r},t)\rangle\langle\hat{A}_{1i}^{(+)}(\vec{r},t)\rangle]\left\langle I_{2s}(\vec{r},t)+\frac{\hbar\omega_{s2}^2}{VK_{2s}}\right\rangle\\
&+\langle I_{1i}(\vec{r},t)\rangle[\langle\hat{A}_{1s}^{(+)}(\vec{r},t)\rangle\langle\hat{A}_{2s}^{(-)}(\vec{r},t)\rangle+\langle\hat{A}_{2s}^{(+)}(\vec{r},t)\rangle\langle\hat{A}_{1s}^{(-)}(\vec{r},t)\rangle]\\
&+\langle I_{2i}(\vec{r},t)\rangle[\langle\hat{A}_{1s}^{(+)}(\vec{r},t)\rangle\langle\hat{A}_{2s}^{(-)}(\vec{r},t)\rangle+\langle\hat{A}_{2s}^{(+)}(\vec{r},t)\rangle\langle\hat{A}_{1s}^{(-)}(\vec{r},t)\rangle]\Big\}\\
\cdot\, Y(\vec{r},\omega)\Big(&\Sigma_P\int_{t_0}^{t'}dt_2'\int_{t_0}^{t_2'}dt_1'\int_{t_0}^{t}dt_1\int_{t_0}^{t'}dt_2|\Sigma(M_{Pi}')|^2\Big),
\end{aligned}
\tag{8}
$$

where we have labeled with i and s the field operators relative to the incident and scattered fields. In Eq. (8)

$$M_{Pi}'=\int_{v'}d\vec{r}_2'\int_{v'}d\vec{r}_1'\int_{v}d\vec{r}_1\int_{v}d\vec{r}_2(M_{Pi}), \tag{9}$$

and $Y(\vec{r},\omega)$ contains the terms given by the space-time integration of the fields terms.

We have supposed that the incident and the scattered fields and the fields from the two separated sources are *statistically independent*. For this reason we have factorized the fields operators:

$$\langle\hat{A}_{Ji}^{(+)}(\vec{r},t)\hat{A}_{Js}^{(-)}(\vec{r},t)\rangle=\langle\hat{A}_{Ji}^{(+)}(\vec{r},t)\rangle\langle\hat{A}_{Js}^{(-)}(\vec{r},t)\rangle\,,\qquad J=1,2$$

and

$$\langle\hat{A}_{1i}^{(-)}(\vec{r},t)\hat{A}_{2i}^{(+)}(\vec{r},t)\rangle=\langle\hat{A}_{1i}^{(-)}(\vec{r},t)\rangle\langle\hat{A}_{2i}^{(+)}(\vec{r},t)\rangle\,. \tag{10}$$

The transition matrix elements do not vanish if we take advantage of the uncertainty over the photon momentum in the interference region. We discuss this point later in Sec. III. We remember that the initial state of the radiation field is a set of coherent states $|\{\alpha_K\}\rangle=\Pi_K|\alpha_K\rangle$; since the photon emission occurs in one of the modes of the field, the scattering is a stimulated scattering, if we suppose that the mode in which the photon is scattered is not $\alpha_K=0$, in the initial state of the field. Then Eq. (8) is more properly the stimulated scattering probability in an interference region, which also contains the spontaneous scattering contributions $\hbar\omega^2/vK$. We note that Eq. (8), which is the total probability for electron scattering in an interference fringe, contains the usual term of interference $\langle\hat{A}_{1i}^{(-)}(\vec{r},t)\rangle\langle\hat{A}_{2i}^{(+)}(\vec{r},t)\rangle$ in the same way as in calculations performed usually, but there are also other interference terms, which are not zero if the average is made on a pure coherent state of the radiation field. The interference terms

$$\langle\hat{A}_{1i}^{(+)}(\vec{r},t)\rangle\langle\hat{A}_{2i}^{(-)}(\vec{r},t)\rangle\propto\langle\hat{a}_{1i}\rangle\langle\hat{a}_{2i}^\dagger\rangle \tag{11}$$

can have a value between zero and a maximum value given by[3]

$$|\langle\hat{a}\rangle|^2\le\langle\hat{a}^\dagger\hat{a}\rangle\,. \tag{12}$$

In our case, i.e., a pure state, we have the maximum value. These interference terms are, on the contrary, zero for thermal sources and for mixed states if the time intervals of integration are longer than the coherence times of the sources, i.e., when the phase of the fields wanders appreciably. For a real laser the interference terms will be a fraction of the maximum interference value.

III. NONRELATIVISTIC FREE-ELECTRON SCATTERING

We now specify the previous relations for the case in which the electron is in a defined final state after the process. In this case the interaction Hamiltonian is

$$H_{\text{int}}=-(e/me^2)(\vec{P}\cdot\vec{A}), \tag{13}$$

if we neglect the second-order term $\propto e^2\vec{A}^2$. The initial state of the radiation field is a coherent state. The matrix elements are not zero if the laws of energy and momentum conservations are respected. They impose that for energy conservation

$$\begin{cases}E_i+\hbar\omega_1=E_p+\hbar\omega_{1s}\,,\\ E_i+\hbar\omega_2=E_p+\hbar\omega_{2s}\,,\end{cases} \tag{14}$$

where E = electron energy, $\hbar\omega$ = photon energy, and the labels i, p, s stand for initial, final, and scattered. If the final state of the electron is fixed and if the fields have initially the same frequency, the law of energy conservation becomes:

$$\begin{aligned} &E_i + \hbar\omega = E_p + \hbar\omega_s \\ &\hbar\omega_1 = \hbar\omega_2 = \hbar\omega; \quad \hbar\omega_{1s} = \hbar\omega_{2s} = \hbar\omega_s\,. \end{aligned} \tag{15}$$

For the momentum conservation

$$\begin{cases} \vec{\mathrm{P}}_i + \vec{\mathrm{K}}_{1i} = \vec{\mathrm{P}}_p + \vec{\mathrm{K}}_{1s}\,, \\ \vec{\mathrm{P}}_i + \vec{\mathrm{K}}_{2i} = \vec{\mathrm{P}}_p + \vec{\mathrm{K}}_{2s}\,, \end{cases} \tag{16}$$

where $\vec{\mathrm{P}}$ = electron momentum and $\vec{\mathrm{k}}$ = photon momentum. If the electron is in a fixed final state $\vec{\mathrm{P}}_p$, we can have $\vec{\mathrm{K}}_{1s} = \vec{\mathrm{K}}_{2s} = \vec{\mathrm{K}}_s$ if we consider the uncertain $\Delta\vec{\mathrm{K}}$ which exists in the interference region for the photon momentum which comes from the two sources (it has been said that the interference is linked with the electron inability to know the origin of the photon). The conservation law of the momentum becomes then

$$\vec{\mathrm{P}}_i + \Delta\vec{\mathrm{K}} = \vec{\mathrm{P}}_p + \vec{\mathrm{K}}_s\,. \tag{17}$$

For long times but always $t < 1/\Delta\omega$ the time integration of Eq. (7) gives a $\delta(E_f - E_a)$ term; then deriving with respect to the time, we have a scattering probability per unit time:

$$\begin{aligned} W = \frac{2\pi}{\hbar} &\left| \Sigma\left(\frac{M_{pp'} M_{p'i}}{E_i - E_{p'}} + \frac{M_{pp''} M_{p''i}}{E_i - E_{p''}} \right) \right|^2 \left(\frac{2\pi\hbar c^2}{\omega_i v} \right) \left(\frac{2\pi\hbar c^2}{\omega_s v} \right) \Sigma_{ki} F_1(\vec{\mathrm{r}}) \\ &\times [\langle \hat{a}^\dagger_{k1i}\hat{a}_{k1i}\rangle\langle 1 + \hat{a}^\dagger_{k1s}\hat{a}_{k1s}\rangle + \langle \hat{a}^\dagger_{k1i}\rangle\langle \hat{a}_{k2i}\rangle\langle \hat{a}_{k1s}\rangle\langle \hat{a}^\dagger_{k2s}\rangle + \langle \hat{a}^\dagger_{k1i}\rangle\langle \hat{a}_{k2i}\rangle\langle 1 + \hat{a}^\dagger_{k1s}\hat{a}_{k1s}\rangle + \langle \hat{a}^\dagger_{k1i}\hat{a}_{k1i}\rangle \\ &\quad \times \langle \hat{a}_{k1s}\rangle\langle \hat{a}^\dagger_{k2s}\rangle + \langle \hat{a}^\dagger_{k2i}\rangle\langle \hat{a}_{k1i}\rangle\langle \hat{a}_{k2s}\rangle\langle \hat{a}^\dagger_{k1s}\rangle + \langle \hat{a}^\dagger_{k2i}\rangle\langle \hat{a}_{k2i}\rangle\langle 1 + \hat{a}^\dagger_{k2s}\hat{a}_{k2s}\rangle + \langle \hat{a}^\dagger_{k2i}\hat{a}_{k2i}\rangle\langle \hat{a}_{k1s}\rangle\langle \hat{a}^\dagger_{k2s}\rangle \\ &\quad + \langle \hat{a}^\dagger_{k2i}\rangle\langle \hat{a}_{k1i}\rangle\langle 1 + \hat{a}^\dagger_{k2s}\hat{a}_{k2s}\rangle + \langle \hat{a}^\dagger_{k1i}\hat{a}_{k1i}\rangle\langle \hat{a}_{k2s}\rangle\langle \hat{a}^\dagger_{k1s}\rangle + \langle \hat{a}^\dagger_{k1i}\rangle\langle \hat{a}_{k2i}\rangle\langle 1 + \hat{a}^\dagger_{k1s}\hat{a}_{k2s}\rangle \\ &\quad + \langle \hat{a}^\dagger_{k1i}\rangle\langle \hat{a}_{k2i}\rangle\langle \hat{a}_{k2s}\rangle\langle \hat{a}^\dagger_{k1s}\rangle + \langle \hat{a}^\dagger_{k1i}\hat{a}_{k1i}\rangle\langle 1 + \hat{a}^\dagger_{k2s}\hat{a}_{k2s}\rangle + \langle \hat{a}^\dagger_{k2i}\rangle\langle \hat{a}_{k1i}\rangle\langle 1 + \hat{a}^\dagger_{k1s}\hat{a}_{k1s}\rangle \\ &\quad + \langle \hat{a}^\dagger_{k2i}\hat{a}_{k2i}\rangle\langle \hat{a}_{k1s}\rangle\langle \hat{a}^\dagger_{k2s}\rangle + \langle \hat{a}^\dagger_{k2i}\hat{a}_{k2i}\rangle\langle 1 + \hat{a}^\dagger_{k1s}\hat{a}_{k1s}\rangle + \langle \hat{a}^\dagger_{k2i}\rangle\langle \hat{a}_{k1i}\rangle\langle \hat{a}_{k1s}\rangle\langle \hat{a}^\dagger_{k2s}\rangle]\, \delta(E_f - E_a)\, \rho(E_f), \end{aligned} \tag{18}$$

where

$$M_{pp'} = -\frac{e}{me^2}(\vec{\epsilon}_s) \int \psi^*_p \frac{\hbar}{i} \vec{\nabla}\psi_{p'}\, d\vec{\mathrm{r}},$$

$$M_{p'i} = -\frac{e}{me^2}(\vec{\epsilon}_i) \int \psi^*_{p'} \frac{\hbar}{i} \vec{\nabla}\psi_i\, d\vec{\mathrm{r}},$$

$$M_{pp''} = -\frac{e}{me^2}(\vec{\epsilon}_i) \int \psi^*_p \frac{\hbar}{i} \vec{\nabla}\psi^*_{p''}\, d\vec{\mathrm{r}},$$

$$M_{p''i} = -\frac{e}{me^2}(\vec{\epsilon}_s) \int \psi^*_{p''} \frac{\hbar}{i} \vec{\nabla}\psi_i\, d\vec{\mathrm{r}}.$$

$$E_f = E_p + \hbar\omega_s\,, \quad E_a = E_i + \hbar\omega,$$

where we have used Eq. (6). We now must look at the average values of the fields operators in Eq. (18). In the first term we have $\langle \hat{a}^\dagger_{k1i}\hat{a}_{k1i}\rangle\langle 1 + \hat{a}^\dagger_{k1s}\hat{a}_{k1s}\rangle$. It is

$$\langle \hat{a}^\dagger_{k1i}\hat{a}_{k1i}\rangle = \langle \hat{n}_{k1i}\rangle = |\alpha_{k1i}|^2 = \overline{n}_{k1i} = \overline{n}_{k1}, \tag{19}$$

because the average is made on coherent states, and $|\alpha_{k1i}|^2$ is just the average numbers of the photons in the incident mode.

Moreover, we have for the number of scattered photons

$$\begin{aligned} \langle 1 + \hat{a}^\dagger_{k1s}\hat{a}_{k1s}\rangle &= 1 + \langle \hat{a}^\dagger_{ks} a_{ks}\rangle = 1 + \langle \hat{n}_{ks}\rangle \\ &= 1 + |\alpha_{ks}|^2 = (1 + \overline{n}_{ks}) = (1 + \overline{n}_s), \end{aligned} \tag{20}$$

where the term 1 is due to the spontaneous emission and $\overline{n}_s$ takes account of the stimulated emission. The second term is an interference term:

$$\langle \hat{a}^\dagger_{k1i}\rangle\langle \hat{a}_{k2i}\rangle\langle \hat{a}_{k1s}\rangle\langle \hat{a}^\dagger_{k2s}\rangle.$$

We know from previous considerations that each average is not zero, and that

$$\begin{aligned} \langle \hat{a}_{k1i}\rangle &= \alpha^*_{k1i} = |\alpha_{k1i}| = (\overline{n}_{k1i})^{1/2}, \\ \langle \hat{a}_{k2i}\rangle &= \alpha_{k2i} = |\alpha_{k2i}| = (\overline{n}_{k2i})^{1/2}, \end{aligned} \tag{21}$$

so that we can write

$$\langle \hat{a}^\dagger_{k1i}\rangle\langle \hat{a}_{k2i}\rangle = \alpha^*_{k1i}\alpha_{k2i} = |\alpha_{k1i}|\,|\alpha_{k2i}| = |\alpha_i|^2 = \overline{n}_i,$$

where $\overline{n}_i$ is the average number of photons in the interference fringe.

Really the field modes have the same frequency and intensity, but have a different wave vector. However, we can accept relations (21) by considering that the modes of the incident fields are indefinite in the interference fringes, and each mode has an uncertainty in the number of the photons proportional to $\Delta n = |\alpha| = \sqrt{\overline{n}}$. The term $\langle \hat{a}_{k1s}\rangle\langle \hat{a}^\dagger_{k2s}\rangle$ for the conservation of energy and momentum is

$$\begin{aligned} \langle a^\dagger_{k2s}\rangle &= \langle \hat{a}_s\rangle\langle \hat{a}^\dagger_s\rangle \\ &= 1 + \langle \hat{a}^\dagger_s\hat{a}_s\rangle = 1 + |\alpha_s|^2 = 1 + \overline{n}_s\,, \end{aligned} \tag{22}$$

where $\overline{n}_s$ is the mean number of scattered photons. We can make the same considerations for each term of Eq. (18) and so arrive at the final expression for the transition probability:

$$W=16[(2\pi)^3(c^2\hbar)^2/\hbar v^2\omega_i\omega_s](1+|\alpha_s|^2)\times(|\alpha_i|^2)|\ldots|^2\delta(E_f-E_a)\,\rho(E_f), \tag{23}$$

if the two sources have equal intensities so that

$|\alpha_{k1i}|^2=|\alpha_{k2i}|^2$.

IV. SCATTERING BY RELATIVISTIC ELECTRONS

We have said that the interference effect is linked with the impossibility of knowing the origin of the photons. To have interference fringes it is necessary that the position of a photon be within a length: $\Delta x<\lambda/\sin\theta$, where θ is the angle between the sources. The uncertainty on the photon momentum is accordingly

$$\Delta P_x=\hbar K\sin\theta. \tag{24}$$

If the electron which must "see" the interference is relativistic, the uncertainty in the photon momentum becomes

$$\Delta x'=\Delta x/\gamma\sim\lambda/\gamma\,\sin\theta\,,\qquad \Delta P_x'=\gamma\hbar K\sin\theta,\quad \gamma=E_i/ne^2, \tag{25}$$

i.e., the uncertainty is increased by a factor γ. It is this uncertainty which preserves the momentum.

Let us now consider the energy of the incident photons. If in the laboratory system the fields have the same frequency, in the electron rest system the energy will be different, due to the Doppler effect:

$$\omega_1=\omega(1-\beta\cos\psi)\gamma,\qquad \omega_2=\omega(1+\beta\cos\psi)\gamma. \tag{26}$$

In the electron rest system we have two kinds of scattering, and in the calculation of the scattering probability, the matrix elements associated with the interference terms are zero.

However, if the angle between the sources, in the laboratory system, is very small ($\cos\psi\rightarrow 90°$), in the electron rest frame the frequencies of the fields can still be considered equal and in this approximation the total scattering probability is given by

$$W=16(2\pi)^3(c^2\hbar)^2(1+|\alpha_s|^2)(|\alpha_i|^2)\times|\ldots|^2\delta(E_f-E_a)\rho(E_f). \tag{27}$$

Equation (27) is similar to Eq. (23), but differs from it in the interaction Hamiltonian

$$H_{\text{int}}=-e\vec{\alpha}\cdot\vec{\mathrm{A}}\,. \tag{28}$$

Also the electron matrix elements are different because they contain the relativistic wave functions for the electron.[8] Of course if in the laboratory system two fields have different frequencies, being

$$\omega_1=[(1+\beta\cos\psi)/(1-\beta\cos\psi)]\,\omega_2 \tag{29}$$

in the electron rest system, the electron "sees" fields of equal frequency. Also in this case the scattering probability is given by Eq. (27).

V. CONCLUSIONS

For two interfering beams, Eqs. (23) and (27) show that the Compton scattering probability respectively for nonrelativistic electrons and equal frequency fields or relativistic electrons and fields shifted in frequency according to Eq. (29) is much larger than would be expected by assuming it as simply proportional to the classical intensity squared in the fringe region. An increase by a factor of 16 is apparent from Eqs. (23) and (27). Therefore, in studying interference between two phase-locked laser beams, the Compton effect is much more effective than the photoelectric effect. This result comes out from the quantum description of the Compton scattering process, which is a second-order process in perturbation theory. On the other hand, the photoelectric effect is a first-order process involving the combination of a number of field operator products, connected to the number of the superimposing fields, higher than the combination number in the photoelectric or scattering effect by only one field. The main issue is that all the matrix elements in the scattering probability do not vanish due to the uncertainty in the photon momentum in the interference region and due to the state of coherence of the radiation fields. This is a result connected to coherence which is not realized by radiation from thermal sources, and it depends only on the magnitude of the expectation values.

Therefore, the behavior of the electron scattering in the interference region that we have shown depends essentially on the kind of interfering fields. We think that this effect could be observed in the interaction between a relativistic electron beam and two powerful optical laser beams locked in phase.

The high scattering rate obtained can be used in the project of a free-electron beam laser at very high frequencies (x- or γ-ray frequencies) in which the periodic magnetic field is replaced by the periodic scattering grating produced by two powerful laser beams locked in phase. A calculation for such a system will be presented in a paper to follow.

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