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Resonance fluorescence in Markovian stochastic fields

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A systematic method is presented for averaging the integral equations of motion for the atomic density matrix operator and its quantum correlation functions over the fluctuations of Markovian exciting fields with arbitrary bandwidth. The theory takes into account the statistics of the field to all orders. The method is applied to the investigation of resonance fluorescence in the presence of three different Markovian fields: (i) a phase-diffusion field, (ii) a chaotic field, and (iii) a Gaussian-amplitude field. It is shown that in the case of a resonant, intense phase-diffusion field a center line dip develops in the spectrum of resonance fluorescence when the Rabi frequency is approximately equal to the bandwidth of the field. In the case of amplitude fluctuations the sidebands of the on-resonance spectrum tend to reproduce the probability distribution for the amplitude of the exciting field.

I. INTRODUCTION

Recently there has been a growing interest in the effects of field fluctuations and of the associated spectral bandwidth in the resonant interaction of intense radiation with matter.[1-18] The main reason for this interest is the stochastic amplitude, phase, and frequency fluctuations in real laser sources. These fluctuations are made very small at the expense of reduced power in well-stabilized, single-mode cw lasers. However, with the exception of very-high-resolution laser spectroscopy, most experiments and applications actually do not require the use of such well-stabilized lasers. Moreover, many experiments and applications require the very high power which is obtained in multimode pulsed operation. In any case, independent of the applied aspects, the role of field fluctuations in resonant multiphoton processes is a very interesting theoretical problem in the physics of interaction of radiation with matter. The field fluctuations add a new dimension to the theory of interaction of radiation with matter, which has many interesting new effects. A number of such new effects due to field fluctuations have been found in the saturation and Stark splitting of an atomic transition. Many more interesting effects will be found in the years ahead as theoreticians and experimentalists investigate the role of field fluctuations in other resonant multiphoton processes.

In a recent paper,[17] we used a diagrammatic method to treat the effects of Markovian field fluctuations in the saturation and Stark splitting of an atomic transition. In this method, the fluctuating field is described statistically in terms of the infinite sequence of field correlation functions.[19] The diagrammatic method, however, becomes very cumbersome in averaging more complicated integral equations for other resonant processes over the field fluctuations. In this paper, we develop a simple and systematic algebraic method for averaging the integral equations of motion for the atomic density matrix operator and its quantum correlation functions over the fluctuations of Markovian exciting fields. The Markovian field is now described statistically in terms of the marginal and the conditional probability densities. The averaging is carried out using the eigenfunctions and eigenvalues of the conditional averaging integral. Generally, if a resonant multiphoton process in the presence of a coherent field is described by a system of N equations, then the same (average) process in the presence of a Markovian stochastic field is described by an infinite number of such coupled systems of equations. The difference stems from the fact that it takes only one parameter to specify the complex amplitude of a coherent field, while it takes an infinite number of parameters (all the moments and correlation times) to specify the complex amplitude of a stochastic field. Note that the phase-diffusion field which is a Markovian field with independent phase increments is an exception.

The method described above is used in this paper to investigate the effects of Markovian field fluctuations in resonance fluorescence. Three different Markovian fields are considered: (a) the phase-diffusion field, (b) the chaotic field, and (c) the Gaussian-amplitude field. The phase-diffusion field undergoes only phase fluctuations and corresponds to an intensity-stabilized single-mode laser field. The chaotic field undergoes both amplitude and phase fluctuations and corresponds to a multimode laser field with a large number of uncorrelated modes. The Gaussian-amplitude field undergoes only amplitude fluctuations.[2] Although pure amplitude fluctuations cannot be produced by a nonadiabatic process,[1] we do consider the Gaussian-amplitude field for two reasons.

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First, because it allows us to isolate those effects due solely to amplitude fluctuations and second, because it is an example of a field which undergoes stronger amplitude (intensity) fluctuations than a chaotic field. By comparing the results for the chaotic and the Gaussian-amplitude fields we can determine the effect of increasing amplitude fluctuations. In Sec. II we describe the statistics of the three model fields and then in Secs. III and IV we investigate their effects on the intensity and the spectrum of resonance flurorescence.

II. STATISTICAL DESCRIPTION OF THE FIELD

In this section we describe the statistics of the fluctuating field which is assumed to be a Gaussian, stationary Markovian stochastic process. The electric field is treated classically and written as

$$E(t)=\overline{\mathcal{E}}(t)e^{-i\omega_0 t}+\overline{\mathcal{E}}^*(t)e^{i\omega_0 t}\ , \tag{1}$$

where ω_0 is the center frequency of the spectrum and $\overline{\mathcal{E}}(t)=\mathcal{E}(t)e^{-i\phi(t)}$, the fluctuating complex amplitude, with $\mathcal{E}(t)$ and $\phi(t)$ being the real amplitude and phase of the field. The mean value of $\overline{\mathcal{E}}(t)$ is assumed to be zero, i.e., $\langle\overline{\mathcal{E}}(t)\rangle=0$, where the angular brackets denote stochastic average. Since $\overline{\mathcal{E}}(t)$ is assumed to be a Gaussian, stationary Markovian process, its first-order correlation is necessarily exponential:

$$\langle\overline{\mathcal{E}}(t_1)\overline{\mathcal{E}}^*(t_2)\rangle=\mathcal{E}_0^2\exp(-\tfrac{1}{2}\gamma|t_1-t_2|)\ , \tag{2}$$

where γ is the full width at half maximum (FWHM) of the Lorentzian spectrum and $\mathcal{E}_0^2=\langle\mathcal{E}^2(t)\rangle$ is the variance of $\mathcal{E}(t)$. Three different stochastic models are used for $\overline{\mathcal{E}}(t)$: (a) the phase-diffusion (PD) model, (b) the chaotic (CH) model, and (c) the Gaussian-amplitude (G) model. The higher-order statistics of each of these fields are given below separately.

A. Phase-diffusion model

The phase-diffusion field has a constant real amplitude but its phase is a Wiener-Levy process (Brownian motion with negligible acceleration or continuous random walk).[20] A Wiener-Levy process is a nonstationary, Gaussian Markovian process whose increments are independent; i.e.,

$$\langle[\phi(t_1)-\phi(t_2)][\phi(t_3)-\phi(t_4)]\rangle=\langle\phi(t_1)-\phi(t_2)\rangle\langle\phi(t_3)-\phi(t_4)\rangle\ ,\quad t_1>t_2\geq t_3>t_4.$$

The nth-order correlation for $\overline{\mathcal{E}}(t)$ is given by[19]

$$\langle\overline{\mathcal{E}}(t_1)\overline{\mathcal{E}}^*(t_2)\cdots\overline{\mathcal{E}}(t_{2n-1})\overline{\mathcal{E}}^*(t_{2n})\rangle^{\rm PD}=\prod_{j\ \rm odd}^{2n-1}\langle\overline{\mathcal{E}}(t_j)\mathcal{E}^*(t_{j+1})\rangle\ , \tag{3}$$

where $t_1>t_2>\cdots>t_{2n-1}>t_{2n}$. The factorization of the nth-order field correlation in terms of first-order field correlations is a consequence of the independence of phase increments. The statistics of a Markovian stochastic process are completely determined if one knows the marginal and the conditional probability densities. The marginal probability density for $\phi(t)$ is given by[20]

$$f(\phi,t)=e^{-\phi^2/2\gamma t}/(2\pi\gamma t)^{1/2} \tag{4}$$

and the conditional probability density by

$$f(\phi_1,t_1|\,\phi_2,t_2)=\exp\left[-\frac{(\phi_1-\phi_2)^2}{2\gamma(t_1-t_2)}\right]\Big/[2\pi\gamma(t_1-t_2)]^{1/2},\quad t_1>t_2\,. \tag{5}$$

As it will be shown in Secs. III and IV, the integral equations of motion for the atomic density matrix operator and its correlation functions can be averaged systematically over Markovian field fluctuations by using the eigenfunctions and eigenvalues of the conditional averaging integral. For the PD field we can show that

$$\langle e^{\pm iN\phi(t_1)}|\,\phi_2,t_2\rangle\equiv\int_{-\infty}^{\infty}e^{\pm iN\phi_1}f(\phi_1,t_1|\,\phi_2,t_2)d\phi_1=e^{-(N^2/2)\gamma(t_1-t_2)}e^{\pm iN\phi(t_2)}\ , \tag{6}$$

where N is a positive integer. Thus $\overline{\mathcal{E}}^N(t_1)$ and $\overline{\mathcal{E}}^{*N}(t_1)$ are eigenfunctions of the conditional averaging integral with eigenvalue $e^{-(N^2/2)\gamma(t_1-t_2)}$. Note that the eigenvalues of the conditional averaging integral are deterministic while the eigenfunctions are random variables.

B. Chaotic model

The chaotic field is a complex Gaussian stochastic process. It can be written as $\overline{\mathcal{E}}(t)=\mathcal{E}_x(t)+i\overline{\mathcal{E}}_y(t)$, where $\mathcal{E}_x(t)$ and $\mathcal{E}_y(t)$ are two real, independent Gaussian processes with zero mean and equal variance. The nth-order correlation of $\overline{\mathcal{E}}(t)$ is given by[19]

$$\langle \bar{\mathcal{E}}(t_1)\bar{\mathcal{E}}^*(t_2)\cdots\bar{\mathcal{E}}(t_{2n-1})\bar{\mathcal{E}}^*(t_{2n})\rangle^{\mathrm{CH}} = \sum_P \prod_{j\ \mathrm{odd}}^{2n-1} \langle \bar{\mathcal{E}}(t_j)\bar{\mathcal{E}}^*(t_{P(j+1)})\rangle , \quad (7)$$

where $t_1 > t_2 > \cdots > t_{2n-1} > t_{2n}$ and P denotes permutation. It should be pointed out here that a chaotic field is not necessarily Markovian. The chaotic property (maximum entropy) is associated with the Gaussian statistics of the field and does not depend on the time evolution of the field. The marginal probability density of a chaotic field is given by[20]

$$f(\mathcal{E},\phi) = \mathcal{E}e^{-\mathcal{E}^2/\mathcal{E}_0^2}/\pi\mathcal{E}_0^2 \quad (8)$$

and the conditional probability density by

$$f(\mathcal{E}_1,\phi_1,t_1|\,\mathcal{E}_2,\phi_2,t_2) = \mathcal{E}_1\exp\left(-\frac{\mathcal{E}_1^2 + r^2\mathcal{E}_2^2 - 2r\mathcal{E}_1\mathcal{E}_2\cos(\phi_1-\phi_2)}{\mathcal{E}_0^2(1-r^2)}\right)\Big/\pi\mathcal{E}_0^2(1-r^2) , \quad (9)$$

where $r = e^{-\gamma(t_1-t_2)/2}$, $t_1 > t_2$, is the correlation coefficient. The conditional probability density for the chaotic field is a generating function for Laguerre polynomials[21] and it can be shown that

$$\langle L_N^m\left(\frac{\mathcal{E}_1^2}{\mathcal{E}_0^2}\right)\bar{\mathcal{E}}_1^m|\,\mathcal{E}_2,\phi_2,t_2\rangle \equiv \int_0^{2\pi} d\phi_1\int_0^\infty d\mathcal{E}_1 L_N^m\left(\frac{\mathcal{E}_1^2}{\mathcal{E}_0^1}\right)\bar{\mathcal{E}}_1^m f(\mathcal{E}_1,\phi_1,t_1|\,\mathcal{E}_2,\phi_2,t_2)$$

$$= e^{-[(2N+m)/2]\gamma(t_1-t_2)} L_N^m\left(\frac{\mathcal{E}_2^2}{\mathcal{E}_0^2}\right)\bar{\mathcal{E}}_2^m , \quad (10)$$

where $L_N^m(x)$ is a generalized Laguerre polynomial. Thus $L_N^m(\mathcal{E}_1^2/\mathcal{E}_0^2)\bar{\mathcal{E}}_1^m$ and $L_N^m(\mathcal{E}_1^2/\mathcal{E}_0^2)\bar{\mathcal{E}}^{*m}$ are eigenfunctions of the conditional averaging integral with eigenvalue $e^{-[(2N+m)/2]\gamma(t_1-t_2)}$.

C. Gaussian-amplitude model

The Gaussian-amplitude field has a constant phase but its real amplitude undergoes Gaussian fluctuations. The chaotic field consists of two such independent fields 90° out of phase. The nth-order correlation of $\mathcal{E}(t)$ is given by[19]

$$\langle \mathcal{E}(t_1)\mathcal{E}(t_2)\cdots\mathcal{E}(t_{2n-1})\mathcal{E}(t_{2n})\rangle = \sum_P \prod_{j\neq k}^{2n} \langle \mathcal{E}(t_j)\mathcal{E}(t_k)\rangle , \quad (11)$$

where $t_1 > t_2 > \cdots > t_{2n-1} > t_{2n}$. Note that while the sum in Eq. (7) involves $n!$ terms, the sum in Eq. (11) involves $n!! = 1.3\ldots(2n-1)$ terms. For $t_1 = t_2 = \cdots = t_{2n}$, Eqs. (7) and (11) give $\langle I^n\rangle^{\mathrm{CH}} = n!\langle I\rangle^n$ and $\langle I^n\rangle^{\mathrm{G}} = n!!\langle I\rangle^n$, where $I = 2\epsilon_0\mathcal{E}^2(t)$ is the intensity of the fields. Thus a Gaussian-amplitude field undergoes stronger intensity fluctuations than a chaotic field. The marginal probability density for $\mathcal{E}(t)$ is given by[20]

$$f(\mathcal{E}) = e^{-\mathcal{E}^2/2\mathcal{E}_0^2}/(2\pi\mathcal{E}_0^2)^{1/2} , \quad (12)$$

and the conditional probability density by

$$f(\mathcal{E}_1,t_1|\,\mathcal{E}_2,t_2) = \exp\left(-\frac{(\mathcal{E}_1 - r\mathcal{E}_2)^2}{2\mathcal{E}_0^2(1-r^2)}\right)\Big/[2\pi\mathcal{E}_0^2(1-r^2)]^{1/2} , \quad (13)$$

where $r = e^{-\gamma(t_1-t_2)/2}$, $t_1 > t_2$, is the correlation coefficient. The conditional density is a generating function for Hermite polynomials[21] and it can be shown that

$$\langle H_N\left(\frac{\mathcal{E}_1}{(2\mathcal{E}_0^2)^{1/2}}\right)|\,\mathcal{E}_2,t_2\rangle \equiv \int_{-\infty}^{\infty} H_N\left(\frac{\mathcal{E}_1}{(2\mathcal{E}_0^2)^{1/2}}\right) f(\mathcal{E}_1,t_1|\,\mathcal{E}_2,t_2)\,d\mathcal{E}_1$$

$$= e^{-(N\gamma/2)(t_1-t_2)} H_N(\mathcal{E}_2/(2\mathcal{E}_0^2)^{1/2}) , \quad (14)$$

where $H_N^{(x)}$ is a Hermite polynomial. Thus $H_N(\mathcal{E}_2/(2\mathcal{E}_0^2)^{1/2})$ is an eigenfunction of the conditional averaging integral with eigenvalue $e^{-(N\gamma/2)(t_1-t_2)}$.

Note that the stochastic average of the eigenfunctions of the conditional averaging integral for the three fields is zero except for $N=0$, i.e.,[22]

$$\langle \bar{\mathcal{E}}^N(t)\rangle^{\mathrm{PD}} = \langle L_N^m(\mathcal{E}^2(t)/\mathcal{E}_0^2)\,\bar{\mathcal{E}}^m(t)\rangle^{\mathrm{CH}} = \langle H_N(\mathcal{E}(t)/\sqrt{2}\,\mathcal{E}_0)\rangle^{\mathrm{G}} = \delta_{N0} .$$

The fact that the eigenfunctions for the chaotic and Gaussian-amplitude fields are polynomials, while those for the PD field are simply powers of $\mathcal{E}(t)$, is related to the form of the field correlation functions. Both the sum over permutations in Eqs. (7) and (11) and the polynomials in Eqs. (10) and (14) reflect the correlation in the intensity fluctuations. On the other hand, the fact that $\bar{\mathcal{E}}^m(t)$ and $\bar{\mathcal{E}}^{*m}(t)$ are eigenfunctions of the conditional averaging integral for the chaotic field is related to the fact that

$$\langle \mathcal{E}(t_1)\mathcal{E}(t_2)\cdots\mathcal{E}(t_m)\rangle^{\rm CH} = \langle \mathcal{E}^*(t_1)\mathcal{E}^*(t_2)\cdots\mathcal{E}^*(t_m)\rangle^{\rm CH} = 0 .$$

III. INTENSITY OF RESONANCE FLUORESCENCE

We consider a two-state atom with ground state $|1\rangle$ and excited state $|2\rangle$ of opposite parity. The electric dipole matrix element is μ_{12} and the transition frequency ω_{21}. The atom is interacting with the Markovian stochastic field of Eq. (1). The equation of motion for the atomic density matrix $\rho(t)$ in the rotating-wave approximation [i.e., $\rho_{12}(t)=\sigma_{12}(t)e^{-i\omega_0 t}$, $\rho_{21}(t)=\sigma_{21}(t)e^{+i\omega_0 t}$, $\rho_{ii}(t)=\sigma_{ii}(t)$, $i=1,2$, where the $\sigma_{ij}(t)$ are slowly varying amplitudes] can be written in the form

$$\frac{d}{dt}\begin{bmatrix}\sigma_{22}(t)\\ \sigma_{11}(t)\\ \sigma_{12}(t)\\ \sigma_{21}(t)\end{bmatrix} = \begin{bmatrix} -\Gamma & 0 & -\frac{1}{2}i\,\omega_R^*(t) & \frac{1}{2}i\,\omega_R(t)\\ \Gamma & 0 & \frac{1}{2}i\,\omega_R^*(t) & -\frac{1}{2}i\,\omega_R(t)\\ -\frac{1}{2}i\,\omega_R(t) & \frac{1}{2}i\,\omega_R(t) & i\Delta-\frac{1}{2}\Gamma & 0\\ \frac{1}{2}i\,\omega_R^*(t) & -\frac{1}{2}i\,\omega_R^*(t) & 0 & -i\Delta-\frac{1}{2}\Gamma \end{bmatrix} \begin{bmatrix}\sigma_{22}(t)\\ \sigma_{11}(t)\\ \sigma_{12}(t)\\ \sigma_{21}(t)\end{bmatrix}, \tag{15}$$

where Γ is the spontaneous decay rate of state $|2\rangle$ and $\Delta=\omega_0-\omega_{21}$, the detuning from resonance. The parameter $\omega_R(t)=2\hbar^{-1}\mu_{12}\mathcal{E}(t)$ is the stochastic Rabi frequency and its root-mean-square (rms) value $\overline{\omega}_R=2\hbar^{-1}\mu_{12}\mathcal{E}_0$ will herein be referred to as the average Rabi frequency. Integrating the system of equations (15) with initial conditions $\sigma_{11}(0)=1$, $\sigma_{22}(0)=\sigma_{12}(0)=\sigma_{21}(0)=0$, we obtain the integral equation

$$n(t) = -1 - \mathrm{Re}\int_0^t e^{\Gamma(t_1-t)}dt_1\int_0^{t_1}\exp[(i\Delta+\tfrac{1}{2}\Gamma)(t_2-t_1)]\,\omega_R(t_1)\omega_R^*(t_2)n(t_2)dt_2 , \tag{16}$$

where $n(t)=\sigma_{22}(t)-\sigma_{11}(t)$ is the population inversion and the normalization condition is $\sigma_{11}(t)+\sigma_{22}(t)=1$.

The average intensity of resonance fluorescence is proportional to the population of the excited state $\sigma_{22}=\frac{1}{2}(1+n)$ averaged over the fluctuations of the driving field. To average Eq. (16) over the Markovian field fluctuations we must multiply both sides of the equation by the joint probability density

$$f(\mathcal{E},\phi,\mathcal{E}_1,\phi_1,\mathcal{E}_2,\phi_2;t,t_1,t_2) = f(\mathcal{E},\phi,t\,|\,\mathcal{E}_1,\phi_1,t_1)f(\mathcal{E}_1,\phi_1,t_1\,|\,\mathcal{E}_2,\phi_2,t_2)f(\mathcal{E}_2,\phi_2,t_2) , \tag{17}$$

where $t>t_1>t_2\geq 0$, and integrate over the random variables $\mathcal{E}$, ϕ, $\mathcal{E}_1$, ϕ_1, $\mathcal{E}_2$, and ϕ_2. Below, we carry out this average for each of the three model fields.

A. Phase-diffusion model

The averaging of Eq. (16) in the case of a PD field was described in detail in Ref. 17 and for completeness we summarize the results in this paper. Using Eq. (6) we can show that

$$\langle\omega_R(t_1)\omega_R^*(t_2)n(t_2)\rangle^{\rm PD} = \langle\omega_R(t_1)\omega_R^*(t_2)\rangle^{\rm PD}\langle n(t_2)\rangle^{\rm PD} ,$$

and then we can solve Eq. (16) for $\langle n(t)\rangle^{\rm PD}$ by Laplace transform. The steady-state value of the average excited-state population is given by

$$\langle\sigma_{22}\rangle^{\rm PD} = \frac{S/2}{1+S} , \tag{18}$$

where

$$S = \frac{\overline{\omega}_R^2}{\Gamma}\,\frac{\frac{1}{2}(\Gamma+\gamma)}{\Delta^2+\frac{1}{4}(\Gamma+\gamma)^2} \tag{19}$$

is the saturation parameter. Note that, compared to a monochromatic field of the same power, a finite bandwidth PD field exactly on resonance ($\Delta=0$) reduces the saturation by $\Gamma/(\Gamma+\gamma)$, while far from resonance ($\Delta\gg\Gamma,\gamma$) it increases the saturation by $(\Gamma+\gamma)/\Gamma$.

B. Chaotic model

Before averaging Eq. (16) for a chaotic field, we multiply both sides of the equation by the Laguerre polynomial

$$L_N^0(\mathcal{E}^2(t)/\mathcal{E}_0^2)\ .$$

If we then apply Eq. (10) twice in succession $(t \to t_1 \to t_2)$, making use of the recursion relations for orthonormal Laguerre polynomials,[21]

$$L_N^{m-1}(x) = L_N^m(x) - L_{N-1}^m(x) \tag{20}$$

and

$$xL_N^{m+1}(x) = (N+m+1)L_N^m(x) - (N+1)L_{N+1}^m(x)\ , \tag{21}$$

we obtain the relation

$$\begin{aligned}\langle\langle n(t)\rangle\rangle_N = -\delta_{N0} - \mathrm{Re}\int_0^t dt_1 e^{(\Gamma+N\gamma)(t_1-t)} \\ \times \int_0^{t_1} dt_2 \exp[(i\Delta+\tfrac{1}{2}\Gamma)(t_2-t_1)]\,\bar{\omega}_R^2\{(N+1)e^{[(2N+1)/2]\gamma(t_2-t_1)}[\langle\langle n(t_2)\rangle\rangle_N - \langle\langle n(t_2)\rangle\rangle_{N+1}] \\ -Ne^{[(2N-1)/2]\gamma(t_2-t_1)}[\langle\langle n(t_2)\rangle\rangle_{N-1} - \langle\langle n(t_2)\rangle\rangle_N]\}\ ,\end{aligned} \tag{22}$$

where $\langle\langle n(t)\rangle\rangle_N \equiv \langle L_N^0(\mathcal{E}^2(t)/\mathcal{E}_0^2)n(t)\rangle$, $N=0, 1, 2, \ldots$, are the coefficients of expansion of $n(t)$ in orthonormal Laguerre polynomials, i.e.,

$$n(t) = \sum_{N=0}^{\infty}\left\langle L_N^0\left(\frac{\mathcal{E}^2(t)}{\mathcal{E}_0^2}\right)n(t)\right\rangle L_N^0\left(\frac{\mathcal{E}^2(t)}{\mathcal{E}_0^2}\right)\ . \tag{23}$$

Taking the Laplace transform of Eq. (22) and calculating the steady-state solution $[\langle\langle n(t=\infty)\rangle\rangle_N = \lim_{p\to 0} p\mathcal{L}\langle\langle n(t)\rangle\rangle_N]$, we obtain the following three-term recursion relation for the expansion coefficients

$$-a_N\langle n\rangle_{N-1} + \langle n\rangle_N - b_N\langle n\rangle_{N+1} = -\delta_{N0}/(1+S_0)\ , \tag{24}$$

where

$$a_N = \frac{\Gamma+(N-1)\gamma}{\Gamma+N\gamma}S_{N-1}\bigg/\left(1+S_N+\frac{\Gamma+(N-1)\gamma}{\Gamma+N\gamma}S_{N-1}\right), \tag{25}$$

$$b_N = S_N\bigg/\left(1+S_N+\frac{\Gamma+(N-1)\gamma}{\Gamma+N\gamma}S_{N-1}\right), \tag{26}$$

and

$$S_N = \frac{(N+1)\bar{\omega}_R^2}{\Gamma+N\gamma}\,\frac{\frac{1}{2}[\Gamma+(2N+1)\gamma]}{\Delta^2+\frac{1}{4}[\Gamma+(2N+1)\gamma]^2}\ . \tag{27}$$

For $N=0$, the recursion relation above gives

$$\langle n\rangle_0 = -\frac{1}{1+S_0}\,\frac{1}{1-b_0\langle n\rangle_1/\langle n\rangle_0}\ ,$$

while for $N \geq 1$ it can be written in the form

$$\frac{\langle n\rangle_N}{\langle n\rangle_{N-1}} = \frac{a_N}{1-b_N\langle n\rangle_{N+1}/\langle n\rangle_N}\ .$$

Iterating the last equation we can obtain a continued fraction for the average population inversion $\langle n\rangle = \langle n\rangle_0$. [Recall that $L_0^m(x)=1$.] The other expansion coefficients $\langle n\rangle_N$, $N \geq 1$, which are needed in the calculation of the spectrum of resonance fluorescence, can be evaluated by subtituting the value for $\langle n\rangle_0$ into Eq. (24). The steady-state value of the average excited-state population is given by

$$\langle\sigma_{22}\rangle^{\mathrm{CH}} = \tfrac{1}{2} - \frac{\frac{1}{2}}{1+S_0}\,\cfrac{1}{1-\cfrac{a_1b_0}{1-\cfrac{a_2b_1}{1-\cfrac{a_3b_2}{1-\cdots}}}}\ . \tag{28}$$

Note that S_0 is identical to the saturation parameter in Eq. (19) for the PD field. The equation for $\langle\sigma_{22}\rangle^{\mathrm{CH}}$ differs from the equation for $\langle\sigma_{22}\rangle^{\mathrm{PD}}$ only by the continued fraction which multiplies the factor $(\frac{1}{2})/(1+S_0)$ in Eq. (28). Since the magnitude of the continued fraction is greater than unity $(0 < a_Nb_{N-1} < \frac{1}{4})$, $\langle\sigma_{22}\rangle^{\mathrm{CH}}$ is generally less than $\langle\sigma_{22}\rangle^{\mathrm{PD}}$. Thus, the intensity of resonance fluorescence in the presence of a chaotic exciting field is less than that in the presence of a phase-diffusion exciting field with the same first-order field correlation function. The continued fraction in Eq. (28) has a different structure from the one in Eq. (33) of Ref. 17, where $\langle\sigma_{22}\rangle^{CH}$ was calculated by diagrammatic summation of an iteration series expansion. The two results, however, are equivalent. Equation (28) is identical with the expression for $\langle\sigma_{22}\rangle^{CH}$ in Ref. 18 which was calculated using the Fokker-Planck formalism.

C. Gaussian-amplitude model

Following the same procedure as in the case of the chaotic field, we multiply both sides of Eq. (16) by the Hermite polynomial

$$H_N(\mathcal{E}(t)/(2\mathcal{E}_0^2)^{1/2})$$

and then take the stochastic average. Applying Eq. (14) twice in succession ($t \to t_1 \to t_2$) and using the recursion relation for orthogonal Hermite polynomials,[21]

$$xH_N(x) = (N+1)H_{N+1}(x) + \tfrac{1}{2}H_{N-1}(x)\,, \tag{29}$$

we obtain the relation

$$\begin{aligned}\langle\langle n(t)\rangle\rangle_N = -\delta_{N0} - \mathrm{Re}\int_0^t dt_1 e^{[\Gamma+(N/2)\gamma](t_1-t)} &\times \int_0^{t_1} dt_2 \exp[(i\Delta+\tfrac{1}{2}\Gamma)(t_2-t_1)] \\ &\times \bar{\omega}_R^2 \{e^{[(N+1)\gamma/2](t_2-t_1)}[2(N+1)(N+2)\langle\langle n(t_2)\rangle\rangle_{N+2} + (N+1)\langle\langle n(t_2)\rangle\rangle_N] \\ &+ e^{[(N-1)\gamma/2](t_2-t_1)}[N\langle\langle n(t_2)\rangle\rangle_N + \tfrac{1}{2}\langle\langle n(t_2)\rangle\rangle_{N-2}]\}\,,\end{aligned} \tag{30}$$

where $\langle\langle n(t)\rangle\rangle_N \equiv \langle\langle H_N(\mathcal{E}(t)/(2\mathcal{E}_0^2)^{1/2})n(t)\rangle\rangle$, $N = 0, 2, 4, \ldots$, are the coefficients of expansion of $n(t)$ in orthogonal Hermite polynomials; i.e.,

$$n(t) = \sum_{K=0}^{\infty}\left\langle H_{2K}\left(\frac{\mathcal{E}(t)}{(2\mathcal{E}_0^2)^{1/2}}\right)n(t)\right\rangle H_{2K}\left(\frac{\mathcal{E}(t)}{(2\mathcal{E}_0^2)^{1/2}}\right). \tag{31}$$

Note that only even-numbered Hermite polynomials, which are functions of the intensity of the field, appear in the equation above. Taking the Laplace transform of Eq. (30) and calculating the steady-state solution, we find the following three-term recursion relation for the expansion coefficients:

$$a_{2K}\langle n\rangle_{2K-2} + \langle n\rangle_{2K} + b_{2K}\langle n\rangle_{2K+2} = -\delta_{K0}/(1+S_0)\,, \tag{32}$$

where

$$a_{2K} = \frac{\dfrac{1}{2(2K-1)}\dfrac{[\Gamma+(K-1)\gamma]}{(\Gamma+K\gamma)}S_{2K-2}}{\left(1+S_{2K}+\dfrac{2K}{2K-1}\dfrac{[\Gamma+(K-1)\gamma]}{(\Gamma+K\gamma)}S_{2K-2}\right)}, \tag{33}$$

$$b_{2K} = \frac{2(K+2)S_{2K}}{\left(1+S_{2K}+\dfrac{2K}{(2K-1)}\dfrac{[\Gamma+(K-1)\gamma]}{(\Gamma+K\gamma)}S_{2K-2}\right)}, \tag{34}$$

and

$$S_{2K} = \frac{(2K+1)\bar{\omega}_R^2}{(\Gamma+K\gamma)}\,\frac{\tfrac{1}{2}[\Gamma+(2K+1)\gamma]}{\Delta^2+\tfrac{1}{4}[\Gamma+(2K+1)\gamma]^2}\,. \tag{35}$$

From the recursion relation above we can obtain a continued fraction for the average value of the population inversion $\langle n\rangle = \langle n\rangle_0$. The steady-state value of the average excited-state population is given by

$$\langle\sigma_{22}\rangle^{G}=\tfrac{1}{2}-\frac{\frac{1}{2}}{1+S_0}\,\cfrac{1}{1-\cfrac{a_2b_0}{1-\cfrac{a_4b_2}{1-\cfrac{a_6b_4}{1-\cdots}}}}\,. \tag{36}$$

The only difference between this equation and Eq. (28) for $\langle\sigma_{22}\rangle^{CH}$ is in the value of the coefficients in the two continued fractions. Comparing Eqs. (27) and (35) we find that $S_K^{CH}/S_{2K}^{G}=(K+1)/(2K+1)$. The factors $(K+1)$ and $(2K+1)$, which multiply $\bar{\omega}_R^2$ in Eqs. (27) and (35), originated in the recursion relations for Laguerre [Eq. (21)] and Hermite [Eq. (29)] polynomials, and are indirectly related to the moments of the intensity for chaotic $(\langle I^{K+1}\rangle^{CH}/\langle I^{K}\rangle^{CH}=K+1)$ and Gaussian-amplitude $(\langle I^{K+1}\rangle^{G}/\langle I^{K}\rangle^{G}=2K+1)$ fields. The coefficients $a_{2K}b_{2K-2}$ in the continued fraction for $\langle\sigma_{22}\rangle^{G}$ are greater $(0<a_{2K}b_{2K-2}<\frac{1}{4})$ than the corresponding coefficients a_Kb_{K-1} in the continued fraction for $\langle\sigma_{22}\rangle^{CH}$. Because of this, the magnitude of the continued fraction in the expression for $\langle\sigma_{22}\rangle^{G}$ is greater than that of the continued fraction in the expression for $\langle\sigma_{22}\rangle^{CH}$. Hence, the intensity of resonance fluorescence in the presence of a Gaussian-amplitude exciting field is less than that in the presence of a chaotic exciting field, which in turn is less than that in the presence of a phase-diffusion exciting field. A precise comparison of the intensities of resonance fluorescence for the three model fields requires numerical evaluation of Eqs. (18), (28), and (36). The continued fractions in the latter two equations are convergent, but the number of fractions required to obtain a certain degree of accuracy increases with the saturation parameter S_0.

Figure 1 shows the dependence of the ratios $\langle\sigma_{22}\rangle^{CH}/\langle\sigma_{22}\rangle^{PD}$ and $\langle\sigma_{22}\rangle^{G}/\langle\sigma_{22}\rangle^{PD}$ on the ratio $\bar{\omega}_R/\Gamma$ for different values of the bandwidth γ, under exact resonance $(\Delta=0)$. The average population of the excited state in the presence of a PD exciting field is always larger than that in the presence of either a chaotic or a Gaussian-amplitude field. As was originally explained in Ref. 17, a field with constant intensity is more effective than a field with fluctuating intensity in saturating a single-photon or multiphoton transition. The atom in essence sees the intensity fluctuations and relaxes between spikes in the intensity, going partially out of saturation. Comparing the curves for $\langle\sigma_{22}\rangle^{CH}/\langle\sigma_{22}\rangle^{PD}$ and $\langle\sigma_{22}\rangle^{G}/\langle\sigma_{22}\rangle^{PD}$ we see that $\langle\sigma_{22}\rangle^{G}$ is always less than $\langle\sigma_{22}\rangle^{CH}$. The physical explanation for this is that the Gaussian-amplitude field undergoes stronger intensity fluctuations than the chaotic field and thus is less effective in keeping the atom excited. Note that by stronger intensity fluctuations we do not mean a shorter correlation time, which is actually the same for the two fields $(1/\gamma)$, but a larger spread in the intensity distribution [see Eqs. (8) and (12)]. The difference between $\langle\sigma_{22}\rangle^{PD}$, $\langle\sigma_{22}\rangle^{CH}$, and $\langle\sigma_{22}\rangle^{G}$ is small when the fields are either very weak $(S_0\ll 1)$ or very strong $(S_0\gg 1)$. In the first case, the excitation depends mainly only on the first-order field correlation, which is the same for the three models. In the second case, the average population of the excited state $\langle\sigma_{22}\rangle$ goes to the limit $\frac{1}{2}$ independently of the statistics of the exciting field, although the rate of approaching this limit depends on the statistics. The largest difference between $\langle\sigma_{22}\rangle^{PD}$, $\langle\sigma_{22}\rangle^{CH}$, and $\langle\sigma_{22}\rangle^{G}$ occurs for intermediate field strengths $(\bar{\omega}_R/\Gamma\sim 1)$. As the bandwidth γ is increased the difference between the three populations decreases. This is because the atom cannot follow the field fluctuations, which become more rapid, and responds to fewer field correlation functions. Therefore, as the bandwidth increases the atom detects less difference in the statistics of the fields.

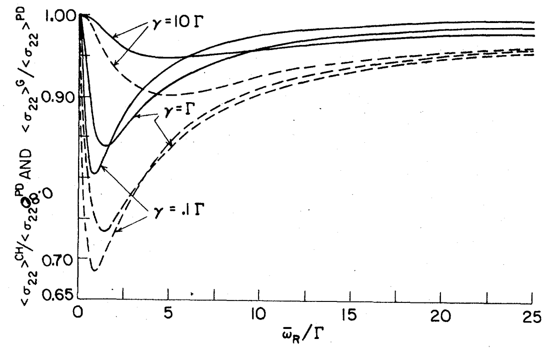

FIG. 1. Plot of the excited-state population ratios $\langle\sigma_{22}\rangle^{CH}/\langle\sigma_{22}\rangle^{PD}$ (solid line) and $\langle\sigma_{22}\rangle^{G}/\langle\sigma_{22}\rangle^{PD}$ (dashed line) versus the ratio $\bar{\omega}_R/\Gamma$ of the average Rabi frequency $\bar{\omega}_R$ to the spontaneous decay rate Γ, for different values of the field bandwidth γ. The center frequency of the fields is tuned on resonance.

IV. SPECTRUM OF RESONANCE FLUORESCENCE

It is well known that the stationary $(t\to\infty)$ spectrum of resonance fluorescence, apart from simple propagation factors, is given by[23]

$$S(\omega)=\int_{-\infty}^{\infty}e^{i(\omega-\omega_0)\tau}\langle\hat{\sigma}_{21}(t)\hat{\sigma}_{12}(t+\tau)\rangle\,d\tau\,, \tag{37}$$

where ω_0 is the center frequency of the exciting field, and $\langle\hat{\sigma}_{21}(t)\hat{\sigma}_{12}(t+\tau)\rangle$ the quantum correlation function of the slowly varying, atomic raising and lowering operators. The equations of motion for the quantum correlation functions $\langle\sigma_{21}(t')\hat{\sigma}_{ij}(t)\rangle$, $i, j=1, 2$, of a two-state atom can be written in the form[24]

$$\frac{d}{dt}\begin{bmatrix}\langle\hat{\sigma}_{21}(t')\hat{\sigma}_{22}(t)\rangle\\ \langle\hat{\sigma}_{21}(t')\hat{\sigma}_{11}(t)\rangle\\ \langle\hat{\sigma}_{21}(t')\hat{\sigma}_{12}(t)\rangle\\ \langle\hat{\sigma}_{21}(t')\hat{\sigma}_{21}(t)\rangle\end{bmatrix}=\begin{bmatrix}-\Gamma & 0 & -\frac{1}{2}i\,\omega_R^*(t) & \frac{1}{2}i\,\omega_R(t)\\ \Gamma & 0 & \frac{1}{2}i\,\omega_R^*(t) & -\frac{1}{2}i\,\omega_R(t)\\ -\frac{1}{2}i\,\omega_R(t) & \frac{1}{2}i\,\omega_R(t) & i\Delta-\frac{1}{2}\Gamma & 0\\ \frac{1}{2}i\,\omega_R^*(t) & -\frac{1}{2}i\omega_R^*(t) & 0 & -i\Delta-\frac{1}{2}\Gamma\end{bmatrix}\begin{bmatrix}\langle\hat{\sigma}_{21}(t')\hat{\sigma}_{22}(t)\rangle\\ \langle\hat{\sigma}_{21}(t')\hat{\sigma}_{11}(t)\rangle\\ \langle\hat{\sigma}_{21}(t')\hat{\sigma}_{12}(t)\rangle\\ \langle\hat{\sigma}_{21}(t')\hat{\sigma}_{21}(t)\rangle\end{bmatrix}, \tag{38}$$

where $t>t'$ and the 4×4 matrix is identical with the matrix in Eq. (15). Integrating the system of Eqs. (38) and eliminating the correlation function $\langle\hat{\sigma}_{21}(t')\hat{\sigma}_{21}(t)\rangle$ we obtain the integral equations

$$g(t,\tau)=\sigma_{22}(t)\exp[(i\Delta-\tfrac{1}{2}\Gamma)\tau]-\frac{i}{2}\int_0^\tau \exp[(-i\Delta+\tfrac{1}{2}\Gamma)(t_1-\tau)]\,\omega_R(t+t_1)h(t,t_1)dt_1\,, \tag{39}$$

and

$$\begin{aligned}\omega_R(t+\tau)h(t,\tau)=&-\omega_R(t+\tau)\sigma_{21}(t)-i\int_0^\tau e^{\Gamma(t_1-\tau)}\exp[(i\Delta-\tfrac{1}{2}\Gamma)t_1]\,\omega_R(t+\tau)\omega_R^*(t+t_1)\sigma_{22}(t)dt_1\\ &-\tfrac{1}{2}\int_0^\tau e^{\Gamma(t_1-\tau)}dt_1\int_0^{t_1}\exp[(-i\Delta+\tfrac{1}{2}\Gamma)(t_2-t_1)]\,\omega_R(t+\tau)\omega_R^*(t+t_1)\omega_R(t+t_2)h(t,t_2)dt_2\\ &-\tfrac{1}{2}\int_0^\tau e^{\Gamma(t_1-\tau)}dt_1\int_0^{t_1}\exp[(i\Delta+\tfrac{1}{2}\Gamma)(t_2-t_1)]\,\omega_R(t+\tau)\omega_R(t+t_1)\omega_R^*(t+t_2)h(t,t_2)dt_2\,,\end{aligned} \tag{40}$$

where we define the quantum correlation functions $g(t,\tau)\equiv\langle\hat{\sigma}_{21}(t)\hat{\sigma}_{12}(t+\tau)\rangle$, $h(t,\tau)\equiv\langle\hat{\sigma}_{21}(t)[\hat{\sigma}_{22}(t+\tau)-\hat{\sigma}_{11}(t+\tau)]\rangle$ and the quantum average $\sigma_{ij}(t)\equiv\langle\hat{\sigma}_{ij}(t)\rangle$. These equations must now be averaged over the fluctuations of the Markovian exciting field, in order to determine the average spectrum $\langle S(\omega)\rangle$. Performing a formal stochastic average on Eq. (39) and then taking the Laplace transform we find

$$G(t,s)=\frac{\langle\sigma_{22}(t)\rangle-\frac{1}{2}iW(t,s)}{s-i\Delta+\frac{1}{2}\Gamma}, \tag{41}$$

where

$$G(t,s)=\int_0^\infty e^{-s\tau}\langle g(t,\tau)\rangle d\tau \tag{42}$$

and

$$W(t,s)=\int_0^\infty e^{-s\tau}\langle\omega_R(t,\tau)h(t,\tau)\rangle d\tau\,. \tag{43}$$

Since in the stationary limit $g(t,-\tau)=g^*(t,\tau)$, the average spectrum $\langle S(\omega)\rangle$ is related to $G(t,s)$ by[23]

$$\langle S(\omega)\rangle=G(t,i\omega_0-i\omega)+\text{c.c.} \tag{44}$$

The averaging of Eq. (40) is more complicated and has to be carried out separately for each stochastic field, by multiplying by the joint probability density $f(\mathcal{E},\phi,\mathcal{E}_1,\phi_1,\mathcal{E}_2,\phi_2;t+\tau,t+t_1,t+t_2)$, $\tau>t_1>t_2\geq0$ [see Eq. (17)], and integrating over the random variables.

A. Phase-diffusion model

In carrying out the integration over the random phases on the right-hand side of Eq. (40), we use the general equation (6) and obtain

$$\begin{aligned}\langle\omega_R(t+\tau)h(t,\tau)\rangle=&-e^{-\gamma\tau/2}\langle\omega_R(t)\sigma_{21}(t)\rangle-i\int_0^\tau \exp[(\Gamma+\tfrac{1}{2}\gamma)(t_1-\tau)]\,\exp[(i\Delta-\tfrac{1}{2}\Gamma)t_1]\overline{\omega}_R^2\langle\sigma_{22}(t)\rangle dt_1\\ &-\tfrac{1}{2}\int_0^\tau \exp[(\Gamma+\tfrac{1}{2}\gamma)(t_1-\tau)]dt_1\int_0^{t_1}\exp[(-i\Delta+\tfrac{1}{2}\Gamma)(t_2-t_1)]\,\overline{\omega}_R^2\langle\omega_R(t+t_2)h(t,t_2)\rangle dt_2\\ &-\tfrac{1}{2}\int_0^\tau \exp[(\Gamma+\tfrac{1}{2}\gamma)(t_1-\tau)]dt_1\int_0^{t_1}\exp[(i\Delta+\tfrac{1}{2}\Gamma+2\gamma)t_2-t_1]\,\overline{\omega}_R^2\langle\omega_R(t+t_2)h(t,t_2)\rangle dt_2\,.\end{aligned} \tag{45}$$

The Laplace transform of the above equation gives

$$W(t,s)=\frac{-\dfrac{(s+\Gamma+\frac{1}{2}\gamma)(s-i\Delta+\frac{1}{2}\Gamma)(s+i\Delta+\frac{1}{2}\Gamma+2\gamma)}{(s+\frac{1}{2}\gamma)}\langle\omega_R(t)\sigma_{21}(t)\rangle-i\bar{\omega}_R^2(s+i\Delta+\frac{1}{2}\Gamma+2\gamma)\langle\sigma_{22}(t)\rangle}{(s+\Gamma+\frac{1}{2}\gamma)(s-i\Delta+\frac{1}{2}\Gamma)(s+i\Delta+\frac{1}{2}\Gamma+2\gamma)+\bar{\omega}_R^2(s+\frac{1}{2}\Gamma+\gamma)}\,, \tag{46}$$

where $W(t,s)$ has been defined in Eq. (43). The stationary average population of the excited state $\langle\sigma_{22}(t)\rangle$, which appears in Eq. (46), is given by Eq. (18). The stationary average $\langle\omega_R(t)\sigma_{21}(t)\rangle$ can be calculated from the equation

$$\sigma_{21}(t)=\tfrac{1}{2}i\int_0^t \exp[(i\Delta+\tfrac{1}{2}\Gamma)(t_1-t)]\,\omega_R^*(t_1)n(t_1)dt_1\,, \tag{47}$$

which is obtained from Eq. (15). If we multiply both sides of Eq. (47) by $\omega_R(t)$ and take the stochastic average and the Laplace transform we find

$$\langle\omega_R(t)\sigma_{21}(t)\rangle=\tfrac{1}{2}i\bar{\omega}_R^2\,\frac{\langle n(t)\rangle}{i\Delta+\frac{1}{2}(\Gamma+\gamma)}\,, \tag{48}$$

where $t\to\infty$. The stationary average spectrum $\langle S(\omega)\rangle^{\rm PD}$ can be evaluated using Eq. (44). It should be pointed out that the average spectrum of resonance fluorescence in the presence of a PD field has been calculated before by Eberly[5] and also be Kimble and Mandel.[6] Both those papers, however, are based on the decorrelation approximation of atomic and field variables, while in this paper the averaging of the spectrum is rigorous. The calculation of $\langle S(\omega)\rangle^{\rm PD}$ by Agarwal[4] is also rigorous, but is based on a different method which cannot be applied in the case of intensity fluctuations. Kimble and Mandel have discussed in detail several of the effects of phase fluctuations.[6] In the end of Sec. IV, we will discuss a new effect caused by phase fluctuations which has not been considered previously.

B. Chaotic model

In order to average Eq. (40) in the case of a chaotic exciting field we proceed as in Sec. III B. Equation (40) is multiplied by the Laguerre polynomial $L_N^1(\mathcal{E}^2(t+\tau)/\mathcal{E}_0^2)$. The choice of the upper index ($m=1$) of the Laguerre polynomial is dictated by the stochastic Rabi frequency $\omega_R(t+\tau)$, which multiplies both sides of Eq. (40). Choosing $m=0$ would lead to a more complicated expression. Taking the stochastic average, using Eqs. (10), (20), and (21), we obtain

$$\begin{aligned}\langle w(t,\tau)\rangle_N^1=&-e^{-(2N+1)\gamma\tau/2}\langle\omega_R(t)\sigma_{21}(t)\rangle_N^1-i\bar{\omega}_R^2\int_0^\tau e^{[\Gamma+(2N+1)\gamma/2](t_1-\tau)}\exp[(i\Delta-\tfrac{1}{2}\Gamma)t_1]\\&\times[e^{-N\gamma t_1}\langle\sigma_{22}(t)\rangle_N-e^{-(N+1)\gamma t_1}\langle\sigma_{22}(t)\rangle_{N+1}]dt_1\\&-\tfrac{1}{2}\bar{\omega}_R^2\int_0^\tau\exp\{[\Gamma+\tfrac{1}{2}(2N+1)\gamma](t_1-\tau)\}dt_1\int_0^{t_1}\exp[(-i\Delta+\tfrac{1}{2}\Gamma)(t_2-t_1)](N+1)\\&\times\{e^{-N\gamma(t_1-t_2)}[\langle w(t,t_2)\rangle_N^1-\langle w(t,t_2)\rangle_{N-1}^1]\\&-e^{(N+1)\gamma(t_1-t_2)}[\langle w(t,t_2)\rangle_{N+1}^1-\langle w(t,t_2)\rangle_N^1]\}dt_2\\&-\tfrac{1}{2}\bar{\omega}_R^2\int_0^\tau\exp\{[\Gamma+\tfrac{1}{2}(2N+1)\gamma](t_1-\tau)\}dt_1\int_0^{t_1}\exp[(i\Delta+\tfrac{1}{2}\Gamma)(t_2-t_1)]\\&\times\{e^{-N\gamma(t_1-t_2)}[N\langle w(t,t_2)\rangle_N^1-(N+1)\langle w(t,t_2)\rangle_{N-1}^1]\\&-e^{-(N+1)\gamma(t_1-t_2)}[(N+1)\langle w(t,t_2)\rangle_{N+1}^1\\&-(N+2)\langle w(t,t_2)\rangle_N^1]\}dt_2\,,\end{aligned} \tag{49}$$

where the average quantities $\langle w(t,\tau)\rangle_N^1\equiv\langle L_N^1(\mathcal{E}^2(t+\tau)/\mathcal{E}_0^2)\,\omega_R(t+\tau)h(t,\tau)\rangle$, $\langle\omega_R(t)\sigma_{21}(t)\rangle_N^1\equiv\langle L_N^1(\mathcal{E}^2(t)/\mathcal{E}_0^2)\,\omega_R(t)\sigma_{21}(t)\rangle$, and $\langle\sigma_{22}(t)\rangle_N\equiv\langle L_N^0(\mathcal{E}^2(t)/\mathcal{E}_0^2)\sigma_{22}(t)\rangle$, $N=0,1,2,\ldots$, are coefficients of expansions of stochastic quantities in terms of stochastic Laguerre polynomials, as in Eq. (23). The Laplace transform of the integral equation above leads to the algebraic, inhomogeneous, three-term recursion relation

$$W_N^1(t,s)\left(s+\Gamma+\tfrac{1}{2}(2N+1)\gamma+\frac{\frac{1}{2}(N+1)\bar\omega_R^2}{s-i\Delta+\frac{1}{2}\Gamma+N\gamma}+\frac{\frac{1}{2}(N+1)\bar\omega_R^2}{s-i\Delta+\frac{1}{2}\Gamma+(N+1)\gamma}+\frac{\frac{1}{2}N\bar\omega_R^2}{s+i\Delta+\frac{1}{2}\Gamma+N\gamma}+\frac{\frac{1}{2}(N+2)\bar\omega_R^2}{s+i\Delta+\frac{1}{2}\Gamma+(N+1)\gamma}\right)$$

$$-W_{N-1}^1(t,s)\left(\frac{\frac{1}{2}(N+1)\bar\omega_R^2}{s-i\Delta+\frac{1}{2}\Gamma+N\gamma}+\frac{\frac{1}{2}(N+1)\bar\omega_R^2}{s+i\Delta+\frac{1}{2}\Gamma+N\gamma}\right)-W_{N+1}^1(t,s)\left(\frac{\frac{1}{2}(N+1)\bar\omega_R^2}{s-i\Delta+\frac{1}{2}\Gamma+(N+1)\gamma}+\frac{\frac{1}{2}(N+1)\bar\omega_R^2}{s+i\Delta+\frac{1}{2}\Gamma+(N+1)\gamma}\right)$$

$$=-\frac{s+\Gamma+\frac{1}{2}(2N+1)\gamma}{s+\frac{1}{2}(2N+1)\gamma}\langle\omega_R(t)\sigma_{21}(t)\rangle_N^1-i\bar\omega_R^2\left(\frac{\langle\sigma_{22}(t)\rangle_N}{s-i\Delta+\frac{1}{2}\Gamma+N\gamma}-\frac{\langle\sigma_{22}(t)\rangle_{N+1}}{s-i\Delta+\frac{1}{2}\Gamma+(N+1)\gamma}\right), \quad (50)$$

where $W_N^1(t,s)$ is the Laplace transform of $\langle w(t,\tau)\rangle_N^1$. The stationary expansion coefficients $\langle\sigma_{22}(t)\rangle_N$ of the stochastic population $\sigma_{22}(t)$ can be calculated from Eq. (24) using the relation

$$\langle\sigma_{22}(t)\rangle_N=\tfrac{1}{2}\langle 1+n(t)\rangle_N=\tfrac{1}{2}[\delta_{N0}+\langle n(t)\rangle_N]\,. \quad (51)$$

The expansion coefficients $\langle\omega_R(t)\sigma_{21}(t)\rangle_N^1$ can be calculated by multiplying Eq. (47) by $L_N^1(\mathcal{E}^2(t)/\mathcal{E}_0^2)\,\omega_R(t)$ and then averaging to obtain

$$\langle\omega_R(t)\sigma_{21}(t)\rangle_N^1=\tfrac{1}{2}i\int_0^t \exp(\{i\Delta+\tfrac{1}{2}[\Gamma+(2N+1)\gamma]\}(t_1-t))\,\bar\omega_R^2(N+1)[\langle n(t_1)\rangle_N-\langle n(t_1)\rangle_{N+1}]dt_1\,. \quad (52)$$

In the stationary limit, the equation above reduces to

$$\langle\omega_R(t)\sigma_{21}(t)\rangle_N^1=\tfrac{1}{2}i(N+1)\bar\omega_R^2\,\frac{(\langle n\rangle_N-\langle n\rangle_{N+1})}{i\Delta+\frac{1}{2}[\Gamma+(2N+1)\gamma]}\;. \quad (53)$$

Because of its complexity, Eq. (50) does not render itself to an analytic solution. For very small average Rabi frequencies ($\bar\omega_R\ll\Gamma$), however, it is a good approximation to neglect all the expansion coefficients $W_N^1(t,s)$ with $N\geq 1$. In that case, Eq. (50) reduces to

$$W_0^1(t,s)=\frac{\left(-\dfrac{(s+\Gamma+\frac{1}{2}\gamma)(s-i\Delta+\frac{1}{2}\Gamma)(s+i\Delta+\frac{1}{2}\Gamma+\gamma)}{(s+\frac{1}{2}\gamma)}\langle\omega_R\sigma_{12}\rangle_0^1-i\bar\omega_R^2(s+i\Delta+\frac{1}{2}\Gamma+\gamma)\langle\sigma_{22}\rangle_0\right)}{\left[(s+\Gamma+\frac{1}{2}\gamma)(s-i\Delta+\frac{1}{2}\Gamma)(s+i\Delta+\frac{1}{2}\Gamma+\gamma)+\frac{1}{2}\bar\omega_R^2\left(3s-i\Delta+\frac{3}{2}\Gamma+\gamma+\dfrac{(s+i\Delta+\frac{1}{2}\Gamma+\gamma)}{(s-i\Delta+\frac{1}{2}\Gamma+\gamma)}(s-i\Delta+\frac{1}{2}\Gamma)\right)\right]}\,. \quad (54)$$

This equation can be obtained directly from Eq. (40) by making the decorrelation approximations

$$\langle\omega_R(t+\tau)\omega_R^*(t+t_1)\omega_R(t+t_2)h(t,t_2)\rangle\simeq\langle\omega_R(t+\tau)\omega_R^*(t+t_1)\rangle\langle\omega_R(t+t_2)h(t,t_2)\rangle+\langle\omega_R(t+t_2)\omega_R^*(t+t_1)\rangle\langle\omega_R(t+\tau)h(t,t_2)\rangle\,,$$

$$\langle\omega_R(t+\tau)\omega_R(t+t_1)\omega_R^*(t+t_2)h(t,t_2)\rangle\simeq\langle\omega_R(t+\tau)\omega_R^*(t+t_2)\rangle\langle\omega_R(t+t_1)h(t,t_2)\rangle+\langle\omega_R(t+t_1)\omega_R^*(t+t_2)\rangle\langle\omega_R(t+\tau)h(t,t_2)\rangle\,,$$

and using the exact relation $\langle\omega_R(t+\tau)h(t,t_2)\rangle=\exp[-\frac{1}{2}\gamma(\tau-t_2)]\langle\omega_R(t+t_2)h(t,t_2)\rangle$. The decorrelation approximations above assume that $h(t,t_2)\propto\omega_R^*(t)$ which corresponds to keeping only the first term in Eq. (40) [for weak fields, indeed, $\sigma_{21}(t)\propto\omega_R^*(t)$]. Clearly, Eq. (54) neglects all field-correlation functions of order greater than 2. The main difference between Eqs. (54) and (46) is in their denominators. For $\Delta=0$, the polynomial in the denominator of Eq. (54) simplifies to

$$p(s)=(s+\Gamma+\tfrac{1}{2}\gamma)(s+\tfrac{1}{2}\Gamma)(s+\tfrac{1}{2}\Gamma+\gamma)+2\bar\omega_R^2(s+\tfrac{1}{2}\Gamma+\tfrac{1}{4}\gamma)\,.$$

If we compare this with the polynomial in the denominator of Eq. (46), we see that $\bar\omega_R^2$ is multiplied by a factor of 2, which indicates the influence of intensity fluctuations. For strong fields this influence becomes stronger. As was shown in the case of double resonance with an intense chaotic field,[16-18] the stochastic Rabi frequency takes all values, from zero to infinity, with probability given by Eq. (8). For strong fields, however, Eq. (50) must be solved numerically keeping several hundred coefficients $W_N^1(t,s)$. The stationary average spectrum $\langle S(\omega)\rangle^{\rm CH}$ can be evaluated then from Eq. (41), with $\langle\sigma_{22}(t)\rangle$ given by Eq. (28) and $W(t,s)=W_0^1(t,s)$. In the end of Sec. IV, we discuss the results of numerical calculations of $\langle S(\omega)\rangle^{\rm CH}$ in strong chaotic fields. The spectrum of resonance fluorescence in the presence of a chaotic field was calculated recently by a different method using the Fokker-Planck operator for a chaotic field.[18] The Fokker-Planck formalism is suitable for averaging one-time stochastic differential equations, while the method developed in this paper, which uses the conditional probability to describe Markovian fields, is suitable for averaging multitime integral equations.

C. Gaussian-amplitude model

Multiplying Eq. (40) by the Hermite polynomial $H_N(\mathcal{E}(t+\tau)/\sqrt{2}\mathcal{E}_0)$ and taking the stochastic average, using Eqs. (14) and (29), we obtain

$$
\begin{aligned}
\langle w(t,\tau)\rangle_N = &-\sqrt{2}\,\bar{\omega}_R\{(N+1)e^{-[(N+1)/2]\gamma\tau}\langle\sigma_{21}(t)\rangle_{N+1}+\tfrac{1}{2}e^{-[(N-1)/2]\gamma\tau}\langle\sigma_{21}(t)\rangle_{N-1}\}\\
&-i\bar{\omega}_R^2\int_0^\tau e^{\Gamma(t_1-\tau)}\exp[(i\Delta-\tfrac{1}{2}\Gamma)t_1]\\
&\quad\times\{e^{[(N+1)/2]\gamma(t_1-\tau)}[2(N+2)(N+1)e^{-[(N+2)/2]\gamma t_1}\langle\sigma_{22}(t)\rangle_{N+2}+(N+1)e^{-(N/2)\gamma t_1}\langle\sigma_{22}(t)\rangle_N]\\
&\quad+e^{[(N-1)/2]\gamma(t_1-\tau)}[Ne^{-(N/2)\gamma t_1}\langle\sigma_{22}(t)\rangle_N+\tfrac{1}{2}e^{-[(N-2)/2]\gamma t_1}\langle\sigma_{22}(t)\rangle_{N-2}]\}dt_1\\
&-\tfrac{1}{2}\bar{\omega}_R^2\int_0^\tau e^{\Gamma(t_1-\tau)}dt_1\int_0^{t_1}\exp[-i\Delta+\tfrac{1}{2}\Gamma)(t_2-t_1)]\\
&\quad\times\{e^{[(N+1)/2]\gamma(t_1-\tau)}[2(N+2)(N+1)e^{[(N+2)/2]\gamma(t_2-t_1)}\langle w(t,t_2)\rangle_{N+2}\\
&\quad+(N+1)e^{(N/2)\gamma(t_2-t_1)}\langle w(t,t_2)\rangle_N]\\
&\quad+e^{[(N-1)/2]\gamma(t_1-\tau)}[Ne^{(N/2)\gamma(t_2-t_1)}\langle w(t,t_2)\rangle_N+\tfrac{1}{2}e^{[(N-2)/2]\gamma(t_2-t_1)}\langle w(t,t_2)\rangle_{N-2}]\}dt_2\\
&-\tfrac{1}{2}\bar{\omega}_R^2\int_0^\tau e^{\Gamma(t_1-\tau)}dt_1\int_0^{t_1}\exp[(i\Delta+\tfrac{1}{2}\Gamma)(t_2-t_1)]\\
&\quad\times\{e^{[(N+1)/2]\gamma(t_1-\tau)}[2(N+2)(N+1)e^{[(N+2)/2]\gamma(t_2-t_1)}\langle w(t,t_2)\rangle_{N+2}\\
&\quad+(N+1)e^{(N/2)\gamma(t_2-t_1)}\langle w(t,t_2)\rangle_N]\\
&\quad+e^{[(N-1)/2]\gamma(t_2-t_1)}[Ne^{(N/2)\gamma(t_2-t_1)}\langle w(t,t_2)\rangle_N+\tfrac{1}{2}e^{[(N-2)/2]\gamma(t_2-t_1)}\langle w(t,t_2)\rangle_{N-2}]\}dt_2\,,
\end{aligned}
\tag{55}
$$

where $\langle w(t,\tau)\rangle_N\equiv\langle H_N(\mathcal{E}(t+\tau)/\sqrt{2}\mathcal{E}_0)\,\omega_R(t+\tau)h(t,\tau)\rangle$, $\langle\sigma_{21}(t)\rangle_N\equiv\langle H_N(\mathcal{E}(t)/\sqrt{2}\mathcal{E}_0)\sigma_{21}(t)\rangle$, and $\langle\sigma_{22}(t)\rangle_N\equiv\langle H_N(\mathcal{E}(t)/\sqrt{2}\mathcal{E}_0)\sigma_{22}(t)\rangle$, $N=0,2,4,\ldots$, are coefficients of expansions in Hermite polynomials, as in Eq. (31). Taking the Laplace transform of the above equation we obtain the inhomogeneous three-term recursion relation

$$
\begin{aligned}
&W_N(t,s)\left[1+\left(\frac{\frac{1}{2}(N+1)\bar{\omega}_R^2}{s+\Gamma+\frac{1}{2}(N+1)\gamma}+\frac{\frac{1}{2}N\bar{\omega}_R^2}{s+\Gamma+\frac{1}{2}(N-1)\gamma}\right)\left(\frac{1}{s-i\Delta+\frac{1}{2}(\Gamma+N\gamma)}+\frac{1}{s+i\Delta+\frac{1}{2}(\Gamma+N\gamma)}\right)\right]\\
&+W_{N-2}(t,s)\left[\frac{\frac{1}{4}\bar{\omega}_R^2}{s+\Gamma+\frac{1}{2}(N-1)\gamma}\left(\frac{1}{s-i\Delta+\frac{1}{2}[\Gamma+(N-2)\gamma]}+\frac{1}{s+i\Delta+\frac{1}{2}[\Gamma+(N-2)\gamma]}\right)\right]\\
&+W_{N+2}(t,s)\left[\frac{(N+2)(N+1)\bar{\omega}_R^2}{s+\Gamma+\frac{1}{2}(N+1)\gamma}\left(\frac{1}{s-i\Delta+\frac{1}{2}[\Gamma+(N+2)\gamma]}+\frac{1}{s+i\Delta+\frac{1}{2}[\Gamma+(N+2)\gamma]}\right)\right]\\
&=-\sqrt{2}\,\bar{\omega}_R^2\left(\frac{(N+1)\langle\sigma_{21}(t)\rangle_{N+1}}{s+\frac{1}{2}(N+1)\gamma}+\frac{\frac{1}{2}\langle\sigma_{21}(t)\rangle_{N-1}}{s+\frac{1}{2}(N-1)\gamma}\right)\\
&\quad-i\bar{\omega}_R^2\left[\frac{1}{s+\Gamma+\frac{1}{2}(N+1)\gamma}\left(\frac{2(N+2)(N+1)\langle\sigma_{22}(t)\rangle_{N+2}}{s-i\Delta+\frac{1}{2}[\Gamma+(N+2)\gamma]}+\frac{(N+1)\langle\sigma_{22}(t)\rangle_N}{s-i\Delta+\frac{1}{2}[\Gamma+N\gamma]}\right)\right.\\
&\quad\left.+\frac{1}{s+\Gamma+\frac{1}{2}(N-1)\gamma}\left(\frac{N\langle\sigma_{22}(t)\rangle_N}{s-i\Delta+\frac{1}{2}(\Gamma+N\gamma)}+\frac{\frac{1}{2}\langle\sigma_{22}(t)\rangle_{N-2}}{s-i\Delta+\frac{1}{2}[\Gamma+(N-2)\gamma]}\right)\right],
\end{aligned}
\tag{56}
$$

where $W_N(t,s)$ is the Laplace transform $\langle w(t,\tau)\rangle_N$. The coefficients $\langle\sigma_{22}(t)\rangle_N$ are related to the coefficients $\langle n\rangle_{2K}$ in Eq. (32) through Eq. (51), which holds for chaotic as well as Gaussian-amplitude fields. The coefficients $\langle\sigma_{21}(t)\rangle_N$ can be calculated by multiplying Eq. (47) by $H_N(\mathcal{E}(t)/\sqrt{2}\mathcal{E}_0)$ and averaging to yield

$$
\langle\sigma_{21}(t)\rangle_N=\frac{i}{\sqrt{2}}\int_0^t\exp\{[i\Delta+\tfrac{1}{2}(\Gamma+N\gamma)](t_1-t)\}\,\bar{\omega}_R[(N+1)\langle n(t)\rangle_{N+1}+\tfrac{1}{2}\langle n(t)\rangle_{N-1}]dt_1\,. \tag{57}
$$

In the stationary limit, the above equation reduces to

$$\langle\sigma_{21}(t)\rangle_N=\frac{i\bar{\omega}_R}{\sqrt{2}}\frac{[(N+1)\langle n(t)\rangle_{N+1}+\frac{1}{2}\langle n(t)\rangle_{N-1}]}{i\Delta+\frac{1}{2}(\Gamma+N\gamma)}\ . \tag{58}$$

For weak fields ($\bar{\omega}_R \ll \Gamma$), it is a good approximation to neglect all the expansion coefficients $W_N(t,s)$ with $N \geq 1$. Under this approximation, Eq. (56) reduces to

$$W_0(t,s)=\frac{-\dfrac{(s+\Gamma+\frac{1}{2}\gamma)(s-i\Delta+\frac{1}{2}\Gamma)(s+i\Delta+\frac{1}{2}\Gamma)}{(s+\frac{1}{2}\gamma)}\langle\omega_R\sigma_{21}\rangle_0-i\bar{\omega}_R^2(s+i\Delta+\frac{1}{2}\Gamma)\langle\sigma_{22}\rangle_0}{(s+\Gamma+\frac{1}{2}\gamma)(s-i\Delta+\frac{1}{2}\Gamma)(s+i\Delta+\frac{1}{2}\Gamma)+\bar{\omega}_R^2(s+\frac{1}{2}\Gamma)}\ , \tag{59}$$

where we have used the relation $\sqrt{2}\,\bar{\omega}_R\langle\sigma_{21}(t)\rangle_1 =\langle\omega_R(t)\sigma_{21}(t)\rangle_0$. Equation (59) can be obtained directly from Eq. (40) by making the decorrelation approximation

$$\langle\omega_R(t+\tau)\omega_R(t+t_1)\omega_R(t+t_2)h(t,t_2)\rangle \simeq\langle\omega_R(t+\tau)\omega_R(t+t_1)\rangle\langle\omega_R(t+t_2)h(t,t_2)\rangle\ .$$

This decorrelation, unlike the decorrelation made in obtaining Eq. (54), neglects even the second-order statistics of the field [see Eq. (11)]. The fact that Eq. (50) to lowest order ($N=0$) contains information about the first- and second-order correlation of the chaotic fields, while Eq. (56) to lowest order contains information about only the first-order correlation of the Gaussian-amplitude field, stems from the different recursion relations for Laguerre and Hermite polynomials. As a consequence of neglecting the second-order field correlation, the bandwidth γ adds only to the diagonal relaxation rate and the factor multiplying $\bar{\omega}_R^2$ in the denominator of Eq. (59) does not show any influence of intensity fluctuations [compare Eqs. (46), (54), and (59)]. The decorrelation approximation above was used by Eberly[5] and Agarwal[15] to study the effects of amplitude fluctuations on the spectrum of resonance fluorescence. These authors, not realizing that the decorrelation is valid only for weak fields ($\bar{\omega}_R \ll \Gamma$), made predictions for the case of strong fields ($\bar{\omega}_R \gg \Gamma,\gamma$) which are not correct. They predicted a triplet structure for the spectrum of resonance fluorescence, while actually the triplet structure is completely washed out in the case of a strong Gaussian-amplitude field. For strong fields Eq. (56) must be solved numerically, retaining several hundred coefficients $W_N(t,s)$.

D. Numerical calculations and discussion

We present now some representative results of numerical calculations of the spectrum of resonance fluorescence in the presence of each of the three models for the stochastic exciting field. Figure 2 shows the spectra $\langle S(\omega)\rangle^{PD}$ (dashed line), $\langle S(\omega)\rangle^{CH}$ (solid line), and $\langle S(\omega)\rangle^{G}$ (dotted line) for an average Rabi frequency $\bar{\omega}_R=10\Gamma$ and zero detuning ($\Delta=0$). The spectra are symmetric about the center frequency of the exciting fields ($\omega=\omega_0$) and only the upper half ($\omega>\omega_0$) is shown. Three different characteristic values were used for the bandwidth $\gamma=0.1\Gamma$, Γ, and 10Γ. In order to show the difference in the wings of the spectra more clearly, a semilog plot is made. Starting with $\langle S(\omega)\rangle^{PD}$, we see that as the bandwidth γ is increased, the familiar triplet structure[23] broadens and the spacing between the two side peaks decreases from its maximum value of $2\bar{\omega}_R(\bar{\omega}_R \gg \Gamma,\gamma)$. The ratio of the heights of the center peak to the side peak decreases.[4-8] When the bandwidth γ becomes $\sim\bar{\omega}_R$, the height of the center peak is smaller than that of the side peaks and a center line dip develops. This effect is reported for the first time in this paper. For $\gamma=\bar{\omega}_R=10\Gamma$, the difference between the center line dip and the

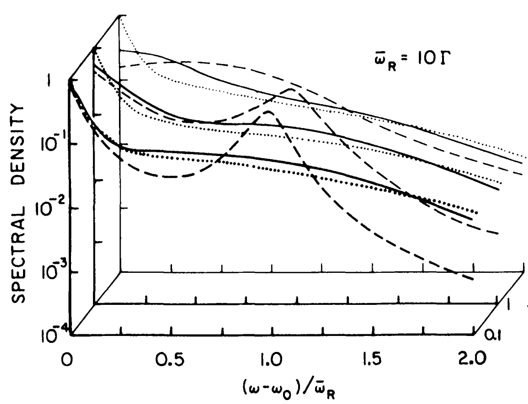

FIG. 2. Resonance fluorescence spectra for a phase-diffusion field (dashed line), a chaotic field (solid line) and a Gaussian-amplitude field (dotted line). The values of the average Rabi frequency $\bar{\omega}_R$ and the bandwidth γ are given in the figure in units of the spontaneous decay rate Γ. The detuning of the fields is zero.

side peak is only 2%, while the spacing between the two side peaks is $\sim 0.64\bar{\omega}_R$. If the bandwidth is increased further $(\gamma > \omega_R)$, the spacing between the two side peaks vanishes and the three spectral components coalesce into a single line. The width of this line decreases with increasing bandwidth. For $\gamma \gg \Gamma, \bar{\omega}_R$, the width of the line tends to Γ.[25] Examining next the curves for $\langle S(\omega)\rangle^{\text{CH}}$ and $\langle S(\omega)\rangle^{G}$, we see that there are no side peaks for $\bar{\omega}_R = 10\Gamma$. The intensity fluctuations in both fields smear out and suppress the sidebands which are associated with the imaginary part of the dipole oscillating at the Rabi frequency.[8] The suppression is stronger for $\langle S(\omega)\rangle^{G}$ because the Gaussian-amplitude field undergoes stronger intensity fluctuations. The central peak, which is associated with the real part of the dipole,[8] is not affected by intensity fluctuations for $\Delta = 0$. Thus, no center line dip develops in the case of intensity fluctuations. The center line dip develops only in the case of phase fluctuations, which couple the real and the imaginary parts of the dipole even for $\Delta = 0$, and suppress the center peak more than the side peaks. It should be mentioned here that for $\gamma = 0.1\Gamma$, Eqs. (50) and (56) were solved with six hundred expansion coefficients. The convergence is excellent ($\langle n\rangle_0^{\text{CH}} = -3.4\times 10^{-2}$, $\langle n\rangle_{600}^{\text{CH}} = -5.7\times 10^{-47}$, $\langle n\rangle_0^{G} = -9.8\times 10^{-2}$, $\langle n\rangle_{600}^{G} = 4.1 \times 10^{-34}$).

Figure 3 shows the resonance fluorescence spectra for $\bar{\omega}_R = 50\Gamma$ and $\Delta = 0$. In the case of the PD field, the triplet structure is resolved more clearly than in the previous figure because of the larger Rabi frequency. A triplet structure develops also in the case of the chaotic field, but the side peaks are much lower and broader than those for the PD field. For $\gamma = 0.1\Gamma$, the spacing between the center peak and the side peaks of $\langle S(\omega)\rangle^{\text{CH}}$ is $\sim 0.69\bar{\omega}_R$. In the limit $(\Gamma+\gamma)/\bar{\omega}_R \to 0$

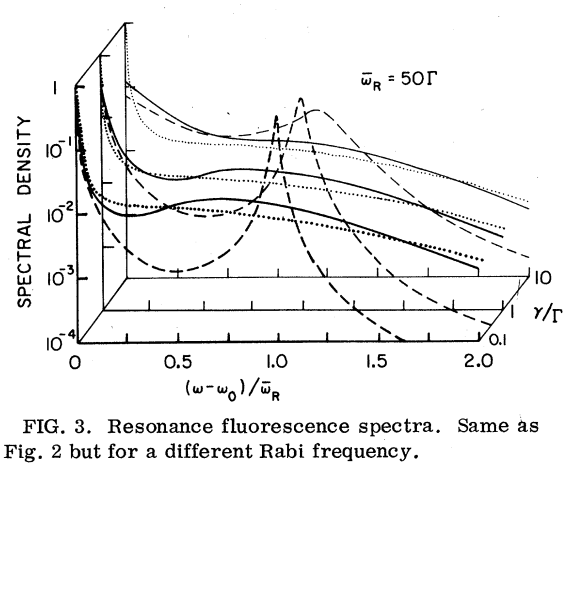

FIG. 3. Resonance fluorescence spectra. Same as Fig. 2 but for a different Rabi frequency.

this spacing tends to $\bar{\omega}_R/\sqrt{2}$, which corresponds to the most probable value in the Rayleigh distribution [Eq. (8)] of the real amplitude of the chaotic field. In fact, the sidebands of $\langle S(\omega)\rangle^{\text{CH}}$ form a two-sided Rayleigh distribution with average value equal to $\bar{\omega}_R$.[16,18] Generally, in the strong field limit and for $\Delta = 0$, the sidebands of the resonance fluorescence spectrum reconstruct the probability distribution of the stochastic field amplitude from the infinite number of moments $\langle \mathcal{E}^{2K}\rangle$, $K = 1, 2, \ldots,$ which enter into the equation for $W(t, s)$. Note that the odd-numbered moments $\langle \mathcal{E}^{2K+1}\rangle$ do not enter in Eq. (50), but they are not needed to reconstruct the two-sided amplitude distribution which is an even function. The reconstruction of the amplitude distribution becomes more accurate as the ratio $(\Gamma+\gamma)/\bar{\omega}_R$ decreases and the atom responds to higher moments of the field amplitude. The amplitude distribution reproduced by the sidebands is partly masked by the center peak which for a chaotic field has a width $\sim(\Gamma+\gamma)$. In the case of *ac* Stark splitting in double resonance the amplitude distribution is reproduced unmasked.[1-3,16-18] The spectrum of resonance fluorescence in the presence of a Gaussian-amplitude field does not exhibit a triplet structure. This is because the amplitude distribution is a Gaussian with zero mean value [Eq. (12)]. The spectrum $\langle S(\omega)\rangle^{G}$ consists of a sharp Lorentzian peak with width Γ and a broad Gaussian peak with width $2\sqrt{2\ln 2}\,\bar{\omega}_R$, both centered at $\omega = \omega_0$. The height of each peak is inversely proportional to its width. A spectrum of this type, but with the Lorentzian peak having width γ, was predicted qualitatively by Avan and Cohen-Tannoudji for the chaotic field.[8] Although their physical arguments were correct, these authors assumed a Gaussian distribution for the amplitude of the chaotic field, which actually is a Rayleigh distribution.

Figure 4 shows the development of the center line dip at $\bar{\omega}_R = 50\Gamma$. For $\gamma = 25\Gamma$ the triplet structure in $\langle S(\omega)\rangle^{\text{PD}}$ is barely resolved. As the bandwidth γ is increased the center line dip appears. For $\gamma = 50\Gamma$, the difference between the center line dip and the side peak is $\sim 11\%$, while the separation of the side peaks is $\sim 0.88\bar{\omega}_R$. As the bandwidth is increased further, the side peaks come together and the center line dip disappears as shown for $\gamma = 100\Gamma$. Note that the center line dip in the spectrum $\langle S(\omega)\rangle^{\text{PD}}$ appears only for $\gamma \sim \bar{\omega}_R \gg \Gamma$. Since phase fluctuations, in general, tend to suppress the center peak, the dip should develop also in the case of other types of phase fluctuations besides the Wiener-Levy type. The spectra $\langle S(\omega)\rangle^{\text{CH}}$ and $\langle S(\omega)\rangle^{G}$ do not exhibit a dip because the intensity fluctuations suppress the

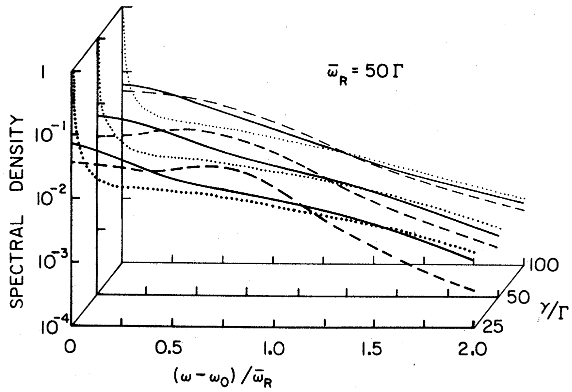

FIG. 4. Resonance fluorescence spectra. Same as Fig. 3 but for different values of the bandwidth.

sidebands. As can be seen from Figs. (2)–(4), $\langle S(\omega_0)\rangle^G$ is always larger than $\langle S(\omega_0)\rangle^{CH}$ and $\langle S(\omega_0)\rangle^{PD}$ for $\bar{\omega}_R \gg \Gamma$. This is because, as we mentioned earlier, intensity (amplitude) fluctuations do not affect the center peak and for the Gaussian-amplitude field the Gaussian sideband has a maximum at $\omega = \omega_0$. For $\bar{\omega}_R \gg \Gamma$ and $\gamma > \Gamma$, $\langle S(\omega_0)\rangle^{CH}$ is larger than $\langle S(\omega_0)\rangle^{PD}$. The width of the center peak in the case of the chaotic field is $\sim(\Gamma+\gamma)$, while in the case of the PD field it is $\sim(\Gamma+2\gamma)$. For $\bar{\omega}_R \gg \Gamma$ and $\gamma < \Gamma$, however, $\langle S(\omega_0)\rangle^{CH}$ can be either larger or smaller than $\langle S(\omega_0)\rangle^{PD}$, depending on the relative contributions of the inelastic sidebands and the elastic component at $\omega = \omega_0$. The sideband contribution is larger for the chaotic field, while the elastic component contribution is larger for the PD field. Note that for $\bar{\omega}_R = 10\Gamma$ and $\gamma = 0.1\Gamma$ (Fig. 2), $\langle S(\omega_0)\rangle^{CH}$ is larger than $\langle S(\omega_0)\rangle^{PD}$, but for $\bar{\omega}_R = 50\Gamma$ and $\gamma = 0.1\Gamma$ (Fig. 3), $\langle S(\omega_0)\rangle^{PD}$ is larger than $\langle S(\omega_0)\rangle^{CH}$.

Figure 5 shows the resonance fluorescence spectra for an average Rabi frequency $\bar{\omega}_R = 10\Gamma$ and detuning $\Delta = 5\Gamma$. As is well known, the off-resonance spectrum for a monochromatic field is symmetric around $\omega = \omega_0$, while for a nonmonochromatic field it becomes asymmetric.[6,7] As was explained in Ref. 12, this asymmetry arises because photons in the tail of the exciting spectrum are in resonance with the atomic transition and excite atoms which subsequently emit within the natural line. This contribution to resonance fluorescence makes the side peak which is closer to the atomic transition frequency more intense than the other side peak, and even the center peak in extreme cases. As can be seen from Fig. 5, for a given detuning the asymmetry depends on the bandwidth γ and the statistics of the exciting field.[18] Off resonance, the generalized Rabi frequency $\Omega_R(t) = [\Delta^2 + |\omega_R(t)|^2]^{1/2}$ is not linear in the field amplitude and the sidebands do not tend to reproduce the amplitude distribution. Because of the detuning, $\Omega_R(t)$ is not completely stochastic and the triplet structure develops even in the case of the Gaussian-amplitude field. The asymmetry ratio (side peak to side peak) for $\langle S(\omega)\rangle^G$ is larger than the asymmetry ratio for $\langle S(\omega)\rangle^{CH}$, which in turn is larger than the ratio for $\langle S(\omega)\rangle^{PD}$. Off resonance, the intensity fluctuations suppress the two side peaks by different amounts. For $\gamma = 10\Gamma$, it is interesting to

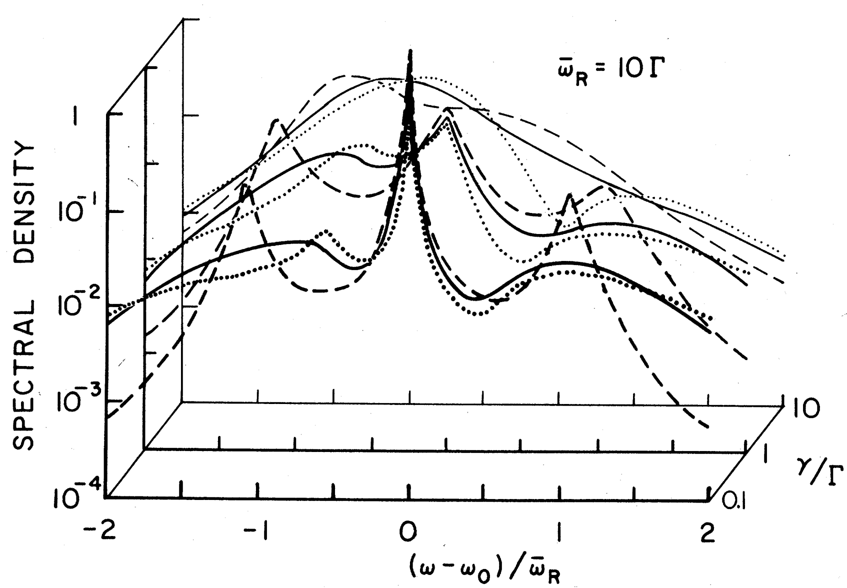

FIG. 5. Resonance fluorescence spectra. Same as Fig. 2 but the detuning is $\Delta = 5\Gamma$.

note that the side peak at $\omega > \omega_0$ in the spectrum $\langle S(\omega)\rangle^G$ has not merged with the center peak as in $\langle S(\omega)\rangle^{CH}$ and $\langle S(\omega)\rangle^{PD}$. This is because, off resonance, the broadening of the central peak by amplitude fluctuations is less than that by phase fluctuations.

V. SUMMARY

A simple and very general method has been developed to treat the effects of Markovian field fluctuations with arbitrary correlation time in resonant multiphoton processes. In this method, a Markovian stochastic field is described statistically by its marginal and conditional probability densities. Using the eigenfunctions and eigenvalues of the conditional averaging integral, one can systematically average the integral equations of motion for the atomic density matrix operator and its quantum correlation functions. The method has been used in this paper to study the effects of Markovian field fluctuations in resonance fluorescence. Three different models were used for the stochastic field: (a) the phase-diffusion field, (b) the chaotic field, and (c) the Gaussian-amplitude field. We have shown that the intensity of resonance fluorescence in the presence of a field with intensity fluctuations is less than that in the presence of a field with only phase fluctuations, having the same average power and bandwidth. The largest difference in the intensity of the fluorescence excited by two such fields occurs for intermediate field strengths (average Rabi frequency ~ spontaneous decay rate). For very strong fields the intensity of resonance fluorescence is independent of the statistics of the exciting field. This is due to the saturation of the atomic transition. We have also shown that the spectrum of resonance fluorescence depends critically on the statistics of the exciting field. For very strong fields the sidebands of the resonance fluorescence spectrum, like the doublet structure in double resonance,[1-3] tend to reproduce the probability distribution for the amplitude of the exciting field.[16-18] Therefore, unlike the total intensity, the spectrum of resonance fluorescence does not become independent of the statistics of the exciting field in the limit of very strong fields. Intensity fluctuations, in general, tend to suppress and broaden the sidebands. Phase fluctuations, on the other hand, tend to suppress the center peak of the spectrum more than the side peaks. Because of the latter effect, a center line dip develops in the spectrum of resonance fluorescence in the presence of a phase-diffusion field. This dip, which is reported for the first time in this paper, occurs for strong fields when the average Rabi frequency is approximately equal to the bandwidth, and the detuning is zero. Off resonance, the spectrum of resonance fluorescence in the presence of a nonmonochromatic exciting field becomes asymmetric.[6,7] Although the asymmetry is basically a bandwidth effect, the degree of asymmetry depends on the higher-order statistics of the exciting field.

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