Photon statistics in multiphoton absorption and emission processes

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Exact solutions of the master equations which describe single-mode m-photon absorption and m-photon emission processes in a two-level atomic system are obtained using a density-matrix approach. Some aspects of the photon statistics, e.g., antibunching, are discussed.

I. INTRODUCTION

In recent years, the study of the photon statistics in multiphoton processes has received a great deal of interest.[1-9] Several authors have discussed the possibility of observing antibunching in multiphoton absorption process.[7,9]

Up to now exact expressions have only been derived for the photon distribution function in a two-photon absorption process using a generating function method and in a two-photon emission process using a Laplace transform method. The generating function approach is rather inconvenient in the study of photon statistics of m-photon processes when $m>2$.

In this paper, we present exact solutions of the master equations which describe m-photon absorption and emission processes, using a density-matrix method. The diagonal elements of the density matrix determine the photon distribution function. By taking the appropriate moments with respect to the distribution function in the m-photon absorption process, we evaluate the photon number fluctuations. With the help of the figures we demonstrate that, for initial coherent and chaotic fields, the fluctuations decrease below unity when $m \geq 2$. This is a manifestation of photon antibunching.

II. MASTER EQUATION FOR THE DENSITY MATRIX OF THE FIELD

Let us consider a coupled system of field and N noninteracting two-level atoms. We assume that the atoms make transitions from lower (upper) level to the upper (lower) level by absorbing (emitting) m photons, one photon in each of the modes $k_1, k_2 \ldots k_m$. The number of atoms in the two levels are assumed to be maintained constant by some external influence. The interaction Hamiltonian for this process is expressed in the form[1]

$$H_I=\sum_{i=1}^{N}\{\eta_m \hat{C}_{2i}^{\dagger}\hat{C}_{1i}\hat{E}_{k_1}^{\dagger}(\vec{\mathbf{r}}_i)\cdots\hat{E}_{k_m}^{\dagger}(\vec{\mathbf{r}}_j)+\text{H.c.}\}, \quad (1)$$

where $\hat{C}_{1i}$, $\hat{C}_{2i}$, $\hat{C}_{1i}^{\dagger}$, and $\hat{C}_{2i}^{\dagger}$ are the annihilation and creation operators for the ith atom in the ground state $|1_i\rangle$ and the excited state $|2_i\rangle$. η_m is the matrix element for the m-photon transition. The positive-frequency part of the electric field at the ith atom is given by

$$\hat{E}_{k_i}^{\dagger}(\vec{\mathbf{r}}_j)=-i(2\pi\hbar\omega_{k_i})^{1/2}U_{k_i}(\vec{\mathbf{r}}_j)\hat{a}_{k_i}, \quad (2)$$

where ω_{k_i} is the frequency of the k_ith mode of the field (with atomic transition frequency $\omega=\omega_{k_1}+\ldots+\omega_{k_m}$), $U_{k_i}(\vec{\mathbf{r}}_j)$ is the mode function of the k_ith mode at the position $\vec{\mathbf{r}}_j$, and $\hat{a}_{k_i}$ is the photon destruction operator of the k_ith mode.

In the interaction picture, the equation of motion for the density operator $\hat{\rho}$ of the atom-field system is given by

$$i\hbar\frac{\partial}{\partial t}\hat{\rho}=[\hat{H}_I,\hat{\rho}]. \quad (3)$$

We assume that at some initial time t_0, the radiation field and the atomic system are decoupled; i.e.,

$$\hat{\rho}(t_0)=\hat{\rho}_F(t_0)\otimes\prod_{i=1}^{N}\hat{\rho}_i(t_0), \quad (4)$$

where $\hat{\rho}_F(t_0)=\mathrm{Tr}_A\hat{\rho}(t_0)$ is the reduced density matrix of the field and $\hat{\rho}_i(t_0)$ is the initial density operator of the ith atom at time t_0.

The equation of motion for $\hat{\rho}_F(t)$ may be obtained using the standard perturbation techniques based on Born and Markoff approximations. The resulting equation is[4]

$$\frac{d\hat{\rho}_F}{dt} = -\beta^{(m)}K_1\left[\left(\prod_{i=1}^{m}\hat{a}_{k_i}^{\dagger}\right)\left(\prod_{j=1}^{m}\hat{a}_{k_j}\right)\hat{\rho}_F - 2\left(\prod_{i=1}^{m}\hat{a}_{k_i}\right)\hat{\rho}_F\left(\prod_{j=1}^{m}\hat{a}_{k_j}^{\dagger}\right) + \hat{\rho}_F\left(\prod_{i=1}^{m}\hat{a}_{k_i}^{\dagger}\right)\left(\prod_{j=1}^{m}\hat{a}_{k_j}\right)\right]$$

$$-\beta^{(m)}K_2\left[\left(\prod_{i=1}^{m}\hat{a}_{k_i}\right)\left(\prod_{j=1}^{m}\hat{a}_{k_j}^{\dagger}\right)\hat{\rho}_F - 2\left(\prod_{i=1}^{m}\hat{a}_{k_i}^{\dagger}\right)\hat{\rho}_F\left(\prod_{j=1}^{m}\hat{a}_{k_j}\right) + \hat{\rho}_F\left(\prod_{i=1}^{m}\hat{a}_{k_i}\right)\left(\prod_{j=1}^{m}\hat{a}_{k_j}^{\dagger}\right)\right], \tag{5}$$

where K_1 and K_2 are the thermal populations of the atomic states $|1\rangle$ and $|2\rangle$; $\beta^{(m)}$ is given by

$$\beta^{(m)} = (2\pi\hbar)^{m-2}(2\pi^2)\left(\prod_{i=1}^{m}\omega_{k_i}\right)|\eta_m|^2 g(\omega)$$

$$\times\int d\vec{r}N(\vec{r})\left(\prod_{i=1}^{m}|U_{k_i}(\vec{r})|^2\right). \tag{6}$$

In Eq. (6) $g(\omega)$ is the line-shape function of the atoms and $N(\vec{r})$ is the atomic density in the medium.

Although the method for the solution of the reduced density matrix of the field in m-photon absorption and emission processes in the following sections can be applied for arbitrary numbers of field modes, we shall restrict ourselves to a single-mode case (with $k_1 = k_2 = \cdots = k_m$) for the sake of mathematical simplicity. Then Eq. (5) becomes

$$\frac{d\hat{\rho}_F}{dt} = -\beta^{(m)}K_1\{\hat{a}^{\dagger m}\hat{a}^m\hat{\rho}_F - 2\hat{a}^m\hat{\rho}_F\hat{a}^{\dagger m} + \hat{\rho}_F\hat{a}^{\dagger m}\hat{a}^m\}$$

$$-\beta^{(m)}K_2\{\hat{a}^m\hat{a}^{\dagger m}\hat{\rho}_F - 2\hat{a}^{\dagger m}\hat{\rho}_F\hat{a}^m + \hat{\rho}_F\hat{a}^m\hat{a}^{\dagger m}\}. \tag{7}$$

It is worthwhile to mention that this equation describes an m-photon laser[10] in linear approximation when the cavity losses are simulated by m-photon absorption process to achieve detailed balance. We shall, however, restrict ourselves in the next sections to m-photon absorption case ($k_1 = 1, k_2 = 0$) and m-photon emission case ($k_1 = 0$, $k_2 = 1$) separately.

III. m-PHOTON ABSORPTION

If we calculate the matrix element of each term in Eq. (7) between Fock states $\langle n|$ and $|n+k\rangle$ ($k = 0,1,2,\ldots$), the matrix element $\rho(n,n+k,\tau) = \langle n|\rho_F(\tau)|n+k\rangle$ of the reduced density matrix ρ_F satisfies an equation of motion of the form (for $K_1 = 1, K_2 = 0$):

$$\partial\rho(n,n+k,\tau)/\partial\tau = a(n,n+k)\rho(n,n+k,\tau)$$

$$+b(n+m,n+k+m)$$

$$\times\rho(n+m,n+k+m,\tau), \tag{8}$$

where $\tau = 2\beta^{(m)}t$ and

$$a(n,n+k) = -\frac{1}{2}\left(\frac{n!}{(n-m)!} + \frac{(n+k)!}{(n+k-m)!}\right), \tag{9a}$$

$$b(n,n+k) = \left(\frac{n!(n+k)!}{(n-m)!(n-m+k)!}\right)^{1/2}. \tag{9b}$$

The matrix element $\rho(n+k,n,\tau)$ is just the complex conjugate of $\rho(n,n+k,\tau)$; i.e.,

$$\rho(n+k,n,\tau) = \rho^*(n,n+k,\tau). \tag{10}$$

The solution of Eq. (8) would, therefore, completely determine the reduced density matrix $\hat{\rho}_F$ of the field.

It is apparent from Eq. (8) that an element $\rho(n+i,k+n+i,\tau)$ ($i = 0,1,\ldots,m-1$) is related only to the elements $\rho(n+i,k+n+i,\tau)$, $\rho(n+i+m,k+n+i+m,\tau)$, and $\rho(n+i+2m,k+n+i+2m,\tau)\cdots$ as shown in Fig. 1. We can therefore replace Eq. (8) by the following set of m independent equations corresponding to m chains in Fig. 1:

$$\partial\rho(nm+i,nm+k+i,\tau)/\partial\tau = a(nm+i,nm+k+i)\rho(nm+i,nm+k+i,\tau)$$

$$+b(nm+m+i,nm+m+k+i)\rho(nm+m+i,nm+m+k+i,\tau), \tag{11}$$

with $i = 0,1,\ldots,m-1$. In matrix notation these equations are written as

$$\frac{\partial}{\partial\tau}\rho_i(\tau) = M_i\rho_i(\tau), \tag{12}$$

where

$$\rho_i(\tau)=\begin{bmatrix}\rho(i,k+i,\tau)\\ \rho(m+i,m+k+i,\tau)\\ \vdots\\ \rho(nm+i,nm+k+i,\tau)\\ \vdots\end{bmatrix}, \tag{13}$$

$$M_i=\begin{bmatrix}a(i,k+i) & b(m+i,m+k+i) & 0 & \\ & a(m+i,m+k+i) & b(2m+i,2m+k+i) & \\ & \ddots & \ddots & \\ 0 & & & \end{bmatrix}. \tag{14}$$

We now define the eigenvalue $\lambda_{l,i}$ of the matrix corresponding to the right eigenstate

$$A_{l,i}=\begin{bmatrix}\alpha_0^{l,i}\\ \alpha_1^{l,i}\\ \vdots\end{bmatrix} \tag{15}$$

and the left eigenstate

$$B_{l,i}=(\beta_0^{l,i}\beta_1^{l,i}\cdots), \tag{16}$$

by the following relations:

$$M_iA_{l,i}=\lambda_{l,i}A_{l,i}, \tag{17}$$

$$B_{l,i}M_i=\lambda_{l,i}B_{l,i}. \tag{18}$$

If we multiply Eq. (17) by $B_{l',i}$ from the left and Eq. (18) for l' by $A_{l,i}$ from the right and subtract the two equations, we get an orthogonality relation:

$$B_{l',i}A_{l,i}=\delta_{ll'}, \tag{19}$$

when $\lambda_{l,i}\neq\lambda_{l',i}$.

In view of Eq. (17), the solution of Eq. (12) is given by

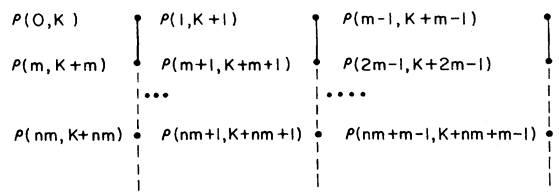

FIG. 1. The chain structure of Eq. (8).

$$\rho_i(\tau)=\sum_{l=0}^{\infty}C_lA_{l,i}e^{\lambda_{l,i}\tau}. \tag{20}$$

In case the eigenvalues are nondegenerate, the coefficient C_l can be determined using the orthogonality condition (16), and we obtain

$$C_l=B_{l,i}\rho_i(0). \tag{21}$$

Equations (20) and (21) can be recast in the functional form by the following equation:

$$\rho(mn+i,mn+k+i,\tau)=\sum_{l=0}^{\infty}\sum_{q=0}^{\infty}\beta_q^{l,i}\alpha_n^{l,i}e^{\lambda_{l,i}\tau}\rho(mq+i,mq+k+i,0). \tag{22}$$

It is therefore evident that a determination of the eigenvalues $\lambda_{l,i}$ and the right and left eigenstates $A_{l,i}$, $B_{l,i}$ of the matrix M_i would completely determine the reduced density matrix $\hat{\rho}_F$.

The eigenvalues $\lambda_{l,i}$ of M_i, which satisfy the equation

$$\det(M_i-\lambda I)=0 \tag{23}$$

(I being the unit matrix), are given by

$$\lambda_{l,i}=a(ml+i,ml+k+i)=-\frac{1}{2}\left(\frac{(ml+i)!}{(ml+i-m)!}+\frac{(ml+k+i)!}{(ml+k+i-m)!}\right). \tag{24}$$

It is easy to verify that $\lambda_{l,i}\neq\lambda_{l',i}$ for $l\neq l'$. On substituting from Eq. (24) into Eq. (17), we get an infinite set of equations whose nth member is the following recursion relation:

$$a(mn-m+i, mn-m+k+i)\alpha_{n-1}^{l,i}+b(mn+i, mn+k+i)\alpha_n^{l,i}=a(ml+i, ml+k+i)\alpha_{n-1}^{l,i}. \tag{25}$$

By iterating this recursion relation, we obtain

$$\alpha_n^{l,i}=\begin{cases}\alpha_0^{l,i}\prod_{r=1}^{n}\dfrac{[a(ml+i, ml+k+i)-a(mr-m+i, mr-m+k+i)]}{b(mr+i, mr+k+i)}, & n\leq l\\ 0, & n>l.\end{cases} \tag{26}$$

In a similar manner, it follows from Eqs. (18) and (24) that

$$\beta_q^{l,i}=\begin{cases}\beta_l^{l,i}\prod_{r=l+1}^{q}\dfrac{b(mr+i, mr+k+i)}{[a(ml+i, ml+k+i)-a(mr+i, mr+k+i)]}, & q\geq l\\ 0, & q<l.\end{cases} \tag{27}$$

To determine $\alpha_0^{l,i}$ and $\beta_l^{l,i}$ we first notice that, in view of Eqs. (15), (16), and (19), the following relationship holds:

$$B_{l,i}A_{l,i}=\beta_l^{l,i}\alpha_l^{l,i}=1. \tag{28}$$

We can therefore choose

$$\alpha_l^{l,i}=1, \tag{29a}$$

$$\beta_l^{l,i}=1. \tag{29b}$$

It then follows from Eqs. (26) and (29a) that

$$\alpha_0^{l,i}=\prod_{r=1}^{l}\frac{b(mr+i, mr+k+i)}{[a(ml+i, ml+k+i)-a(mr-m+i, mr-m+k+i)]}. \tag{30}$$

The choices (29b) and (30) enable us to combine the m equation in Eq. (22) corresponding to m values of i (i.e., $i=0, 1, \ldots, m-1$).

On substituting from Eqs. (24), (26), (27), (29b), and (30) into Eq. (22), we obtain, after some rearrangement,

$$\rho(n, n', \tau)=\sum_{l=n}^{q}{}'\sum_{q=l}^{\infty}{}'\frac{\left(\prod_{r=n+m}^{q}{}' b(r, r+n'-n)\right)e^{a(l,\,l+n'-n)\tau}\rho(q, q+n'-n, 0)}{\left(\prod_{\substack{r=n\\ r\neq l}}^{q}{}' [a(l, l+n'-n)-a(r, r+n'-n)]\right)}, \tag{31}$$

where the primes on the summation and product signs mean that the increment is in steps of m. This equation combined with the expressions of $a(r, r')$ and $b(r, r')$, i.e.,

$$a(r, r')=-\frac{1}{2}\left(\frac{r!}{(r-m)!}+\frac{r'!}{(r'-m)!}\right), \tag{32}$$

$$b(r, r')=\left(\frac{r!\,r'!}{(r-m)!\,(r'-m)!}\right)^{1/2}, \tag{33}$$

completely determines the evolution of the reduced density matrix of the field in the m-photon absorption process.

The photon distribution function $p(n,\tau)$, which is obtained by putting $n=n'$ in Eq. (31), is given by

$$p(n,\tau)=\sum_{l=n}^{\infty}{}'\sum_{q=l}^{\infty}{}'\frac{\left(\prod_{r=n+m}^{q}{}' b(r,r)\right)e^{a(l,\,l)\tau}p(q,0)}{\left(\prod_{r=n,\,r\neq l}^{q}{}' [a(l,l)-a(r,r)]\right)}. \tag{34}$$

It is not difficult to verify that, for the two-photon absorption process (i.e., for $m=2$), this expression for $p(n,\tau)$ reduces identically to the one obtained by Simann and Loudon[7] using the generating function method. In Figs. 2–7, we have plotted $p(n,\tau)$ versus n for various values of τ in two-, three-,

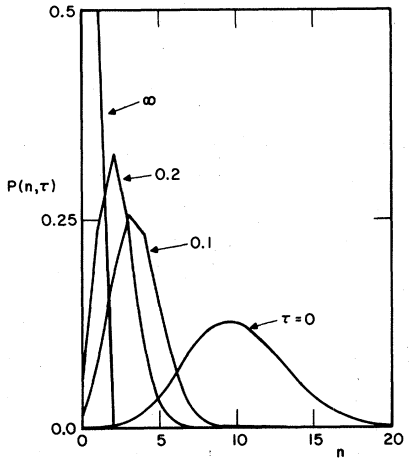

FIG. 2. $p(n,\tau)$ versus n for two-photon absorption process with initial coherent state (with $\bar{n}_0=10$) at $\tau=0$, 0.1, 0.2, and ∞, respectively.

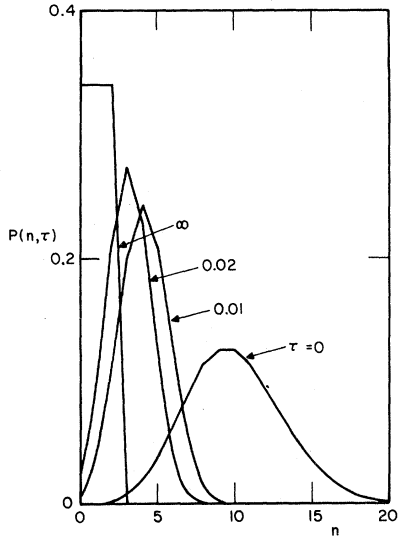

FIG. 3. $p(n,\tau)$ versus n for three-photon absorption process with initial coherent state (with $\bar{n}_0 = 10$) at $\tau = 0$, 0.01, 0.02, and ∞, respectively.

and four-photon processes when the initial photon distributions are coherent and chaotic, i.e.,

$$p_{\mathrm{coh}}(n,0) = \frac{\bar{n}_0{}^n e^{-\bar{n}_0}}{n!}, \tag{35}$$

$$p_{\mathrm{cha}}(n,0) = \frac{\bar{n}_0{}^n}{(1+\bar{n}_0)^{1+n}}. \tag{36}$$

With the expression (34) for $p(n,\tau)$, the expectation values $\langle n^r\rangle$ ($r = 1, 2, \ldots$) for the photon numbers can be evaluated using the formula

$$\langle n^r\rangle = \sum_{n=0}^{\infty} n^r p(n,0). \tag{37}$$

In Figs. 8 and 9, we have plotted the quantity

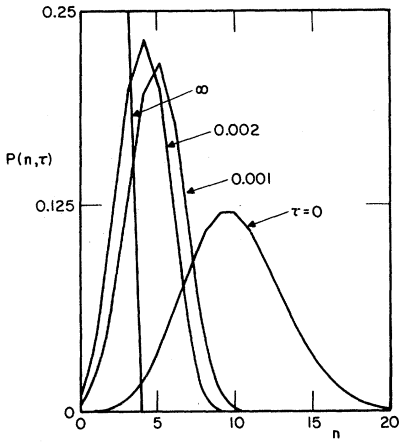

FIG. 4. $p(n,\tau)$ versus n for four-photon absorption process with initial coherent state (with $\bar{n}_0 = 10$) at $\tau = 0$, 0.001, 0.002, and ∞, respectively.

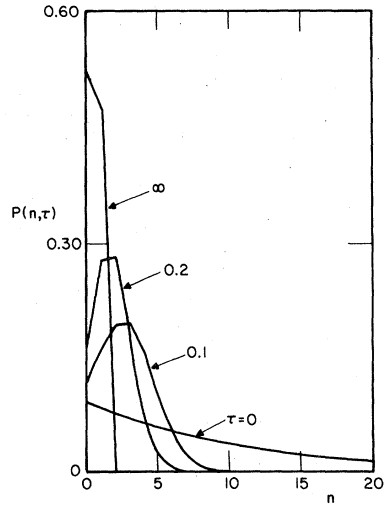

FIG. 5. $p(n,\tau)$ versus n for two-photon absorption process with initial chaotic state (with $\bar{n}_0 = 10$) at $\tau = 0$, 0.1, 0.2, and ∞, respectively.

$$g_2(\tau) = (\langle n^2\rangle - \langle n\rangle^2)/\langle n\rangle \tag{38}$$

versus τ in two-, three-, and four-photon absorption processes for initial coherent and chaotic distributions. (Caution must be used in interpreting the results from these figures because $\tau = 2\beta^{(m)}t$ depends upon m.)

It is evident from the figures that, in many cases, $g_2(\tau) < 1$. This is a manifestation of the photon antibunching. Simaan and Loudon[7] discussed this effect in two-photon absorption process and Paul, Mohr, and Brunner[9] predicted this effect in an arbitrary m-photon absorption process (with $m > 2$) using a perturbative method.

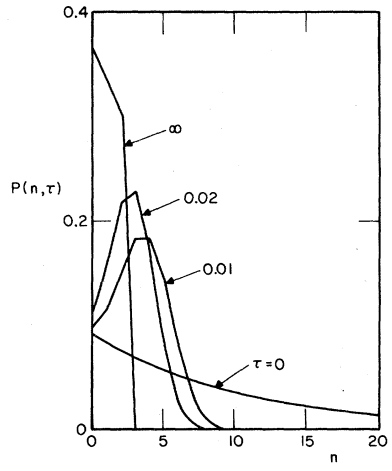

FIG. 6. $p(n,\tau)$ versus n for three-photon absorption process with initial chaotic state (with $\bar{n}_0 = 10$) at $\tau = 0$, 0.01, 0.02, and ∞, respectively.

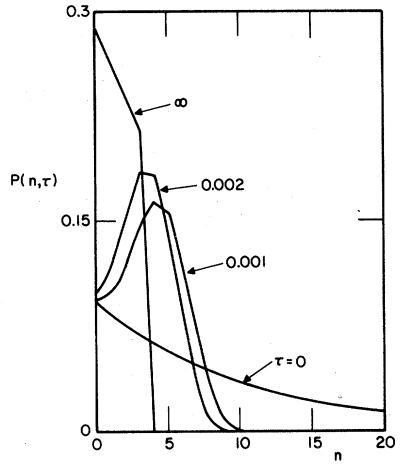

FIG. 7. $p(n,\tau)$ versus n for four-photon absorption process with initial chaotic state (with $\bar{n}_0=10$) at $\tau=0$, 0.001, 0.002, and ∞, respectively.

It follows from Eq. (34) that, in the steady state ($\tau\to\infty$), we obtain

$$p(n,\infty)=\begin{cases}0, & n\geq m\\ \sum_{q=n}^{\infty}{}' \prod_{r=n+m}^{q}{}' \dfrac{b(r,r)}{[a(n,n)-a(r,r)]}, & n<m\,.\end{cases} \tag{39}$$

This expression for $p(n,\infty)$ is according to our expectation that the steady state photon distribution function is nonvanishing only for $n<m$. In Table I we have presented the values of $g_2(\infty)$ and $\langle n\rangle_{\tau=\infty}$ in two-, three-, and four-photon absorption processes with initial coherent and chaotic photon distribution functions.

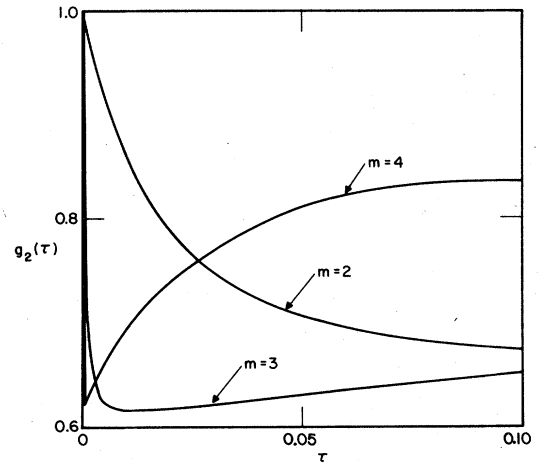

FIG. 8. $g_2(\tau)$ versus τ for two-, three-, and four-photon absorption processes with initial coherent state (with $\bar{n}_0=10$).

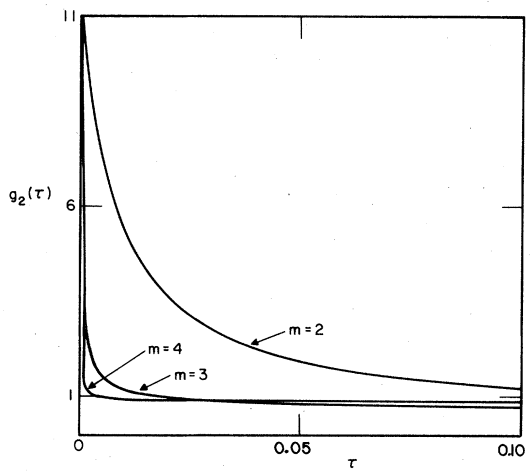

FIG. 9. $g_2(\tau)$ versus τ for two-, three-, and four-photon absorption processes with initial chaotic state (with $\bar{n}_0=10$).

IV. m-PHOTON EMISSION

We now consider the equation of motion of the matrix element $\rho(n,n+k,\tau)$ of the reduced density matrix $\hat{\rho}_F$ in the case of the m-photon emission process which is obtained from Eq. (7) with $K_1=0$ and $K_2=1$. It is given by

$$\begin{aligned}\partial\rho(n,n+k,\tau)/\partial\tau &= c(n,n+k)\rho(n,n+k,\tau)\\ &\quad +d(n-m,n+k-m)\\ &\quad \times\rho(n-m,n+k-m)\,,\end{aligned} \tag{40}$$

where

$$c(n,n+k)=-\frac{1}{2}\left(\frac{(n+m)!}{n!}+\frac{(n+k+m)!}{(n+k)!}\right), \tag{41}$$

$$d(n,n+k)=\left(\frac{n!\,(n+k)!}{(n-m)!\,(n+k-m)!}\right)^{1/2}. \tag{42}$$

Following the method outlined in the previous section, we first replace Eq. (40) by the following set of m independent equations, for $i=0,1,2,\ldots,m-1$:

TABLE I. The value of $g_2(\infty)$ and $\langle n\rangle_{\tau=\infty}$ for several multiphoton absorption processes with coherent and chaotic initial photon distributions.

	Coherent $\bar{n}_0=10$		Chaotic $\bar{n}_0=10$	
	$g_2(\infty)$	$\langle n\rangle_{\tau=\infty}$	$g_2(\infty)$	$\langle n\rangle_{\tau=\infty}$
Two-photon process	0.500	0.5	0.524	0.476
Three-photon process	0.667	1.0	0.709	0.936
Four-photon process	0.833	1.5	0.898	1.381

$$\partial\rho(nm+i,nm+k+i,\tau)/\partial\tau = c(nm+i,nm+k+i)\rho(nm+i,nm+k+i,\tau) + d(nm-m+i,nm-m+k+i)\rho(nm-m+i,nm-m+k+i,\tau)\,. \tag{43}$$

As before, their solution can be expressed as

$$\rho(nm+i,nm+k+i,\tau) = \sum_{l=0}^{\infty}\sum_{q=0}^{\infty} \gamma_q^{\,l,i}\delta_n^{\,l,i} e^{\mu_{l,i}\tau}\rho(mq+i,mq+k+i,0)\,, \tag{44}$$

where $\mu_{l,i}$, $\delta_n^{\,l,i}$, and $\gamma_q^{\,l,i}$ are the eigenvalue, the nth element of the right eigenstate, and the qth element of the left eigenstate, respectively, of the matrix

$$N_i = \begin{bmatrix} c(i,k+i) & & & 0 \\ d(i,k+i) & c(m+i,m+k+i) & & \\ & \ddots & & \\ & & d(m+i,m+k+i) & \\ & & \ddots & \\ 0 & & & \end{bmatrix}. \tag{45}$$

It can be shown that they are given by the following equations:

$$\mu_{l,i} = c(ml+i,ml+k+i)\,, \tag{46}$$

$$\delta_n^{\,l,i} = \begin{cases} \prod_{r=l+1}^{n} \dfrac{d(mr-m+i,mr-m+k+i)}{[c(ml+i,ml+k+i)-c(mr+i,mr+k+i)]}, & n \geq l \\ 0, & n<l \end{cases} \tag{47}$$

$$\gamma_q^{\,l,i} = \begin{cases} \prod_{r=q}^{l-1} \dfrac{d(mr+i,mr+k+i)}{[c(ml+i,ml+k+i)-c(mr+i,mr+k+i)]}, & q \leq l \\ 0, & q>l \end{cases}. \tag{48}$$

It follows, on substituting from Eqs. (46), (47), and (48) into Eq. (44) that

$$\rho(nm+i,nm+k+i,\tau) = \sum_{l=0}^{n}\sum_{q=0}^{l} \frac{\left(\prod_{r=q}^{n-1} d(mr+i,mr+k+i)\right) e^{c(ml+i,ml+k+i)\tau}\rho(mq+i,mq+k+i,0)}{\left(\prod_{r=q,\,r\neq l}^{n} [c(ml+i,ml+k+i)-c(mr+i,mr+k+i)]\right)}. \tag{49}$$

As before, the photon distribution function for m-photon emission process can be obtained by putting $k=0$ in Eq. (49).

V. CONCLUSION

We have obtained, using a density-matrix approach, the exact expressions for the density matrix in single-mode m-photon absorption and emission processes. With the help of the figures, we demonstrated the possibility of observing photon antibunching in an arbitrary m-photon absorption process (with $m \geq 2$).

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