Theory of laser scattering in plasmas

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(Received 27 April 1979)

The corrected formula for the laser scattering intensity in collision-dominated plasmas is given. Expressions for the ratios of the ion to electron peak intensities and integrated intensities are obtained. These ratios and the intensity profile are compared to experimental data with good agreement.

The complete scattering formulas in the collisionless limit were derived in work[1-3] previous to the work described here. The theory of laser scattering in a collision-dominated plasma ($\Lambda \gtrsim 1$) was developed in a series of papers by DuBois *et al.*[4-7] In Ref. 6 it was shown that the intensity of the scattering is proportional to $\mathrm{Im}[S(k,\omega)]/\omega$, where $S(k,\omega)$ is the electron structure factor

$$S(k,\omega)=(T/n)\{\Pi_e(k,\omega)-[\Pi_e(k,\omega)]^2\Gamma(k,\omega)\}\,. \quad (1)$$

This is Eq. (A10) in Ref. 3 where $4\pi e^2\Pi_e(k,\omega) = Q_e^*(k,\omega)$. Here k is the wave number ($k = 4\pi\sin(\frac{1}{2}\theta)/\lambda_{\mathrm{laser}}$), $\Pi_{e(i)}$ is the electron (ion) polarization operator, n is the electron density, T is the temperature, and Γ is the total vertex part,[8] i.e., the screened interaction potential energy, given by

$$\Gamma(k,\omega)=\frac{4\pi e^2}{k^2+4\pi e^2[\Pi_e(k,\omega)+\Pi_i(k,\omega)]}\,. \quad (2)$$

For simplicity, a totally ionized hydrogen plasma is considered with $T_e = T_i$.

In their next equation (A11), Dubois and Gilinsky give

$$\mathrm{Im}S(k,\omega)=\frac{T}{n}\left(\mathrm{Im}\Pi_e(k,\omega)-\frac{4\pi e^2[\Pi_e(k,\omega)]^2\mathrm{Im}[\Pi_e(k,\omega)+\Pi_i(k,\omega)]}{|k^2+4\pi e^2[\Pi_e(k,\omega)+\Pi_i(k,\omega)]|^2}\right). \quad (3)$$

This does not mathematically follow from Eq. (A10) in Ref. 6, unless Eq. (A10) is treated as if it reads

$$S(k,\omega)=(T/n)[\Pi_e(k,\omega)-|\Pi_e(k,\omega)|^2\Gamma(k,\omega)]\,. \quad (4)$$

A related error appears in Eq. (2) of Nimura *et al.*[9] (who base their work on Ref. 6). His equation (in our notation) is

$$\mathrm{Im}S(k,\omega)=\frac{T}{n}\left(\mathrm{Im}\Pi_e(k,\omega)-\frac{4\pi e^2|\Pi_e(k,\omega)|^2\mathrm{Im}[\Pi_e(k,\omega)+\Pi_i(k,\omega)]}{|k^2+4\pi e^2[\Pi_e(k,\omega)+\Pi_i(k,\omega)]|^2}\right). \quad (5)$$

Expression (5) is equivalent to expression (4).

The correct expression for the electron structure factor is Eq. (1) of this paper [Eq. (A10) in Ref. 6]. The DuBois and Gilinsky paper is correct to this point. This equation was independently obtained by Adamyan *et al.*[10]

When we compare expression (1) and expression (4) with experimental data, we find good agreement in the ion region. In Fig. 1 the dashed curve is the theoretical prediction for ruby laser scattering from a plasma at $\theta = 90°$ with a density of $n = 2.2\times10^{16}$ cm^{-3} and temperature of $T = 2$ eV. The experimental points were collected by Anderson.[11]

However, further analysis of Eq. (4) shows that there exists a frequency at which the scattering cross section becomes zero and then negative. Under ordinary experimental conditions this occurs between the ion peak ($\omega \approx kv_i$) and the electron satellite ($\omega \approx \omega_{pe}$). Thus expression (4) fails to correctly describe the region beyond the ion peak

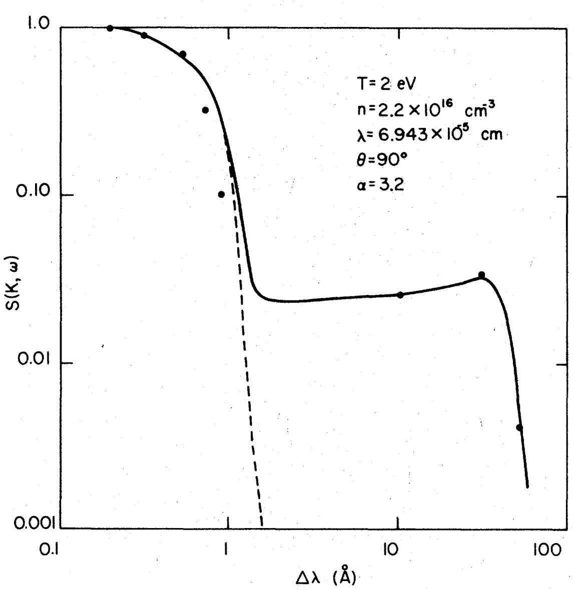

FIG. 1. Plot of $S(k,\omega)$ showing ion and electron peaks derived from Eq. (1) (dashed line) and from Eq. (3) (solid line) with experimental points from Anderson (Ref. 11).

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and in particular the electron satellites (see Fig. 1).

The results are similar in the ion region because, as $\omega \to 0$, $\mathrm{Re}\Pi_e(k, \omega) \to 0$. This fact is enough to make Eq. (1) equal to Eq. (4), but this is not true when $\omega \approx \omega_{pe}$.

To further check expression (1) against expression (4) we have calculated the differential scattering cross section at different densities and temperatures in both the ion and the electron peak regions. We have used the random-phase-approximation (RPA) expression $[e^2k/T \ll 1$ (Ref. 3)] for the polarization operators:

$$\Pi_\alpha(k, \omega) = (n/T)[1 + \zeta_\alpha Z(\zeta_\alpha)], \quad \alpha = i, e, \tag{6}$$

where $\zeta_\alpha = (\omega/k)(m_\alpha/2T)^{1/2}$ and $Z(\zeta_\alpha)$ is the plasma dispersion function.[12] Note that Eqs. (1) and (6) are valid if the density and the temperature satisfy the following conditions:

$$T \gg (\hbar^2/m_e)(n)^{2/3}, \tag{7}$$

$$e^2/Tl \ll 1. \tag{8}$$

Condition (7) implies that the electron gas can be treated as a classical Boltzmann gas, while Eq. (8) requires that the electrostatic energy of two electrons at the distance $l = (\frac{3}{4}\pi n)^{1/3}$ be much less than their thermal energy. Therefore, this treatment readily applies to a plasma with $T > 1$ eV and $n \lesssim 10^{18}$ cm^{-3}. As the temperature is increased, even higher densities can be modeled.

As can be seen from Fig. 1, we have obtained good agreement (the solid line) with experimental data in the frequency range 0.1 to 100 Å and have successfully described both the ion and the electron features. Figure 2 compares the scattering cross section from Eq. (1) and experimental data[13] for $\theta = 90°$, $n = 5.0 \times 10^{17}$ cm^{-3}, $\lambda_{\text{laser}} = 6.943 \times 10^{-5}$ cm, and $T = 4.5$ eV. Figure 3 compares Eq. (1) to data[14] for $\theta = 45°$, $n = 6 \times 10^{15}$ cm^{-3}, $\lambda_{\text{laser}} = 6.943 \times 10^{-5}$ cm, and $T = 103$ eV in the area of the electron peak. [Eq. (4) gives no prediction for the electron peak.]

The expression in Eq. (1) is used to compute the solid curves in Figs. 4 and 5. Figure 4 represents the ratio (p) of the electron-satellite peak frequency (ω_{peak}) to the electron plasma frequency

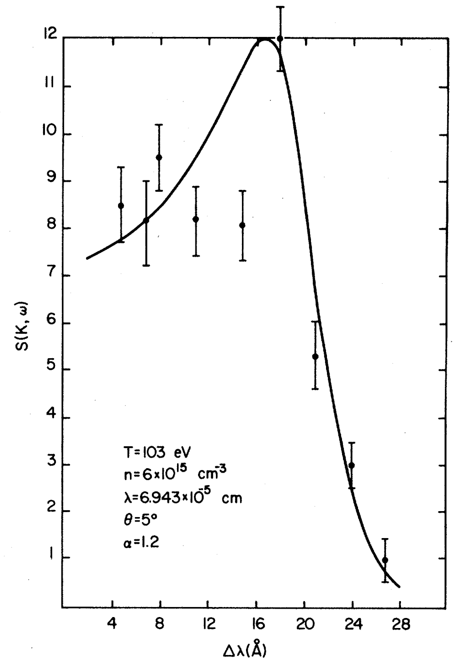

FIG. 3. Plot of $S(k,\omega)$ showing electron feature from Eq. (3) (line) with experimental points from D. Evans and Katzenstein (Ref. 14).

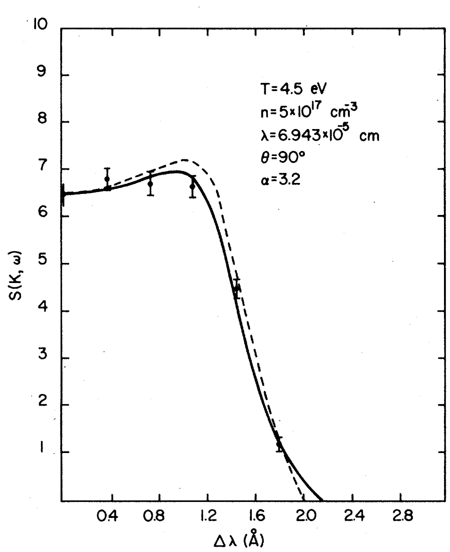

FIG. 2. Plot of $S(k,\omega)$ showing ion feature derived from Eq. (1) (dashed line) and from Eq. (3) (solid line) with experimental points from Röhr (Ref. 13).

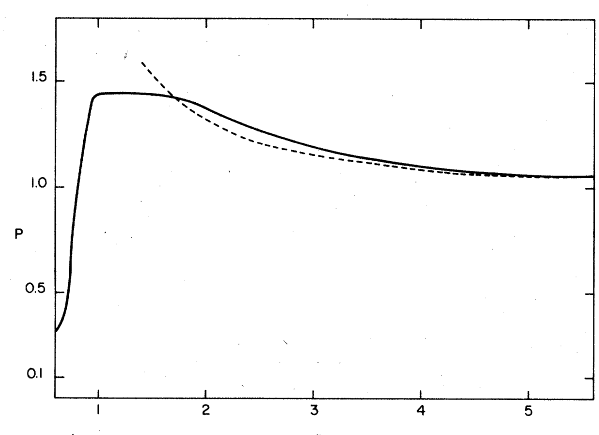

FIG. 4. Computer derived curve for $P = \omega_{\text{peak}}/\omega_{\text{pe}}$ as a function of α. The dashed line is $p = (1 + 3/\alpha^2)^{1/2}$.

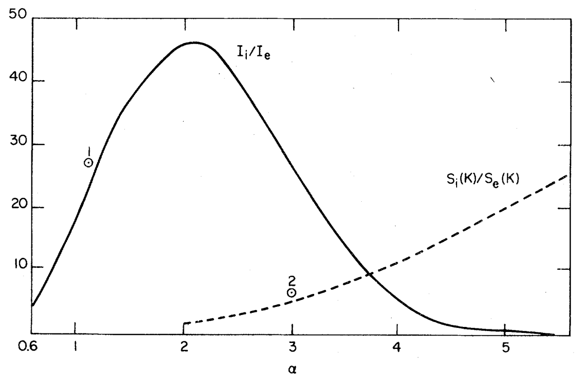

FIG. 5. Solid curve is the ratio of ion to electron peak intensities I_i/I_e. Dashed curve is the ratio of integrated peak intensities $S_i(k)/S_e(k)$. Point 1 is experimental I_i/I_e from Anderson (Ref. 11), and point 2 is $S_i(k)/S_e(k)$ from Ramsden and Davies (Ref. 15).

(ω_{pe}) as a function of α. The dashed curve [$p = (1+3/\alpha^2)^{1/2}$] in Fig. 4 is the thermally corrected kinetic-theory approximation for the ratio where $\alpha = (k\lambda_D)^{-1} = (4\pi n e^2/Tk^2)^{1/2}$.

The solid curve in Fig. 5 gives the ratio of the ion-center ($\omega = 0$) peak intensity (I_i) to the electron-satellite peak intensity (I_e) as a function of α. This curve [generated from Eq. (1)] can be estimated by

$$\frac{I_i}{I_e} \approx \frac{p^2\pi\alpha^2(M_i/M_e)^{1/2}e^{-\alpha^2p^2/2}}{2(2+1/\alpha^2)^2}, \tag{9}$$

where p is obtained from Fig. 4. Equation (9) is accurate to within 16% for $\alpha > 2$ and to within a factor of 4 for $\alpha < 2$. Below $\alpha = 0.6$ the collective effects cease to be important, the scattering profile becomes Gaussian, and there are no longer any electron peaks.

It is important to note that above $\alpha \approx 3$ the electron peak value grows rapidly but the full width at half maximum (FWHM) soon becomes experimentally unresolvable. Therefore, it is useful to examine the ratio of the integrated areas of the two peaks $S_i(k)/S_e(k)$.

Because of the Lorenztian nature of the electron peak, the FWHM ($\Delta\omega_e$) and the area can be easily shown to be

$$\Delta\omega_e = (\pi/2)^{1/2}\alpha^2p^4\omega_{pe}e^{-\alpha^2p^2/2} \tag{10}$$

and

$$S_e(k) = \frac{p^2}{2\alpha^2}\left(\frac{Vnr_0^2(1+\cos^2\theta)}{2}\right), \tag{11}$$

where $r_0 = e^2/m_ec^2$, V is the scattering volume, and θ is the scattering angle. If we take the integrated ion peak area to be $S_i(k) = I_ikv_i$ or

$$S_i(k) = \frac{\sqrt{\pi}}{(2+1/\alpha^2)^2}\left(\frac{Vnr_0^2(1+\cos^2\theta)}{2}\right), \tag{12}$$

we find that the ratio of integrated areas is

$$\frac{S_i(k)}{S_e(k)} = \frac{2\sqrt{\pi}\,\alpha^2}{p^2(2+1/\alpha^2)^2}. \tag{13}$$

Equation (13) is given by the dashed line in Fig. 5 for $\alpha > 2$. For large α, this coincides with Salpeter's prediction[3] for the ratio. The two experiments we found which give enough information to calculate $S_i(k)/S_e(k)$ (Ref. 15) or I_i/I_e,[11] are also shown in Fig. 5 for comparison.

In conclusion, we would like to point out that the minor error of DuBois and Gilinsky,[6] to which we refer in this paper, has had a tendency to be the starting point of subsequent works (e.g., Nimura[15]). Only minor modifications are necessary to recover the total validity of these papers. It is useful to note that the DuBois and Gilinsky treatment is valid not only in collision-dominated plasmas but also in collisionless plasmas, within the parameters given in (7) and (8).

Finally, the extremely useful graphs in Figs. 4 and 5 show excellent agreement with available experimental data. However, there is an obvious need for well documented data in the range of $\alpha \gtrsim 3.5$.

We wish to thank A. DeSilva and J. Huba for their very useful discussions. This work was supported in part by the National Science Foundation. One of the authors (I.M.T) wishes to thank the University of Maryland for its hospitality.

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