Development of overvoltage breakdown at high gas pressure

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A model for the development of electrical breakdown in dense gases is presented. It describes the initial phase of breakdown in the regime where the Townsend avalanche mechanism does not apply. The main features of the model are as follows: (1) It gives a continuous picture of the development both in the structure of the breakdown and the physics of the processes, and (2) it is based on electron kinetics, so that the theory is general in scope. In light of this model a brief discussion of experimental results is given.

I. INTRODUCTION

Two basic models have been proposed to explain the initial phase of electrical breakdown in gases at high pressures. These are commonly known as the Townsend avalanche model[1] and the streamer model.[2] The mechanism playing a role in the first model involves the interaction of primary and secondary processes of which cathode processes are generally found to predominate in uniform fields.[3] Photoionization of the gas in the interelectrode volume forms the basis for the second model.[2] Over the last 30 years, considerable effort has gone into establishing the regime of validity of both models. Experimental evidence was sought for the transition between the two regimes, such as an abrupt decrease in the formative time of the breakdown as the voltage was increased above but near the self-breakdown voltage. The results were not conclusive.[4]

The Townsend avalanche model was initially thought to apply only at low pressures and for voltages near self-breakdown. This regime, however, has recently been expanded by the experimental and theoretical works at Swansea[5,6] and by the computer simulations of Ward,[7] to include the high-pressure, small overvoltage regime ($\lesssim 20\%$). This was the regime for which the streamer model was originally proposed. Unlike the Townsend avalanche theory, the foundations of the streamer theory have been very vague from the start.[1] It is important to note that the "streamer model" is actually more than one model, each based on different hypotheses.[8] The clearest physical picture of the processes associated with a streamer model has been given by Lozanskii,[9] who proposed that associative ionization of atoms excited by photons of lower energy than those required to ionize, are responsible for the formation and propagation of the streamer. The initial support for the streamer model came from experiments by Rogowski[10] and Raether,[2] and later by Fletcher,[11] who measured values for the formation time somewhat in agreement with those obtained theoretically. Luminosity measurements initially used by many workers as further evidence in support of the theory, have been shown, in certain cases, to be very misleading.[12] Moreover, as pointed out by Mesyats,[13] agreement in the value of the formative time and, we may add, in the structure of the breakdown process, is a necessary but not a sufficient criterion for the validity of the theoretical premises. Agreement in the formation time was also obtained by Dickey,[14] although his theory was based on a completely different hypothesis.

At very high overvoltages (two to three times self-breakdown voltage), Stankevich and Kalinin[15] have observed that the structure of the breakdown, once again, changes; i.e., a broad channel breakdown is observed as opposed to a filamentary channel. This has been explained by Babich and Stankevich[16] in terms of runaway electrons from the main avalanche. We must point out that there are a few inconsistencies in the paper of Babich *et al.* regarding the criterion for the existence of these runaway electrons. Experimental evidence of these runaway electrons exists.[17] These experiments, however, were not time resolved so that it is not known when during the breakdown runaway electrons were produced.

At voltages below this regime, Mesyats[18] obtained values for the formative time which were not in agreement with those obtained using the streamer model. He then proposed the avalanche chain model, where a chain of avalanches originating at the cathode and supported by the photoelectric effect at the cathode is responsible for bridging the gap.

Thus, at present, the streamer model is thought to qualitatively describe the breakdown processes at high pressures and overvoltages (above 20% self-breakdown), but below Mesyats' avalanche chain regime.

In this paper, we present a model to describe the breakdown processes for all voltages and pressures above the Townsend avalanche domain. We call it the

two-group model due to the similarity between this problem and the neutron transport problem in nuclear reactor physics. It is based on electron kinetics, and it is thus applicable to a broad range of conditions (breakdown in pure gases, for example). The model offers the attractive feature of the unification of all the proposed breakdown phenomena; that is, it provides a continuous picture of the development of the initial phase of breakdown above the Townsend regime both in the structure of the breakdown and the physics of the processes. It merges into the Townsend avalanche picture as the voltage is reduced.

In Sec. II, we will present the model, devoting our attention to the physical processes that occur and that lead to the formation of breakdown. We then proceed, in Sec. III, to attempt to formulate a mathematical framework for the theory. In Sec. IV, we discuss some experimental results in light of the theory presented.

II. TWO-GROUP MODEL OF BREAKDOWN

We will develop a physical picture of the electrical breakdown of a gas at high pressures and for voltages so that the effects of the produced space charge cannot be neglected. This regime lies above the Townsend avalanche regime. It is our purpose in this section to give a clear physical picture of the processes responsible for increasing the propagation speed of the avalanche towards the anode and for causing the filamentary appearance of the evolving avalanche.

Our model is based on electron kinetics rather than photon induced processes. Electrons with a broad spectrum of energies are always present in an avalanche. On the other hand, the existence of photons with enough energy and range to photoionize the gas outside the avalanche has always been a questionable matter, especially in relatively pure gases.[9] The possibility of multiphoton ionization would require photon fluxes not available in the avalanche stage.

A sketch of the experimental situation we want to consider is shown in Fig. 1. Also shown is an avalanche propagating towards the anode. The shape of the avalanche is indicative of the processes at play in the development up to its present location in the gap. The avalanche is divided into three stages (see Fig. 1). This division is strictly for illustrative purposes, since a continuous transformation is actually taking place. In stage I, the radial dimensions of the avalanche is determined primarily by diffusion processes. The avalanche radius is given by[1]

$$r_d = (6Dt)^{1/2}, \tag{1}$$

where D is the electron diffusion coefficient. In

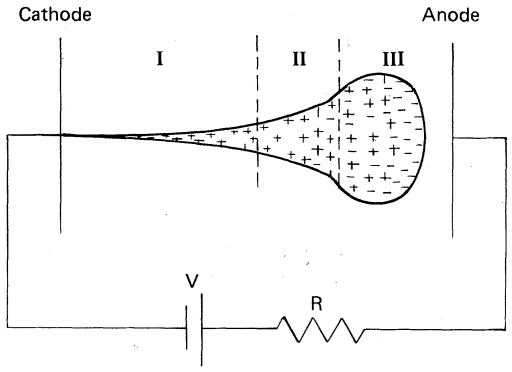


FIG. 1. Sketch of the developed avalanche in the cathode-anode volume.

the regime of interest (i.e., for voltages >20% self-breakdown), the time of development is very short so that very little expansion occurs. We can estimate the value of the radius for the case of breakdown in nitrogen, at atmospheric pressure. Here, $D \simeq 862$ cm^2/sec, $t \sim 10^{-8}$ sec, and from (1), $r_d \sim 7.2 \times 10^{-3}$ cm.

As the number of electrons in the avalanche increases, electrostatic repulsion begins to play a role in the expansion of the avalanche. In this regime (stage II), the avalanche radius increases exponentially with time. This radius may be estimated using the expression[19]

$$r_e = \left(\frac{3e}{4\pi\epsilon_0 E_0}\right)^{1/3} e^{\alpha z/3}, \tag{2}$$

where e is the electron charge, E_0 is the external field, α is the Townsend primary ionization coefficient, z is the average position along the gap axis of the avalanche with $z = 0$ being the cathode position, and ϵ_0 is the permittivity of free space. For breakdown in nitrogen at 1 atm and an applied field of 40 kV/cm (i.e., ~25% above self-breakdown), for which $\alpha \sim 56$ cm^{-1}, the radius of the avalanche at $z = 0.05$ cm is $r_e = 0.14$ cm. The radial expansion is somewhat slowed down in stage III as space-charge neutralization begins to take place.

Throughout these three stages, as the electron and ion densities in the avalanche increase, two things are happening: First, a highly nonequilibrium electron energy distribution is evolving[20] and second, the external field is becoming distorted due to the presence of the space charge. It is the combination of these two effects that play the major role in the following stage, i.e., stage IV. A qualitative picture of the longitudinal field at the axis of the avalanche as a function of distance and just prior to the time the avalanche "enters" into stage IV is given in Fig. 2. The $x = 0$ plane corresponds to the front of the avalanche.

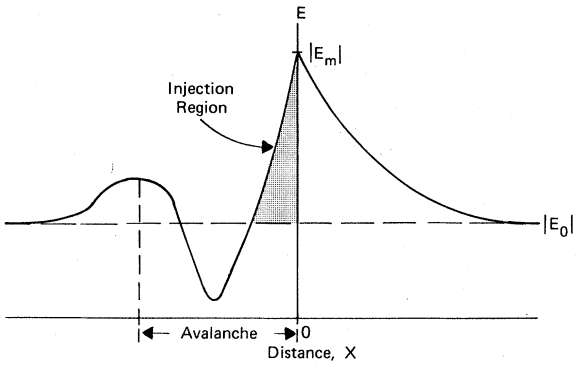


FIG. 2. Sketch of the magnitude of the longitudinal electric field along the axis of the avalanche.

Here, the field attains the maximum average value it can have in the gap E_m, and it decreases away from this plane. The rate of decrease is slower for $x>0$.

The electron energy distribution in the avalanche is far from equilibrium. It is enriched with high-energy electrons, a consequence of the large electric field. Furthermore, it is anisotropic in the high-energy region; it acquires a directed character along the field (this will be further discussed). For the purpose of discussion, we may think of the electron energy distribution as being formed of two groups of electrons: the "main" electron distribution and the "fast" electrons.

It is possible for some of these fact electrons to become runaways, that is, they may continuously gain energy from the field. This happens because the effective retarding force on an electron moving through a neutral gas decreases with increasing velocity, in the case of electrons possessing a sufficiently high energy (i.e., $u \gtrsim 3-5\epsilon_i$, where ϵ_i is the ionization energy). The energy threshold for electron runaway is determined by the magnitude of the electric field. The larger the field, the lower the threshold energy and consequently the larger number of electrons can run away.

In Fig. 2, the region where electrons are most likely to run away is labeled the "injection region." In this region of high electric field, the runaway threshold energy is the lowest and is determined by the maximum field intensity E_m. Since the distribution is anisotropic in this energy regime, the runaway electrons are accelerated out of this region and are injected into the region ahead of the avalanche where the field is decreasing. The number of particles injected is maximum at the axis. Here, the space-charge fields and the external field are colinear, thus giving the maximum total field, hence, the lowest injection energy. As we move away from the axis, the space-charge field and the external field are no longer collinear,

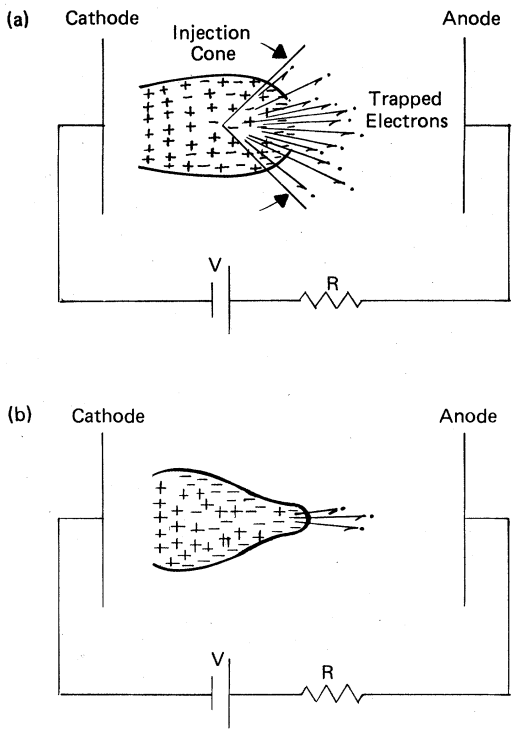


FIG. 3. Development of a filamentary channel ahead of the avalanche.

so that the total field is reduced, thus increasing the runaway threshold energy. We thus observe that there is an injection cone with a maximum on the axis [see Fig. 3(a)].

Once injected, most of the fast electrons no longer meet the runaway conditions and become "trapped"; that is, the energy they gain along their trajectory is not enough to overcome the losses (see Sec. III). The trapping distance (i.e., the distance from the avalanche front to where they become trapped) is for a given gas a function of the initial injection energy of the electrons and the slope of the electric field ahead of the avalanche.

Thus, just prior to stage IV, we may think of the avalanche as a localized distribution of electrons. The beginning of stage IV [Fig. 3(a)] is marked by the "burst" of the avalanche along its axis, followed by the ejection of high-velocity, electrons and their subsequent capture at varying distance from the original avalanche position where they start to generate avalanches of their own.

The captured electrons ionize the gas extending the boundary of the main avalanche along a filamentary channel centered at the axis of the avalanche [see Fig. 3(b)]. Once the channel starts to narrow, the electrical field just ahead of the tip increases. Fast electrons are continuously injected into the region ahead of the tip and again

are trapped. This process accelerates the development of the avalanche tip towards the anode. This was shown by Raether,[2] although his explanation was based on the electrons being produced by photoionization ahead of the avalanche rather than by captured runaway electrons. Thus a filamentary channel (note that the channel is maintained narrow due to the injection cone phenomenon) evolves from the main avalanche whose structure is similar to the classical streamer, although the physics of its formation and subsequent development is quite different. It is possible that for certain gas mixtures, photoionization may be present; however, the electron kinetic effects we have described still play the fundamental role.

Once the avalanche front makes contact with the anode, the field in the cathode side of the avalanche is greatly enhanced. This happens because a conducting path now exists between the cathode tip of the avalanche and the anode. Moreover, the field lines converge toward this tip, so that subsequent avalanches emerging from the cathode can complete the final bridging of the gap.

As the applied voltage increases, two things happen: The avalanche "bursts" closer to the cathode, and the number of untrapped runaway electrons increases. These runaways have been experimentally observed for voltages two to three times the self-breakdown voltage.[17]

III. MATHEMATICAL FOUNDATIONS OF THE TWO-GROUP MODEL

From the discussion in Sec. II it is evident that the breakdown of a gas at higher overvoltages (i.e., $V > 20\%$ self-breakdown) is governed by electron kinetics in a highly inhomogeneous, three dimensional field. A full treatment of the problem would require a knowledge of the space-time-dependent electron energy distribution function and of the space-charge fields in the region bounded by the electrodes, subject to the boundary conditions defined by the secondary processes occurring at the electrodes and by the external supply circuit. This problem is indeed formidable, and the only possibility of it being solved is via computer analysis. In this paper we will not give either an analytical or a computational description of the physics presented in Sec. II; however, we will give a mathematical justification of the hypothesis presented.

Our point of departure is the state of the avalanche just prior to stage IV (see Sec. II and Fig. 1). At this point, we have a weakly ionized medium (typical values for the density of electrons are 10^{11}–10^{12} particles/cm^3) in the presence of an external electric field. From Gurevich's work[20] we say that the electron energy distribution function can be thought of as being formed of two groups: the main distribution electrons and the fast electrons. This discontinuity is again of a mathematical nature to make the problem tractable.

Although the conditions existing during breakdown are transient in character and bounded in space, we will assume that the fast electron distribution just prior to stage IV is, to zeroth order, given by the quasistatic distribution obtained by Gurevich[20] for the case of a weakly ionized plasma in an infinite domain; that is,

$$f_f(v,t) \simeq f(v_0,t) \exp\left[-\frac{2m}{\epsilon_i}\int_{v_0}^{v} v\,dv\left(1-\frac{eE}{F(v)}\right)\right], \qquad (3)$$

where v_0 is the lower boundary of the fast electron distribution, $v_0 \simeq (5\epsilon_i/m)^{1/2}$; ϵ_i is the ionization energy; $F(v)$ is the effective retarding force on an electron moving with a velocity v in a neutral gas; and $f(v_0, t)$ is the main distribution evaluated at $v = v_0$ (and it may be taken as Maxwellian). The fast electron distribution if valid for $v > v_0$.

Instead of the analytical expression used by Gurevich[20] for the retarding force $F(v)$ (see Ref. 21 for further discussion of this retarding force), we retain the same functional dependence on v and introduce two parameters A and B which are then determined from experimental data.[22] The reason for this is that there is a disagreement between the values obtained experimentally and analytically for the position and height of the maximum of the retarding force. Thus, we take $F(v)$ to be

$$F(v) = A'p \ln(v^2/B')/v^2,$$

or letting $u = \frac{1}{2}mv^2$ (4)

$$F(u) = Ap \ln(u/B)/u.$$

The function $F(u)$ is sketched in Fig. 4. We can now find the injection energy, i.e., the minimum value of energy that a fast electron must have to run away from the avalanche and be injected into the decreasing field region ahead of the avalanche. From Eq. (3) the runaway condition is defined as

$$F(u) = eE \qquad (5)$$

and yields the minimum energy u_i for $F(u_i) = eE_m$ (see Fig. 4), where E_m is the maximum field at the front of the avalanche (Fig. 2). Electrons with energy $u \geq u_i$ can run away from the avalanche. Gurevich[20] found that the flux of runaways falls off rapidly with increasing ratio F_{max}/eE, where F_{max} is the maximum value of the retarding force, so that the greatest number of runaway electrons will be found near u_i.

In a reference frame moving with the main avalanche, the energy change of one of these injected electrons per unit distance away from the avalanche front is given by

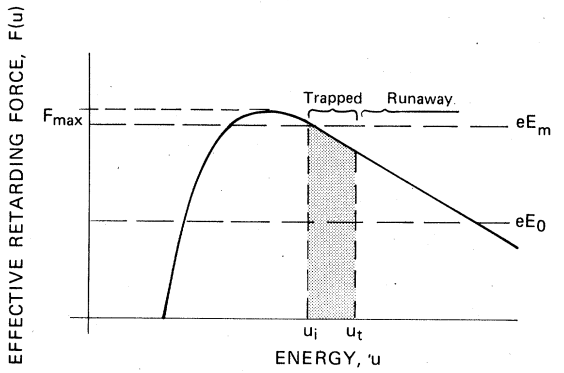


FIG. 4. Effective retarding force as a function of electron energy.

$$du/dx = eE(x) - F(u), \tag{6}$$

where u is the fast electron energy in eV. At $x = 0$, u = injection energy of a given fast electron.

From Eq. (6) and from Fig. 5, we note that an injected particle with energy $u_0 > u_i$ will start to gain energy because $du/dx|_{x=0} > 0$. However, if the rate of energy gained from the field is less than the rate of energy loss due to collisions, the particle will slow down and be trapped. This is due to the fact that the particle is moving in a region of decreasing field with a larger gradient than that of the loss function F.

In (u, x) space, the equation

$$eE(x) = F(u) \tag{7}$$

defines the locus of points for which $du/dx = 0$. A particle, starting with energy u_0, whose trajectory crosses this line will become trapped. Thus there is a range of injected particle energies $u_i < u < u_t$ which will slow down and become trapped. u_t is the injection energy of a particle whose trajectory is tangent to the locus curve [Eq. (7)] for $x \to \infty$. The electrons in this range of energy become trapped at various distances from the avalanche. Thus a narrow channel, as described in Sec. II, evolves from the main avalanche. To show this effect, it is necessary to solve Eq. (6) with $F(u)$ given by Eq. (4). To do so, we must know the dependence of the electric field with x.

It is common practice to assume that the avalanche may be replaced by either a spherical cloud of electrons[2] or by a conducting sphere.[16] We, however, will assume that just prior to stage IV, the avalanche may be represented by a conductor with an excess of electrons. The reasoning is as follows: n_1 electrons after traveling a certain distance produce n_2 ions and n_2 electrons. The $n_1 + n_2$ electrons and n_2 ions act collectively to establish a minimum field configuration inside the avalanche. This configuration can be thought of as being formed by the superposition of a dipole charge distribution (i.e., similar to the uncharged conducting sphere approach) and an electron layer (similar to the electron-cloud approach). The axial field at a distance x from the boundary of the avalanche may then be written as[23]

$$E(x) = E_0 + \tfrac{3}{8}\pi E_0 \left(\frac{r_e}{r_e + x}\right)^2 + 2E_0 \left(\frac{r_e}{r_e + x}\right)^3, \tag{8}$$

where r_e is the radius of the avalanche just prior to the transition into stage IV and E_0 is the applied electric field. As the applied field increases (e.g. 50% overvoltages), the transition to stage IV occurs closer to the cathode. As this happens, the contribution to Eq. (8), due to the $(n_1 - 1)$ ions left behind by the avalanche, must be taken into consideration. Their effect is to reduce the value of the second term in the right-hand side of Eq. (8). At higher fields ($\gtrsim$80% overvoltages), the transition occurs very close to the cathode. For these cases, the whole space charge has a dipole structure so that the second term in Eq. (8) disappears.

Numerical solutions of Eqs. (4), (6), and (8) are shown in Fig. 5 for the case of breakdown in a 1-cm gap in nitrogen at atmospheric pressure. The dotted curve is the locus curve, i.e., Eq. (7). The differences between Figs. 5(a), 5(b), 5(c), and 5(d) are in the values of r_e and E_0 used. In Fig. 6, we show the minimum injection energy u_i as a function of the maximum electric field E_m for this example. The values for the parameters A and B in Eq. (4) were found from the data in.[22]

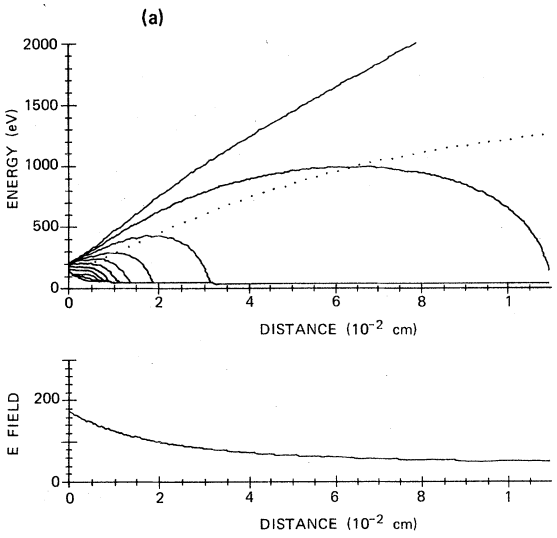
The results are qualitatively similar: Injected fast electrons with energies in the range $u_i < u < u_t$ become trapped at various distances ahead of the avalanche. Electrons with $u > u_t$ will actually run away. Their number increases as the applied field increases. When $E_m > F_{max}/e$, the runaway flux is substantial. Under these conditions, the fast electron distribution is no longer given by Eq. (3). A solution to the Boltzmann equation when strong electric fields are present must be obtained.

The minimum injection energy and the trapping distance are a function of r_e and E_0. The consequence of this dependence is discussed in the next section.

IV. DISCUSSION

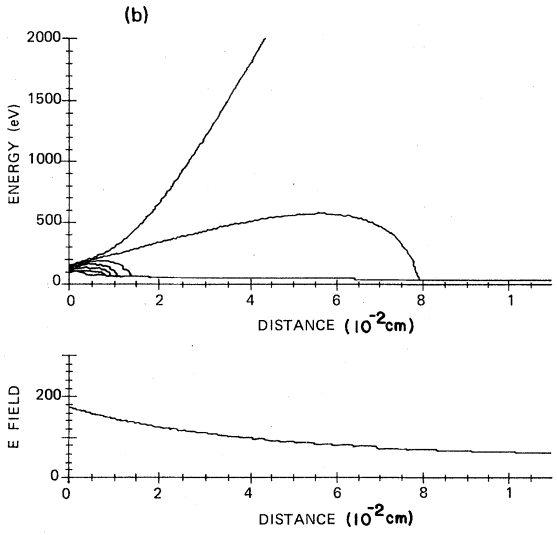
It is impossible to apply a fundamental model alone, as presented in Sec. III, to a specific experiment. In general, each experiment involves the fundamental physical processes and processes related to idiosyncrasies of a particular experiment. It is sometimes difficult to isolate the contribution from each process thus leading to misinterpretation. In the early days of the streamer

GAS = NITROGEN
PRESSURE = 750 Torr
MINIMUM INJECTION ENERGY, U_i = 112.6 eV
E_m = 175.1 kV/cm



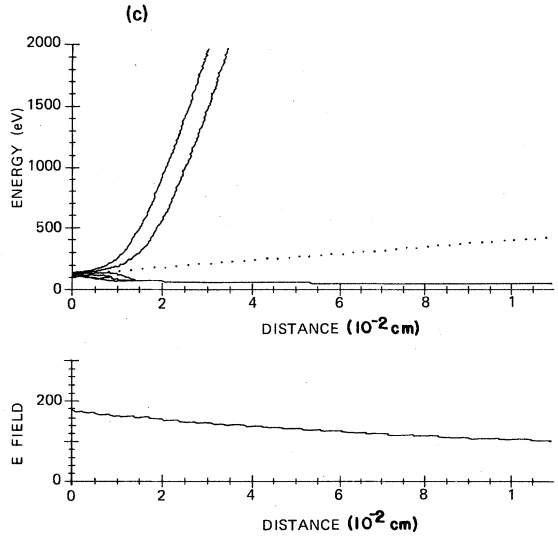
AVALANCHE RADIUS, R_e = 0.05 cm
EXTERNAL E FIELD, E_o = 42 kV/cm

GAS = NITROGEN
PRESSURE = 750 Torr
MINIMUM INJECTION ENERGY, U_i = 112.6
E_m = 175.1 kV/cm



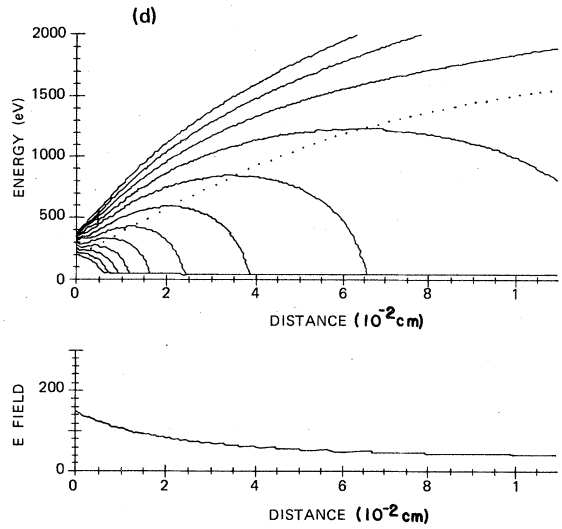
AVALANCHE RADIUS, R_e = 0.1 cm
EXTERNAL E FIELD, E_o = 42 kV/cm

GAS = NITROGEN
PRESSURE = 750 Torr
MINIMUM INJECTION ENERGY, U_i = 112.6 eV
E_m = 175.1 kV/cm



AVALANCHE RADIUS, R_e = 0.3 cm
EXTERNAL E FIELD, E_o = 42 kV/cm

GAS = NITROGEN
PRESSURE = 750 Torr
MINIMUM INJECTION ENERGY, U_i = 197.0 eV
E_m = 150. kV/cm



AVALANCHE RADIUS, R_e = 0.05 cm
EXTERNAL E FIELD, E_o = 42 kV/cm

FIG. 5. Evolution of injected electron energy as a function of distance away from the avalanche head.

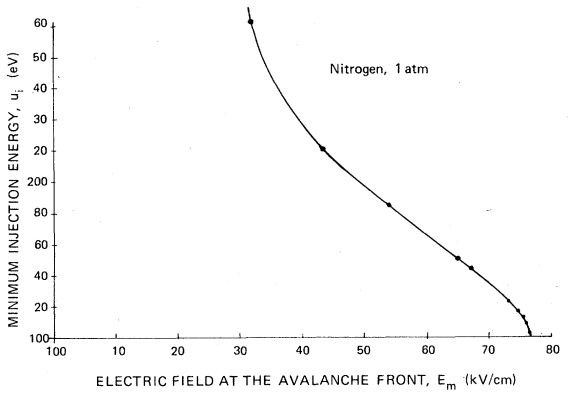


FIG. 6. Minimum injection energy as a function of maximum electric field.

model, Dickey[14] questioned the necessity for introducing a "new" model, besides the Townsend model, to explain the short time lag experimentally observed by Rogowski[10] and later by Fletcher.[11] He argued that these time lags were due to the particular experimental arrangement and not due to any new physics in the breakdown process.

In the discussion of our model, we will concentrate on the general structure of the observed breakdowns. We will also examine the transitions that may occur in any specific experimental arrangement due to the fundamental processes in the gas alone.

In the regime of interest to our model (i.e., $V > 20\%$ self-breakdown), two kinds of breakdown development have been observed: (1) filamentary breakdown and (2) broad breakdown. Broad breakdown has been observed either at very large voltages[17] ($V > 2$ times self-breakdown) or at lower voltages when the initiating electrons are generated over a large surface of the cathode.[24] In all other cases, filamentary breakdown occurs.

The principal parameters that we need to describe the structure of the breakdown are (1) the applied electric field E_0, (2) the spatial dependence of the electric field just ahead of the avalanche, and (3) the frictional force, $F(u)$, on an electron as a function of its energy for a given gas. Note that $F(u)$ also depends on the pressure. From Fig. 5 we note that if the avalanche is initially small in cross section (this occurs when the electrons initiating the avalanche are generated in a small area of the cathode), the inhomogeneity in the electric field is large. This implies that the energy spectrum of the trapped electrons is broad, i.e., many electrons will be trapped. Moreover, these electrons will be injected along the axis due to the "escape cone" phenomenon. This behavior corresponds to the range of parameters where filamentary breakdown has been observed.

As the field is increased, we reach the regime described by Mesyats' avalanche chain model.[18] He proposed that the main avalanche slows down and that a new avalanche develops from the previous one. This process is repeated a number of times until an avalanche chain is formed.

In our model, the process is continuous. At the higher field values, the width of the energy spectrum of the trapped electrons decreases due to the fact that the decrease in the injection energy u_i will be smaller than the decrease on the lower energy limit for runaway, i.e., u_t. Accordingly, the range of the trapped electrons decreases. The avalanche is extended via these short-range electrons [see Fig. 5(a)]. The number of runaway electrons and their effect in this regime is still small. However, as the field increases further (i.e. $\gtrsim 2$ times self-breakdown), the injected electrons are no longer trapped. In this regime $eE(x) > F(u(x))$ and electrons continuously gain energy; they all run away [compare Figs. 5(a) and 5(d)]. This is the regime discussed by Babic *et al.*[16] Our model is, in this regime, similar to theirs; however, we do not assume that the injected electrons have zero initial velocity and we are consistent in our definition of runaway electrons. This discharge in this regime has a diffuse or multichannel character.

A situation similar to the above, i.e., no trapped electrons, exists when the breakdown is initiated by a large number of electrons distributed over a large area.[24] In this case, the inhomogeneity ahead of the avalanche is small so that once again injected electrons actually runaway [see Fig. 5(c)]. The result, as in the very high field conditions, is a broad breakdown due to ionization and radiation generated by these runaways.

A subject of interest has been whether or not an abrupt decrease in the formative time would be observed as the percent overvoltage above self-breakdown is increased. In this respect the results have been mixed.[25] In some cases, in oxygen, and nitrogen, for example, a decrease was observed for large gap spacing. In hydrogen, however, it was not. This behavior can be explained by considering the magnitude of the maximum of the retarding force for each gas and the percent overvoltage used. The number of injected electrons is a function of E_{cr}, where $E_{cr} = F_{max}/e$ (see Fig. 4).[20] If $E_m \ll E_{cr}$ (see Sec. III) the number of injected electrons is practically zero so that no transition will be observed. As E_m increases, the number of injected electrons increases. Accordingly a continuous change in the slope of the time lag versus percent overvoltage curve may be observed. This was observed experimentally by Phillip and Allen.[25] We were not able to check our

model for all the gases they used due to lack of data on the frictional force. For the cases of hydrogen, oxygen, and nitrogen, for which data is available, the observed time lag versus overvoltage curve may be explained by our model.

In the case of hydrogen, the applied overvoltages used were too low, so that no change in the slope was observed. The maximum of the retarding force for hydrogen is relatively high, while the breakdown voltage for a 1-cm gap is small (i.e., ~15 kV). Therefore, large percentage overvoltages (~3 times self-breakdown) will be required before injection can begin.

For the cases of oxygen and nitrogen, changes were observed for a 3-cm gap at atmospheric pressure. The changes were more pronounced in oxygen than in nitrogen. The maximum retarding force for oxygen is less than for nitrogen; moreover, the breakdown voltage for oxygen is higher. This combination may account for the observed differences.

As the gap width decreased for the same range of overvoltages, the change in the slope decreased. For a gap width of 1 cm, no change was observed in nitrogen for up to 25% overvoltages. As for hydrogen, these values are again too low for injection to occur. No experiments were performed for oxygen in 1-cm gaps although the trend seems to indicate that a similar behavior would be obtained. The changes observed for 3-cm gaps in these two gases at these lower overvoltages may be due to large space-charge distortion which lowers the injection energy. This may occur if the magnitude of the "free charge" just prior to stage IV is large (see Sec. III), in which case the second term in Eq. (8) will be larger.

The next step in the development of the model is to do a numerical analysis of a fluid model for various particle species. The important point is that the electron fluid must be divided into two groups: the fast and slow electrons. A number of experiments should be carried out to test the validity of the theory and its basic assumptions. In particular, since the retarding force in Eq. (6) is a function of the properties of the gas,[21] the appearance of the runaway electrons will depend strongly on the gas used. A series of experiments should be carried out to test this dependence. Moreover, experimental verification of the time of appearance of the runaway electrons as a function of percent overvoltage should be provided.

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