Generation of an ion-acoustic pulse by two electromagnetic pulses at difference frequencies in a collisionless plasma

M. S. Sodha, Md. Salimullah, and R. P. Sharma
Centre of Energy Studies and Physics Department, Indian Institute of Technology, Delhi, New Delhi 110029, India
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This paper presents an investigation of the generation of an ion-acoustic pulse by two electromagnetic (EM) pulses in a collisionless hot unmagnetized plasma at the difference frequency of the two EM pulses. On account of the interaction of the two EM pulses, a ponderomotive force at the difference frequency becomes finite and leads to the generation of an ion-acoustic pulse. When the two EM pulses have a Gaussian intensity distribution in time and uniform intensity distribution in space, the generated ion-acoustic pulse is also Gaussian in time with a pulse width $=[t_{10}^2 t_{20}^2/(t_{10}^2+t_{20}^2)]^{1/2}$, where t_{10} and t_{20} are the initial pulse widths of the incident EM pulses. Moreover, if the incident EM pulses have a Gaussian intensity distribution in space and time, the nonuniform intensity distribution of the EM pulses in a plane transverse to the direction of propagation leads to the redistribution of electrons and ions, and transient (time-dependent) cross focusing of the pulses may occur for appropriate initial powers of the EM pulses. The ion-acoustic-pulse generation is seen to be drastically modified by cross focusing of the two EM pulses.

I. INTRODUCTION

When two electromagnetic (EM) waves of frequencies ω_1 and ω_2 interact in a collisionless plasma, the ponderomotive force at the difference frequency $\Delta\omega=\omega_1-\omega_2$ becomes finite and leads to the resonant excitation of an electrostatic wave at the difference frequency, when the difference frequency $\Delta\omega$ and the difference of the propagation vectors satisfy the dispersion relation of the electrostatic waves.[1-3]

In the present paper, we have investigated the excitation of an ion-acoustic pulse at the difference frequency $\Delta\omega$ by two Gaussian (in space and time) laser pulses of frequencies ω_1 and ω_2 in a collisionless, hot, and unmagnetized plasma. When the two EM pulses have a uniform intensity distribution in space and a Gaussian distribution in time, the ponderomotive force at the difference frequency excites an ion-acoustic pulse at the difference frequency. But the ponderomotive force at the difference frequency is time dependent when the incident pulses have a Gaussian intensity distribution in time; therefore, the generated pulse is also Gaussian in time with a pulse width equal to $[t_{10}^2 t_{20}^2/(t_{10}^2+t_{20}^2)]^{1/2}$, where t_{10} and t_{20} are the initial pulse durations of the EM pulses.

When the incident pulses are Gaussian in space and time, the ponderomotive force from the nonuniform intensity distributions of the EM pulses in a plane transverse to the direction of propagation becomes finite[4,5] and leads to the redistribution of the electrons and ions, and, if the initial pulse powers are appropriate, the transient cross focusing of the EM pulses occurs. As time elapses, the intensities of the incident pulses change, and thereby the cross focusing of the pulses and ion-acoustic pulse power are affected. This is relevant to wave-interaction studies in the field of laser thermonuclear fusion, where high-power short-duration laser pulses are frequently used.

In Sec. II we have studied the time-dependent behavior of the ponderomotive nonlinearity in the presence of the two EM pulses in a collisionless plasma. The transient cross focusing of the EM pulses has been studied in Sec. III. In Sec. IV we have investigated the generation of the ion-acoustic pulse at the difference frequency. A brief discussion of the results is presented in Sec. V.

II. TIME-DEPENDENT BEHAVIOR OF PONDEROMOTIVE NONLINEARITY

We consider the propagation of two coaxial Gaussian EM pulses (Gaussian in space and time) along the z axis in a collisionless, hot, unmagnetized, and homogeneous plasma. The intensity distributions of the pulses at $z=0$ are given by

$$E_1E_1^*|_{z=0}=E_{10}^2(t)\exp(-\gamma^2/\gamma_{10}^2)\,, \tag{1}$$

$$E_2E_2^*|_{z=0}=E_{20}^2(t)\exp(-\gamma^2/\gamma_{20}^2)\,, \tag{2}$$

$$E_{10}^2(t)=E_{100}^2\exp(-t^2/t_{10}^2)\,, \tag{3}$$

$$E_{20}^2(t)=E_{200}^2\exp(-t^2/t_{20}^2)\,, \tag{4}$$

where $\gamma^2=x^2+y^2$ and γ_{10}, γ_{20} are the initial pulse widths and t_{10}, t_{20} are the inital pulse durations. On account of nonuniform intensity distributions of the pulses in a plane transverse to the direction of propagation, the ponderomotive force becomes nonzero and leads to the diffusion of the electrons and ions.[4-6] Hence, in the presence of the EM pulses, the background electron and ion densities must be modified. Siegrist[7] has studied numerically the change in the background electron density by the ponderomotive force in the presence of a Gaussian pulse. Here we have derived an analytical expression for the modified electron density in the presence of two Gaussian EM pulses.

Following Sodha, Ghatak, and Tripathi,[4] Schmidt,[2] and Sodha *et al.*,[5] the expression for ponderomotive force can be written as

$$\vec{F}=-\frac{e^2}{4m_e}\left(\frac{1}{\omega_1^2}\vec{\nabla}(\vec{E}_1\cdot\vec{E}_1^*)+\frac{1}{\omega_2^2}\vec{\nabla}(\vec{E}_2\cdot\vec{E}_2^*)\right). \quad (5)$$

In the quasihydrodynamic two-fluid approximation, one obtains the following equations for diffusion of electrons and ions:

$$m_e\frac{\partial\vec{V}_e}{\partial t}=-e\vec{E}_s-\frac{k_BT_0}{N_e}\vec{\nabla}N_e -\frac{e^2}{4m_e}\left(\frac{1}{\omega_1^2}\vec{\nabla}(\vec{E}_1\cdot\vec{E}_1^*)+\frac{1}{\omega_2^2}\vec{\nabla}(\vec{E}_2\cdot\vec{E}_2^*)\right), \quad (6)$$

$$m_i\frac{\partial\vec{V}_i}{\partial t}=e\vec{E}_s-\frac{k_BT_0}{N_i}\vec{\nabla}N_i\,, \quad (7)$$

where the indices e and i refer to electrons and ions, respectively; m_e and m_i are the masses of electron and ion, respectively; $\vec{E}_s$ is the space-charge field; T_0 is the temperature of the plasma; k_B is the Boltzmann constant; ω_1 and ω_2 are the angular frequencies of the EM pulses; and $-e$ is the electronic charge. It must be mentioned here that in writing Eq. (7), the ponderomotive force on ions is not taken into account because its magnitude is less by a factor of m_e/m_i in comparison to that on electrons. Owing to space-charge effects, electrons and ions move almost with the same velocity; i.e., $V_e\cong V_i=V$ in an almost electrically neutral medium $N_e\cong N_i=N$, where V is the diffusion velocity of electrons and ions and N is the modified electron and ion density in the presence of the pulses. Adding Eqs. (6) and (7), and neglecting electron mass m_e as compared to the ion mass m_i, we obtain

$$\frac{\partial\vec{V}}{\partial t}=-\frac{2k_BT_0}{m_iN}\vec{\nabla}N -\frac{e^2}{4m_em_i}\left(\frac{1}{\omega_1^2}\vec{\nabla}(\vec{E}_1\cdot\vec{E}_1^*)+\frac{1}{\omega_2^2}\vec{\nabla}(\vec{E}_2\cdot\vec{E}_2^*)\right). \quad (8)$$

For further analysis, Eq. (8) may be supplemented by the equation of continuity

$$\frac{\partial N}{\partial t}+\vec{\nabla}\cdot(N\vec{V})=0\,. \quad (9)$$

Equations (8) and (9) may be combined to give

$$\frac{\partial^2}{\partial t^2}\left(\ln\frac{N}{N_0}\right)=\frac{2k_BT_0}{m_i}\nabla^2\left(\ln\frac{N}{N_0}\right) +\frac{e^2}{4m_em_i}\left(\frac{1}{\omega_1^2}\nabla^2(\vec{E}_1\cdot\vec{E}_1^*)+\frac{1}{\omega_2^2}\nabla^2(\vec{E}_2\cdot\vec{E}_2^*)\right), \quad (10)$$

where N_0 is the electron density in the absence of the pulses. This differential equation is a wave equation with a velocity of propagation $V_a=(2k_BT_0/m_i)^{1/2}$ of the order of ion-acoustic velocity. When a single pulse is propagating in the plasma, Eq. (8) reduces to Eq. (10) of Siegrist[7] derived from a slightly different approach. For stationary conditions, the time derivative on the left-hand side of Eq. (10) disappears, and one obtains the following expression for the modified electron density on account of the ponderomotive force[5]:

$$N=N_0\exp\left[-\frac{e^2}{8m_ek_BT_0}\left(\frac{E_1E_1^*}{\omega_1^2}+\frac{E_2E_2^*}{\omega_2^2}\right)\right]. \quad (11)$$

An analytical expression for N from Eq. (10) can be obtained as follows. To keep the analysis more general we add a damping term phenomenologically in Eq. (10). Taking the Fourier transform of the resulting equation with respect to space coordinates would give an equation of the form

$$\frac{d^2\phi}{dt^2}+\nu\frac{d\phi}{dt}+\omega'^2\phi=F(t)\,, \quad (12)$$

where ϕ is the Fourier transform of $\ln(N/N_0)$ and $F(t)$ is the Fourier transform of the second term on the right-hand side of Eq. (10) given by

$$F(t)=-\frac{e^2(K_1^2+K_2^2)}{8m_em_i}\times\left[\frac{\gamma_{10}^2}{\omega_1^2}E_{10}^2(t)\exp\left(-(K_1^2+K_2^2)\frac{\gamma_{10}^2f_1^2}{4}\right)+\frac{\gamma_{20}^2}{\omega_2^2}E_{20}^2(t)\exp\left(-(K_1^2+K_2^2)\frac{\gamma_{20}^2f_2^2}{4}\right)\right].$$

A Green's-function solution of Eq. (12) is

$$\phi=\frac{1}{\Omega}\int_{-\infty}^{t}\sin[\Omega(t-t')]\exp[-\tfrac{1}{2}\nu(t-t')]F(t')\,dt'\,, \quad (13)$$

where $\Omega=\frac{1}{2}(4\omega'^2-\nu^2)^{1/2}$. Inversion of the Fourier transform would give an equation for N. Equation (13) gives N as

$$N=N_0\exp\left[-\frac{e^2}{16\pi m_em_iV_{\rm th}}\left(\frac{\gamma_{10}^2I_1}{\omega_1^2}+\frac{\gamma_{20}^2I_2}{\omega_2^2}\right)\right], \quad (14)$$

where

$$I_{1,2}=\int_{-\infty}^{+\infty}dK_1\int_{-\infty}^{+\infty}dK_2\int_0^{\infty}dt\,\sin[(K_1^2+K_2^2)^{1/2}V_{\rm th}t_1](K_1^2+K_2^2)^{1/2} \times\exp[-(K_1^2+K_2^2)\tfrac{1}{4}\gamma_{10,20}^2f_{1,2}^2(t-t_1,z)]\exp[-i(K_1x+K_2y)]E_{10,20}^2(t-t_1)\,, \quad (15)$$

where the subscript "th" denotes thermal speeds. It must be mentioned here that in deriving Eq. (15), the intensity distributions of the pulses in the plasma are consistent with Eqs. (25)–(28).

III. TRANSIENT CROSS FOCUSING OF THE TWO EM PULSES

The electric vectors of the pulses in the plasma are governed by the wave equation

$$\frac{\partial^2}{\partial x^2}E_{1,2}+\frac{\partial^2}{\partial y^2}E_{1,2}+\frac{\partial^2}{\partial z^2}E_{1,2}=\frac{1}{c^2}\frac{\partial^2}{\partial t^2}E_{1,2}+\frac{4\pi}{c^2}\frac{\partial}{\partial t}J_{1,2}\,. \tag{16}$$

In writing Eq. (16) we have neglected the $\vec{\nabla}(\vec{\nabla}\cdot\vec{E}_{1,2})$ term which is justified as long as[4]

$$\frac{\omega_p^2}{\omega_{1,2}^2}\frac{1}{\epsilon_{1,2}}\ln\epsilon_{1,2}\ll 1\,.$$

However, for a two-dimensional beam in space and $\partial/\partial y=0$, this is identically zero. In Eq. (16), J_1 and J_2 are the total current densities in the plasma in the presence of first and second pulse, respectively. Assuming the variation of $E_{1,2}$ in space and time as[4]

$$\vec{E}_{1,2}=\vec{A}_{1,2}(x,y,z,t)\exp[i(\omega_{1,2}t-k_{1,2}z)]\,, \tag{17}$$

where

$$k_{1,2}=(\omega_{1,2}/c)(1-\omega_p^2/\omega_{1,2}^2)^{1/2}$$

and $A_{1,2}$ is a complex function of space and time, the current densities in the plasma can be written as[4]

$$\vec{J}_{1,2}\cong\frac{Ne^2}{m_e i\omega_{1,2}}\left(\vec{A}_{1,2}+\frac{i}{\omega_{1,2}}\frac{\partial}{\partial t}\vec{A}_{1,2}\right)\times\exp[i(\omega_{1,2}t-k_{1,2}z)]\,. \tag{18}$$

Substituting the expression for $\vec{J}_{1,2}$ from Eq. (18) and $\vec{E}_{1,2}$ from Eq. (17) in Eq. (16), we obtain

$$-2ik_{1,2}\frac{\partial}{\partial z}A_{1,2}+\frac{\partial^2}{\partial x^2}A_{1,2}+\frac{\partial^2}{\partial y^2}A_{1,2}\cong\frac{\omega_{1,2}^2}{c^2}\frac{\omega_p^2}{\omega_{1,2}^2}\left(\frac{N-N_0}{N_0}\right)A_{1,2}+\frac{2i\omega_{1,2}}{c^2}\frac{\partial}{\partial t}A_{1,2}\,. \tag{19}$$

Further, assuming the variation of $A_{1,2}$ as[4-6]

$$\vec{A}_{1,2}=\vec{A}_{10,20}(x,y,z,t)\times\exp[-ik_{1,2}S_{1,2}(x,y,z,t)]\,, \tag{20}$$

where $A_{10,20}$ and $S_{1,2}$ are the real functions of space and time; substituting for $\vec{A}_{1,2}$ from Eq. (20) in Eq. (19) and separating the real and imaginary parts of the resulting equation, we obtain

$$\frac{1}{v_{g_{1,2}}}\frac{\partial}{\partial t}A_{10,20}^2+\frac{\partial}{\partial z}A_{10,20}^2+\frac{\partial}{\partial x}S_{1,2}\frac{\partial}{\partial x}A_{10,20}^2+\frac{\partial}{\partial y}S_{1,2}\frac{\partial}{\partial y}A_{10,20}^2+\left(\frac{\partial^2}{\partial x^2}S_{1,2}+\frac{\partial^2}{\partial y^2}S_{1,2}\right)A_{10,20}^2=0 \tag{21}$$

and

$$\frac{2}{v_{g_{1,2}}}\frac{\partial}{\partial t}S_{1,2}+2\frac{\partial}{\partial z}S_{1,2}+\left(\frac{\partial}{\partial x}S_{1,2}\right)^2+\left(\frac{\partial}{\partial y}S_{1,2}\right)^2=\frac{\omega_p^2}{\omega_{1,2}^2}\frac{N_0-N}{N_0\epsilon_{1,2}}+\frac{1}{k_{1,2}^2A_{1,2}}\left(\frac{\partial^2}{\partial x^2}A_{10,20}+\frac{\partial^2}{\partial y^2}A_{10,20}\right), \tag{22}$$

where

$$v_{g_{1,2}}=c\epsilon_{1,2}^{1/2}\quad\text{and}\quad\epsilon_{1,2}=1-\omega_p^2/\omega_{1,2}^2\,.$$

On transforming the variables (z,t) to $z(=z)$ and $\xi_{1,2}(=t-z/v_{g_{1,2}})$, Eqs. (21) and (22) assume the following form:

$$\frac{\partial}{\partial z}A_{10,20}^2+\frac{\partial}{\partial x}A_{10,20}^2+\frac{\partial}{\partial y}S_{1,2}\frac{\partial}{\partial y}A_{10,20}^2+\left(\frac{\partial^2}{\partial x^2}S_{1,2}+\frac{\partial^2}{\partial y^2}S_{1,2}\right)A_{10,20}^2=0 \tag{23}$$

and

$$2\frac{\partial}{\partial z}S_{1,2}+\left(\frac{\partial}{\partial x}S_{1,2}\right)^2+\left(\frac{\partial}{\partial y}S_{1,2}\right)^2=\frac{\omega_p^2}{\omega_{1,2}^2}\frac{N_0-N}{N_0\epsilon_{1,2}}+\frac{1}{k_{1,2}^2A_{10,20}}\left(\frac{\partial^2}{\partial x^2}A_{10,20}+\frac{\partial^2}{\partial y^2}A_{10,20}\right)\,. \tag{24}$$

The solution of these equations can be written as

$$A_{10,20}^2=\frac{E_{10,20}^2(\xi_{1,2})}{f_{1,2}^2(z,\xi_{1,2})}\times\exp\left(-\frac{\gamma^2}{\gamma_{10,20}^2f_{1,2}^2(z,\xi_{1,2})}\right), \tag{25}$$

$$S_{1,2}=\tfrac{1}{2}\gamma^2\beta_{1,2}(z,\xi_{1,2})+\Phi_{1,2}(z,\xi_{1,2}), \tag{26}$$

$$\beta_{1,2}=\frac{1}{f_{1,2}}\frac{d}{dz}f_{1,2}\,, \tag{27}$$

where $f_{1,2}$ are the dimensionless beam-width parameters. The equation governing the beam-width parameters $f_{1,2}$ can be obtained from Eq. (24) by substituting for $A_{10,20}^2$ and $S_{1,2}$ from Eqs. (25)–(27) and equating the coefficients of γ^2 $(=x^2+y^2)$ in the resulting equation (within the paraxial-ray approximation)

$$\frac{d^2}{dz^2} f_{1,2} = \frac{1}{k_{1,2}^2 \gamma_{10,20}^4} \frac{1}{f_{1,2}^3} - \frac{\omega_P^2}{\epsilon_{1,2}} \left(\frac{e^2}{16\pi m_e m_i V_{\rm th}^2}\right) f_{1,2} \left(\frac{\gamma_{10}^2 I_3}{\omega_1^2} + \frac{\gamma_{20}^2 I_4}{\omega_2^2}\right) \times \exp\left[-\left(\frac{e^2}{16\pi m_e m_i V_{\rm th}^2}\right)\left(\frac{\gamma_{20}^2 I_3'}{\omega_1^2} + \frac{\gamma_{20}^2 I_4'}{\omega_2^2}\right)\right], \tag{28}$$

where

$$I_{3,4} = \int_{-\infty}^{+\infty} dK_1 \int_{-\infty}^{+\infty} dK_2 \int_0^{\infty} dt_1 \sin[(K_1^2+K_2^2)^{1/2} V_{\rm th}\, t_1] K_1^2 (K_1^2+K_2^2)^{1/2} \times \exp[-\tfrac{1}{4}(K_1^2+K_2^2)\gamma_{10,20}^2 f_{1,2}^2(z, \xi - t)] E_{10,20}^2(\xi_{1,2} - t_1), \tag{29}$$

$$I_{3,4}' = \int_{-\infty}^{+\infty} dK_1 \int_{-\infty}^{+\infty} dK_2 \int_0^{\infty} dt_1 \sin[V_{\rm th} (K_1^2+K_2^2)^{1/2} t_1] (K_1^2+K_2^2)^{1/2} \times \exp[-\tfrac{1}{4}(K_1^2+K_2^2)\gamma_{10,20}^2 f_{1,2}^2(z, \xi_{1,2} - t_1)] E_{10,20}^2(\xi_{1,2} - t_1). \tag{30}$$

To have a numerical appreciation of the results, we have plotted f_1 and f_2 in Fig. 1 for the set of parameters mentioned in the figure caption.

IV. GENERATION OF THE ION-ACOUSTIC PULSE AT DIFFERENCE FREQUENCIES

The generation of the ion-acoustic pulse by two EM pulses in a collisionless plasma is governed by the fluid equations[8]

$$\frac{\partial}{\partial t} N_{e,i} + \vec{\nabla} \cdot (N_{e,i} \vec{V}_{e,i}) = 0, \tag{31}$$

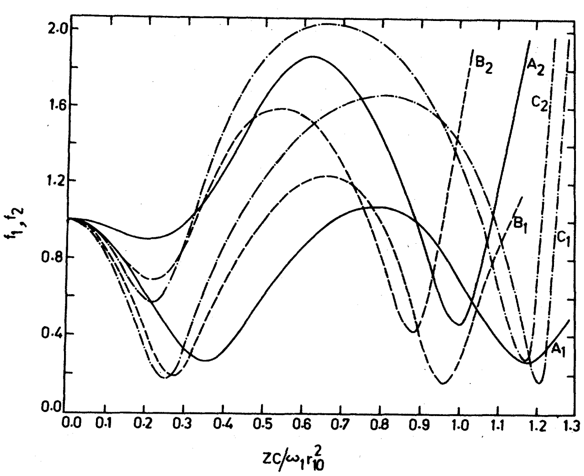

FIG. 1. Variation of f_1 and f_2 with $zc/\omega_1\gamma_{10}^2$ at different times for the following parameters: $\omega_1 = 1.963 \times 10^{14}$ rad sec^{-1}, $\omega_2 = 1.778 \times 10^{14}$ rad sec^{-1}, $\gamma_{10} = 40\pi c/\omega_1$, $\gamma_{10}/\gamma_{20} = 2$, $\omega_P = 0.5(\omega_1 - \omega_2)$, $E_{100}^2 \cong 3.98 \times 10^8$ (in cgs units), $E_{200}^2 \cong 3.27 \times 10^8$ (cgs), $t_{10}/t_{20} = 0.5$. Curves A_1, A_2 correspond to $t/t_{10} = 0.0$. Curves B_1, B_2 correspond to $t/t_{10} = 0.8$. Curves C_1, C_2 correspond to $t/t_{10} = 1.0$.

$$\frac{\partial}{\partial t} \vec{V}_{e,i} + (\vec{V}_{e,i} \cdot \vec{\nabla})\vec{V}_{e,i} = \frac{q}{m_{e,i}} \left(\vec{E} + \frac{1}{c} \vec{V}_{e,i} \times \vec{B}\right) - 2\Gamma_{e,i} \vec{V}_{e,i} - \frac{\gamma_{e,i} \vec{\nabla} P_{e,i}}{m_{e,i} N_{e,i}}, \tag{32}$$

where the subscripts e and i stand for electrons and ions, respectively, $q = -e$ for the electron-fluid case, and $q = +e$ for the ion case. In the above equations $N_{e,i}$ are the instantaneous particle densities, $\vec{V}_{e,i}$ their fluid velocities, $P_{e,i}$ the hydrodynamic pressures, $\gamma_{e,i}$ the ratio of specific heats of the electron and ion gases, and c is the velocity of light in vacuum. The damping coefficients $\Gamma_{e,i}$ are given by[9]

$$\Gamma_e = \frac{\pi}{8}^{1/2} \frac{\omega_{Pe}}{(k\lambda_D)^3} \exp\left(-\frac{1}{2k^2\lambda_D^2} - \frac{3}{2}\right) \tag{33}$$

and

$$\Gamma_i = \frac{k}{(1+k^2\lambda_D^2)^2} \left(\frac{\pi k_B T_e}{8 m_i}\right)^{1/2} \times \left[\left(\frac{m_e}{m_i}\right)^{1/2} + \left(\frac{T_e}{T_i}\right)^{3/2} \exp\left(-\frac{T_e}{2T_i} \Big/ (1+k^2\lambda_D^2)\right)\right], \tag{34}$$

where T_e and T_i are the electron and ion gas temperatures, $\vec{k}$ is the ion-acoustic wave vector, and $\lambda_D = (k_B T_e / 4\pi e^2 N_0)^{1/2}$ is the Debye length, and the other symbols have their usual meaning. It may be observed that the $\vec{V}_{e,i} \times \vec{B}$ and $(\vec{V}_{e,i} \cdot \vec{\nabla})\vec{V}_{e,i}$ terms of Eq. (32) give rise to ion-acoustic-pulse excitation at the difference frequency $\omega_1 - \omega_2$.

Combining Eqs. (31) and (32), we obtain

$$\frac{\partial^2}{\partial t^2}N_e + 2\Gamma_e\frac{\partial}{\partial t}N_e - v_{\text{th}}^2\nabla^2 N_e - \frac{e}{m_e}\vec{\nabla}\cdot(N_e\vec{E}) = \vec{\nabla}\cdot[\tfrac{1}{2}N_e\vec{\nabla}(\vec{V}_e\cdot\vec{V}_e) - \vec{V}_e\partial N_e/\partial t] \quad (35)$$

and

$$\partial^2 N_i/\partial t^2 + 2\Gamma_i\partial N_i/\partial t - V_{\text{th}}^2\nabla^2 N_i + \frac{e}{m_i}\vec{\nabla}\cdot(N_i\vec{E}) = \vec{\nabla}\cdot[\tfrac{1}{2}N_i\vec{\nabla}(\vec{V}_i\cdot\vec{V}_i) - \vec{V}_i\partial N_i/\partial t], \quad (36)$$

for electron and ion fluids, respectively, where

$$v_{\text{th}} = (\gamma_e k_B T_e/m_e)^{1/2} \text{ and } V_{\text{th}} = (\gamma_i k_B T_i/m_i)^{1/2}$$

for the electron and ion, respectively. In writing Eqs. (35) and (36), we have made use of Maxwell's equation

$$\vec{\nabla}\times\vec{E} = -\frac{1}{c}\frac{\partial\vec{B}}{\partial t}.$$

To solve for $N_{e,i}$ we assume, in the perturbation approximation,

$$\begin{aligned}&\vec{V}_{e,i} = \vec{V}_{e1,i1} + \vec{V}_{e2,i2} + \vec{v}_{e,i}, \quad \vec{v}_{e,i} \ll \vec{V}_{e1,i1}, \vec{V}_{e2,i2},\\ &\vec{E}(t) = \vec{E}_1(t) + \vec{E}_2(t) + \vec{E}_L(t), \quad \vec{E}_L \ll \vec{E}_1, \vec{E}_2,\\ &N_{e,i}(t) = N_{0e,0i}(t) + N_{e1,i1}(t), \quad N_{e1,i1} \ll N_{0e,0i},\end{aligned} \quad (37)$$

where $N_{0e}(t) \cong N_{0i}(t)$ is the electron density in the presence of the two Gaussian pulses given by Eq. (14), $\vec{V}_{e1,i1}$ are the electron and ion velocities in the high-frequency fields $\vec{E}_1$ and $\vec{E}_2$, $\vec{v}_{e,i}$ and $N_{e1,i1}$ are the perturbations in the velocity and density varying at difference frequencies $(\omega_1 - \omega_2)$, and $\vec{E}_L$ is the electric field associated with the generated ion-acoustic pulse and is given by Poisson's equation,

$$\vec{\nabla}\cdot\vec{E}_L = 4\pi e(N_{i1} - N_{e1}). \quad (38)$$

The equations for the perturbed quantities N_{e1} and N_{i1} are given by

$$\partial^2 N_{e1}/\partial t^2 - v_{\text{th}}^2\nabla^2 N_{e1} + 2\Gamma_e\partial N_{e1}/\partial t + \omega_{Pe}^2(N_{e1} - N_{i1})N_{0e}/N_0 \cong \vec{\nabla}\cdot[\tfrac{1}{4}N_{0e}\vec{\nabla}(\vec{V}_{e1}\cdot\vec{V}_{e2}^*)] \quad (39)$$

and

$$\partial^2 N_{i1}/\partial t^2 - V_{\text{th}}^2\nabla^2 N_{i1} + 2\Gamma_i\partial N_{i1}/\partial t + \omega_{Pi}^2(N_{i1} - N_{e1})N_{0i}/N_0 \cong \vec{\nabla}\cdot[\tfrac{1}{4}N_{0i}\vec{\nabla}(\vec{V}_{i1}\cdot\vec{V}_{i2}^*)], \quad (40)$$

where only terms oscillating at frequency $(\omega_1 - \omega_2)$ have been retained in the right-hand side of Eqs. (39) and (40). It follows from Eq. (33) that the electron plasma pulse is heavily damped when $k\lambda_D \cong 1$. The ion-acoustic pulse can still propagate if $T_e \gg T_i$. Henceforth, only Eq. (40) need be considered.

Equation (40) is a coupled equation for electron and ion perturbations. Now, taking the variation of N_{e1} as $\exp[i(\omega t - \Delta kz)]$ in Eq. (39), we obtain

$$N_{e1} \cong \frac{\vec{\nabla}\cdot[\tfrac{1}{4}N_{0e}\vec{\nabla}(\vec{V}_{e1}\cdot\vec{V}_{e2}^*)] + \omega_{Pe}^2 N_{i1}\frac{N_{0e}}{N_0}}{\left[-\omega^2 + (\Delta k)^2 v_{\text{th}}^2 + \omega_{Pe}^2\frac{N_{0e}}{N_0}\right]}, \quad (41)$$

where

$$\begin{aligned}&\omega = \omega_1 - \omega_2 \equiv \Delta\omega,\\ &\Delta k = k_1 - k_2.\end{aligned} \quad (42)$$

Substituting N_{e1} from Eq. (41) into Eq. (40), we obtain

$$\frac{\partial^2}{\partial t^2}N_{i1} - V_{\text{th}}^2\nabla^2 N_{i1} + 2\Gamma_i\frac{\partial}{\partial t}N_{i1} + \frac{\omega_{Pi}^2 N_{i1}}{D_{\gamma1}}[(\Delta k)^2 v_{\text{th}}^2 - \omega^2]\frac{N_{0i}}{N_0} \cong \vec{\nabla}\cdot[\tfrac{1}{4}N_{0i}\vec{\nabla}(\vec{V}_{i1}\cdot\vec{V}_{i2}^*)] + \frac{\omega_{Pi}^2}{D_{\gamma1}}\frac{N_{0i}}{N_0}\vec{\nabla}\cdot[\tfrac{1}{4}N_{0i}\vec{\nabla}(\vec{V}_{e1}\cdot\vec{V}_{e2}^*)], \quad (43)$$

where

$$D_{\gamma1} \equiv [-\omega^2 + (\Delta k)^2 v_{\text{th}}^2 + \omega_{Pe}^2 N_{0e}/N_0]. \quad (44)$$

We assume the solution of Eq. (44) to be of the form[4-6]

$$N_{i1} = N'_{i1}(x,y,z,t)\exp[i(\omega t - kz)] + N'_{i2}(x,y,z,t)\exp[i(\omega t - \Delta kz)], \quad (45)$$

where $k \cong \omega/C_s$, and N'_{i1} and N'_{i2} are slowly varying complex functions of space and time. Substituting for N_{i1} from Eq. (45), we obtain the following equation for N'_{i1} and N'_{i2}:

$$(-\omega^2 + k^2 V_{\text{th}}^2)N'_{i1} - V_{\text{th}}^2\left(-2ik\frac{d}{dz}N'_{i1} + \frac{\partial^2}{\partial x^2}N'_{i1} + \frac{\partial^2}{\partial y^2}N'_{i1}\right) + 2i\omega\Gamma_i N'_{i1} + 2(\Gamma_i + i\omega)\frac{\partial}{\partial t}N'_{i1} + \frac{\omega_{Pi}^2}{D_{\gamma1}}[(\Delta k)^2 v_{\text{th}}^2 - \omega^2]\frac{N_{0i}}{N_0}N'_{i1} \cong 0, \quad (46)$$

and

$$[-\omega^2+(\Delta k)^2V_{th}^2]N'_{i2}-V_{th}^2\left(-2i(\Delta k)\frac{\partial}{\partial z}N'_{i2}+\frac{\partial^2}{\partial x^2}N'_{i2}+\frac{\partial^2}{\partial y^2}N'_{i2}\right)+2i\omega\Gamma_i N'_{i2}+2(\Gamma_i+i\omega)\frac{\partial}{\partial t}N'_{i2}$$
$$+\frac{\omega_{Pi}^2}{D_{\gamma 1}}[(\Delta k)^2v_{th}^2-\omega^2]\frac{N_{0i}}{N_0}N'_{i2}=-\frac{N_{0i}}{4}\frac{\left\{e^2E_{100}E_{200}\exp\left[-\frac{t^2}{2}\left(\frac{1}{t_{10}^2}+\frac{1}{t_{20}^2}\right)\right]\right\}}{\omega_1\omega_2 f_1 f_2}\exp\left[-\frac{\gamma^2}{2}\left(\frac{1}{\gamma_{10}^2f_1^2}+\frac{1}{\gamma_{20}^2f_2^2}\right)\right]$$
$$\times\left((\Delta k)^2+\frac{2}{\gamma_{10}^2f_1^2}+\frac{2}{\gamma_{20}^2f_2^2}\right)\left(\frac{1}{m_i^2}+\frac{\omega_P^2}{D_{\gamma 1}}\frac{N_{0i}}{N_0}\frac{1}{m_e^2}\right). \tag{47}$$

Substituting further for N'_{i1} as[4-6]

$$N'_{i1}=N''_{i1}(x,y,z,t)\exp[-ikS(x,y,z,t)]\,. \tag{48}$$

In Eq. (46) we obtain, after separating real and imaginary parts,

$$\frac{\partial}{\partial z}N''^2_{i1}+\frac{1}{v'_g}\frac{\partial}{\partial t}N''^2_{i1}+\frac{\partial}{\partial x}S\frac{\partial}{\partial x}N''^2_{i1}+\frac{\partial}{\partial y}S\frac{\partial}{\partial y}N''^2_{i1}+\left(\frac{\partial^2}{\partial x^2}S+\frac{\partial^2}{\partial y^2}S\right)N''^2_{i1}+\frac{2\Gamma_i\omega N''^2_{i1}}{kV_{th}^2}=0\,, \tag{49}$$

and

$$2\frac{\partial S}{\partial z}+\frac{2}{v'_g}\frac{\partial S}{\partial t}+\left(\frac{\partial S}{\partial x}\right)^2+\left(\frac{\partial S}{\partial y}\right)^2=\frac{1}{k^2N''_{i1}}\left(\frac{\partial^2}{\partial x^2}N''_{i1}+\frac{\partial^2}{\partial y^2}N''_{i1}\right)+\left(\frac{\omega^2}{k^2V_{th}^2}-1\right)-\frac{\omega_{Pi}^2}{k^2V_{th}^2D_{\gamma 1}}[(\Delta k)^2v_{th}^2-\omega^2]\frac{N_{0i}}{N_0}\,, \tag{50}$$

where

$$v'_g=\frac{ikV_{th}^2}{(\Gamma_i+i\omega)}\cong\frac{kV_{th}^2}{\omega}\,. \tag{51}$$

We now make the transformation of variables from the set (z,t) to $(z,\xi'\equiv t-z/v'_g)$. Then Eqs. (49) and (50) reduce to

$$\frac{\partial}{\partial z}N''^2_{i1}+\frac{\partial S}{\partial x}\frac{\partial}{\partial x}N''^2_{i1}+\frac{\partial S}{\partial y}\frac{\partial}{\partial y}N''^2_{i1}+\left(\frac{\partial^2 S}{\partial x^2}+\frac{\partial^2 S}{\partial y^2}\right)N''^2_{i1}+\frac{2\Gamma_i\omega N''^2_{i1}}{kV_{th}^2}=0 \tag{52}$$

and

$$2\frac{\partial S}{\partial z}+\left(\frac{\partial S}{\partial x}\right)^2+\left(\frac{\partial S}{\partial y}\right)^2=\frac{1}{k^2N''_{i1}}\left(\frac{\partial^2}{\partial x^2}N''_{i1}+\frac{\partial^2}{\partial y^2}N''_{i1}\right)+\left(\frac{\omega^2}{k^2V_{th}^2}-1\right)-\frac{\omega_{Pi}^2}{k^2V_{th}^2D_{\gamma 1}}[(\Delta k)^2V_{th}^2-\omega^2]\frac{N_{0i}}{N_0}\,. \tag{53}$$

The solution of Eqs. (52) and (53) is given by[4-6,9]

$$N''^2_{i1}=\frac{B'^2}{f^2(z,\xi')}\exp\left(-\frac{\gamma^2}{a_0^2f^2(z,\xi')}\right)\exp(-2k_iz)\,, \tag{54}$$

$$S(z,\xi')=\tfrac{1}{2}\gamma^2\beta(z,\xi')+\Phi(z,\xi')\,, \tag{55}$$

$$\beta(z,\xi')=\frac{1}{f(z,\xi')}\frac{d}{dz}f\,, \tag{56}$$

$$k_i=\frac{2\Gamma_i\omega}{kV_{th}^2}\,, \tag{57}$$

where B' and a_0 are the unknowns to be evaluated from the boundary condition. The equation for the beam-width parameter of the generated ion-acoustic pulse is obtained from Eq. (53) by substituting for S and N''_{i1} from Eqs. (55) and (54) and equating the coefficients of γ^2 on both sides in the resulting equation:

$$\frac{d^2}{dz^2}f=\frac{1}{k^2a_0^4f^3}-\left[\frac{\omega_{Pi}^2[(\Delta k)^2V_{th}^2-\omega^2]}{k^2V_{th}^2D_{\gamma 1}}f\left(\frac{e^2}{16\pi m_em_iV_{th}^2}\right)\left(\frac{\gamma_{10}^2I_3}{\omega_1^2}+\frac{\gamma_{20}^2I_4}{\omega_2^2}\right)\right]\times\exp\left[-\left(\frac{e^2}{16\pi m_em_iV_{th}^2}\right)\left(\frac{\gamma_{10}^2I'_3}{\omega_1^2}+\frac{\gamma_{20}^2I'_4}{\omega_2^2}\right)\right], \tag{58}$$

where $I_{3,4}$ and $I'_{3,4}$ are given by Eqs. (29) and (30), respectively.

Substituting for N'_{i2} as (Refs. 4–6, and 9)

$$N'_{i2}=N''_{i2}(x,y,z,t)\times\exp\{-i[k_1S_1(x,y,z,t)-k_2S_2(x,y,z,t)]\} \tag{59}$$

in Eq. (47), we obtain

$$N''_{i2}=-\frac{N_{0i}}{4D_{\gamma 2}}\left(\frac{1}{m_i^2}+\frac{\omega_{Pi}^2}{D_{\gamma 1}}\frac{N_{0i}}{N_0}\frac{1}{m_e^2}\right)\times\left(\frac{e^2E_{10}(t)E_{20}(t)}{\omega_1\omega_2f_1f_2}\right)\left((\Delta k)^2+\frac{2}{\gamma_{10}^2f_1^2}+\frac{2}{\gamma_{20}^2f_2^2}\right)\times\exp\left[-\frac{\gamma^2}{2}\left(\frac{1}{\gamma_{10}^2f_1^2}+\frac{1}{\gamma_{20}^2f_2^2}\right)\right], \tag{60}$$

where

$$D_{\gamma 2}=-\omega^2+(\Delta k)^2V_{\text{th}}^2+(\omega_{Pi}^2/D_{\gamma 1})[(\Delta k)^2v_{\text{th}}^2-\omega^2][N_{0i}/N_0]. \quad (61)$$

To find an expression for the electric vector of the ion-acoustic pulse, we employ the equation of continuity,

$$\partial N_{i1}/\partial t+\vec{\nabla}\cdot(N_{0i}\vec{V}_{i1})=0, \quad (62)$$

where $\vec{V}_{i1}\cong e\vec{E}_L/m_i i\omega$ and N_{i1} is given by Eq. (45). Thus,

$$E_L=\frac{m_i i\omega^2}{eN_{0i}}\left(\frac{N'_{i1}}{k}\exp(-ikz)+\frac{N'_{i2}}{\Delta k}\exp(-i\Delta kz)\right), \quad (63)$$

where

$$N'_{i1}=\frac{B'}{f(z,\xi')}\exp\left(-\frac{\gamma^2}{2a_0^2f^2}-k_iz-ikS\right) \quad (64)$$

and

$$N'_{i2}=-\frac{N_{0i}}{4D_{\gamma 2}}\left(\frac{1}{m_i^2}+\frac{\omega_{Pi}^2}{D_{\gamma 1}}\frac{N_{0i}}{N_0}\frac{1}{m_e^2}\right)\times\left((\Delta k)^2+\frac{2}{\gamma_{10}^2f_1^2}+\frac{2}{\gamma_{20}^2f_2^2}\right)\left(\frac{e^2E_{10}(t)E_{20}(t)}{\omega_1\omega_2f_1f_2}\right)\times\exp\left[-\frac{\gamma^2}{2}\left(\frac{1}{\gamma_{10}^2f_1^2}+\frac{1}{\gamma_{20}^2f_2^2}\right)\right]. \quad (65)$$

Hence,

$$E_L\cong i\left(\frac{m_i\omega^2}{eN_{0i}}\right)\left\{F_1\exp\left(-\frac{\gamma^2}{2a_0^2f^2}\right)\exp[-ik(z+S)]+F_2\exp\left[-\frac{\gamma^2}{2}\left(\frac{1}{\gamma_{10}^2f_1^2}+\frac{1}{\gamma_{20}^2f_2^2}\right)\right]\times\exp[-i(\Delta kz+k_1S_1-k_2S_2)]\right\}, \quad (66)$$

where

$$F_1=\frac{B'\exp(-k_iz)}{kf(z,\xi')}, \quad (67)$$

and

$$F_2=-\frac{N_{0i}}{4(\Delta k)D_{\gamma 2}}\left(\frac{1}{m_i^2}+\frac{\omega_{Pi}^2}{D_{\gamma 1}}\frac{N_{0i}}{N_0}\frac{1}{m_e^2}\right)\times\left((\Delta k)^2+\frac{2}{\gamma_{10}^2f_1^2}+\frac{2}{\gamma_{20}^2f_2^2}\right)\times\left(\frac{e^2E_{10}(t)E_{20}(t)}{\omega_1\omega_2f_1f_2}\right). \quad (68)$$

In the expression for N'_{i1} [Eq. (64)], B' and a_0 are two unknowns and have been obtained from the assumption that the electric field of the generated ion-acoustic pulse is zero at $z=0$. Thus,

$$E_L=0 \quad \text{at } z=0, \quad (69)$$

which yields

$$\frac{1}{a_0^2}=\frac{1}{\gamma_{10}^2}+\frac{1}{\gamma_{20}^2}$$

and (70)

$$B'=\frac{N_{0i}kf(0,\xi')}{4(\Delta k)D_{\gamma 2}}\left(\frac{1}{m_i^2}+\frac{\omega_{Pi}^2}{D_{\gamma 1}}\frac{N_{0i}}{N_0}\frac{1}{m_e^2}\right)\left((\Delta k)^2+\frac{2}{\gamma_{10}^2f_1^2}+\frac{2}{\gamma_{20}^2f_2^2}\right)\left(\frac{e^2E_{10}(t)E_{20}(t)}{\omega_1\omega_2f_1f_2}\right)\times\exp(ikS)\exp\left[-\frac{\gamma^2}{2}\left(\frac{1}{\gamma_{10}^2f_1^2}+\frac{1}{\gamma_{20}^2f_2^2}-\frac{1}{a_0^2f^2}\right)\right], \quad (71)$$

Therefore,

$$E_LE_L^*(t)=\left(\frac{m_i\omega^2}{eN_{0i}}\right)^2\left\{F_1^2\exp\left(-\frac{\gamma^2}{a_0^2f^2}\right)+F_2^2\exp\left[-\gamma^2\left(\frac{1}{\gamma_{10}^2f_1^2}+\frac{1}{\gamma_{20}^2f_2^2}\right)\right]+2\cos[k(z+S)-\Delta kz-(k_1S_1-k_2S_2)]\times F_1F_2\exp\left[-\frac{\gamma^2}{2}\left(\frac{1}{a_0^2f^2}+\frac{1}{\gamma_{10}^2f_1^2}+\frac{1}{\gamma_{20}^2f_2^2}\right)\right]\right\}. \quad (72)$$

This is the intensity of the generated ion-acoustic pulse at the difference frequency, when the two EM pulses are having Gaussian intensity distribution in space and time.

Hence, the total integrated power associated with the generated ion-acoustic pulse across the transverse cross section at z is given by

$$P=\frac{V_{\text{th}}}{16}\left(\frac{m_i\omega^2}{eN_{0i}}\right)^2\left[a_0^2f^2F_1^2+\frac{\gamma_{10}^2\gamma_{20}^2f_1^2f_2^2}{\gamma_{10}^2f_1^2+\gamma_{20}^2f_2^2}F_2^2+\frac{4F_1F_2\cos(kz-\Delta kz)}{\left(\frac{1}{a_0^2f^2}+\frac{1}{\gamma_{10}^2f_1^2}+\frac{1}{\gamma_{20}^2f_2^2}\right)}\right], \quad (73)$$

where N_{0i} is given by Eq. (14) and F_1 and F_2 are given by Eqs. (67) and (68).

V. DISCUSSION

To have more physical insight into the present mechanism of the generation of ion-acoustic pulse by two EM pulses at the difference frequency, we concentrate our attention on Eqs. (63)–(70). In Eq. (63) the first term within the bracket is on account of the natural mode of the hot plasma at the difference frequency, and the second term arises on account of the ponderomotive force at the difference frequency. The term which is due to the hot plasma can also be Landau-damped when the phase velocity ω/k is comparable to the thermal velocity of the particles, and under such conditions

$$E_L E_L^*(t) \cong F_2^2\left(\frac{m_i\omega^2}{eN_{0i}}\right)^2 \exp\left[-\gamma^2\left(\frac{1}{\gamma_{10}^2 f_1^2}+\frac{1}{\gamma_{20}^2 f_2^2}\right)\right], \tag{74}$$

where F_2 is defined in Eq. (68).

Moreover, if the incident EM pulses have a uniform intensity distribution in space ($\gamma_{10}, \gamma_{20}=\infty$, $f_1, f_2=1$ and $N_{0e}/N_0=N_{0i}/N_0=1$), the intensity of the generated ion-acoustic pulse at the difference frequency is

$$E_L E_L^*(t) = I_0 \exp[-t^2(1/t_{10}^2+1/t_{20}^2)], \tag{75}$$

where

$$I_0=\left[\left(\frac{m_i\omega^2}{eN_0}\right)\left(\frac{N_0\Delta k}{4D_{\gamma 2}}\right)\left(\frac{1}{m_i^2}+\frac{\omega_{Pi}^2}{D_{\gamma 1}}\frac{1}{m_e^2}\right)\times\left(\frac{e^2E_{100}E_{200}}{\omega_1\omega_2}\right)\right]^2,$$

$$D_{\gamma 1}=[-\omega^2+(\Delta k)^2 v_{\rm th}^2+\omega_{Pe}^2], \tag{76}$$

$$D_{\gamma 2}=\{-\omega^2+(\Delta k)^2 V_{\rm th}^2+(\omega_{Pi}^2/D_{\gamma 1})[(\Delta k)^2 v_{\rm th}^2-\omega^2]\},$$

$$\Delta k=\left[\frac{\omega_1}{c}\left(1-\frac{\omega_P^2}{\omega_1^2}\right)^{1/2}-\frac{\omega_2}{c}\left(1-\frac{\omega_P^2}{\omega_2^2}\right)^{1/2}\right].$$

It is obvious from the expression of the intensity of the generated pulse (Eq. 75) that the intensity has the Gaussian intensity distribution in time with a pulse width equal to

$$[t_{10}^2 t_{20}^2/(t_{10}^2+t_{20}^2)]^{1/2}.$$

The maximum intensity I_0 is a function of the density of the plasma, frequencies of the incident EM pulses, and the temperature of the plasma. But for the propagating pulses [$\omega_p \ll \omega_1$, ω_2, $\Delta\omega$] and for the nonrelativistic plasma ($v_{\rm th} \ll c$)

$$I_0 \cong \left(\frac{1}{4c}\frac{\omega_{Pe}^2}{\omega_1-\omega_2}\frac{eE_{100}E_{200}}{m_e\omega_1\omega_2}\right)^2. \tag{77}$$

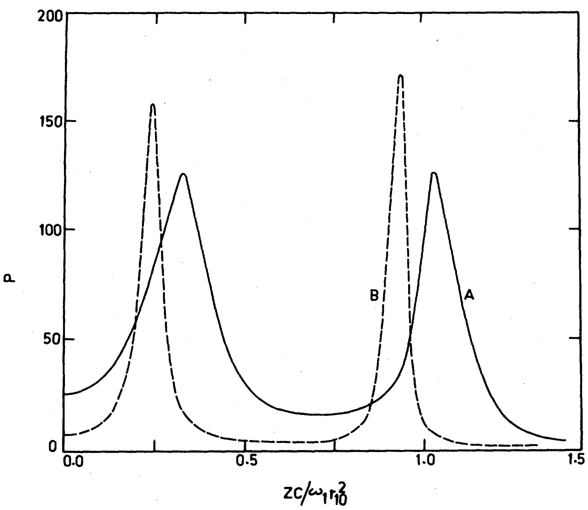

FIG. 2. Variation of the power P (in cgs units) of the generated ion-acoustic pulse as a function of $zc/\omega_1\gamma_{10}^2$ for the parameters as in Fig. 1. Curve A represents $t/t_{10}=0.0$. Curve B represents $t/t_{20}=1.0$.

For such a plasma the peak intensity is proportional to the square of the electron and ion density, $(E_{100}E_{200})^2$ and inversely proportional to $[\omega_1\omega_2(\omega_1-\omega_2)]^2$. Moreover, for a fixed ratio of ω_1 and ω_2, the peak intensity is inversely proportional to ω_1^3. Therefore, if the EM pulses are having their frequencies in the microwave range, the peak intensity will drastically increase in comparison to that for the laser pulses. For example, for two CO_2-laser pulses ($\omega_1=1.963\times 10^{14}$ rad sec^{-1} and $\omega_2=1.778\times 10^{14}$ rad sec^{-1}), the peak intensity is 10^9 times less in comparison to when we choose $\omega_1=1.963\times 10^{11}$ rad sec^{-1} and $\omega_2=1.778\times 10^{11}$ rad sec^{-1}.

When the incident EM pulses also have Gaussian intensity distributions in space, the transient cross focusing of the two pulses may occur, and the power of the generated pulse gets drastically affected with the distance of propagation. The results of calculations are depicted in Figs. 1 and 2. The calculations have been made for the following set of parameters: $\omega_1=1.963\times 10^{14}$ rad sec^{-1}, $\omega_2=1.778\times 10^{14}$ rad sec^{-1} (CO_2-laser pulses), $t_{10}/t_{20}=0.5$, $k_BT_0=3$ keV, $\gamma_{10}=1.9\times 10^{-2}$ cm, $\gamma_{10}/\gamma_{20}=2$, peak power fluxes of the two lasers are $\cong 4.8\times 10^{10}$ watt/cm^2 and $\cong 3.5\times 10^{10}$ watt/cm^2, and $\omega_P/\omega=0.5$.

Figure 1 depicts the variation of the dimensionless beam-width parameters f_1 and f_2 of the pump pulses with the normalized distance of propagation at different times. It is observed that two first pulse exhibits oscillatory focusing while the second pulse exhibits focusing and defocusing with the distance of propagation. As the time elapses the intensities of the two incident EM pulse changes and hence the cross focusing also gets affected. Therefore, the power of the generated pulse also exhibits maxima and minima with the distance of propagation, as is depicted in Fig. 2.

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