PHYSICAL REVIEW A VOLUME 20, NUMBER 2 AUGUST 1979

Rebuttal to "Comments on 'Power conversion of energy fluctuations'"

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(Received 24 July 1978; revised manuscript received 25 April 1979)

The conclusion predicting no practical power output from the power conversion of energy fluctuations is shown to be the result of fundamental errors in physics and in the theory of the master equation. One decisive error was to compute the output voltage by assuming that no energy for the electron barrier crossings of the cold diodes is supplied by the fluctuation energy of the heated diode. Another fundamental error in the physics was to assume that the total available fluctuation power is small, whereas it is orders of magnitude larger than the radiated power alone. Computations using the master equation corrected for errors in physics give a maximum output power that is within (91–99)% of the Carnot-cycle efficiency for this reversible cycle. Physically realizable diode design options are noted (thin film, quantum effect, thermionic) that can enable the high power output and high-efficiency potential of this approach to be achieved with small material cost.

The comments of EerNisse[1] on the power conversion of energy fluctuations cannot be answered briefly since these comments not only state errors exist in the basic theory developed in the 1974 paper,[2] but predict that no power output can be achieved from the power conversion of energy fluctuations using practical components. Although the conclusion is based, in part, on assumptions concerning the physics of the basic approach that were not part of Ref. 2, this rebuttal will respond to each comment. This will be done because, as stated by EerNisse, there is a broad interest in this basic approach to power conversion. This broad interest results from the theoretical potential of the approach as an alternative energy source to achieve maximum efficiency, maximum power, minimum size, and minimum costly material. This theoretical potential exists for widespread applications over wide temperature ranges in reversible power conversion, heat pumping, refrigeration, and air conditioning.

To summarize, the conclusions in Ref. 1 are, first, that errors exist in the basic theory in Ref. 2 that leads to significant errors as the capacitance of the circuit increases, and second, that the corrected equations show no practical amount of power can be achieved from the approach at efficiencies that are not too low to be useful and for circuit capacitances that are not too small to be practical.

In response to the first conclusion, it will be shown that the conclusion is incorrect by using the performance equations of Ref. 2 to obtain the correct macroscopic performance in the limit as the circuit capacitance increases. The basic assumptions of the Alkemade diode used by van Kampen[3] are fulfilled in the derivation of the master equation for the circuit so as to provide the essential *repeated randomness assumptions* of the master equation.[4] This proof follows the approach of Landauer in his definitive work on fluctuations in bistable tunnel-diode circuits.[5,6] The recursion equation of the equilibrium state given by Eq. (15) of Ref. 2 is

$$P(N)=\frac{K\exp\left[-\alpha(N-\frac{1}{2}+n)\right]+\exp\left[-\beta(N-\frac{1}{2}-m)\right]}{K+1}\times P(N-1)\,, \tag{1}$$

where

$$\alpha=q^2/kT_rC,\quad \beta=q^2/kT_cC,\quad K=A/G\,,$$

$$n=V_rC/q,\quad m=V_cC/q,\quad V=V_c+V_r\,.$$

Then if we let $C\to\infty$ and subtract $P(N)$ from each side of the equation, we have

$$P(N+1)-P(N)=\frac{K\alpha(N+\frac{1}{2}+n)+\beta(N+\frac{1}{2}-m)}{K+1}\times P(N)\,. \tag{2}$$

In this equation for the macroscopic behavior of the circuit, we can replace the summation by integration following Landauer. Then, dropping the negligible term $\frac{1}{2}$ for the macroscopic limit, we have

$$\sum_0^N\frac{P(N+1)-P(N)}{P(N)}\simeq\int_0^N d\ln P(N)\simeq\int_0^N-\frac{K\alpha(N+n)+\beta(N-m)}{K+1}\times dN\,. \tag{3}$$

Then after integration we obtain

$$P(N) \simeq P(0) \exp\left[\frac{(\beta m - K\alpha n)^2}{2(K+1)(K\alpha+\beta)^2} - \frac{K\alpha+\beta}{2(K+1)} \times \left(N - \frac{\beta m - K\alpha n}{K\alpha+\beta}\right)^2\right]. \quad (4)$$

It can be easily shown the mean value of N for this Gaussian distribution is identical to that derived by solving for N using the macroscopic diode equations to compute the dc current in the circuit for an input dc voltage V.

It can also be easily shown that the variance of N given by this Gaussian equation is identical to that derived by solving for $\sigma_r^2 + \sigma_c^2$ using the macroscopic equations for the diode circuit using Eqs. (27)–(30) of Ref. 2.

Therefore, there is no error that results in the behavior given by the equations in Ref. 2 as $C \to \infty$ and no error is proved or even defined by the results of EerNisse. In fact, EerNisse does not derive the probability distribution as $C \to \infty$ from the master equation.

One error in the theory of the master equation made in Ref. 1 is in the *ab initio* mixing of macroscopic diode equations into the microscopic equations. In general, as van Kampen has pointed out,[3,7] the use of macroscopic results in the master equation to compute the microscopic results will give incorrect results. This error in Ref. 1 results from the assumption that the performance of macroscopic diodes cannot be derived from solving for the steady-state solution of the master equation for microscopic diodes in the limit as $C \to \infty$. This assumption is shown to be incorrect by the above derivation of Eq. (4) for the steady-state distribution for macroscopic diodes.

It is also incorrect as stated in Ref. 1 that serious errors can follow for the steady-state solution given in Ref. 2 as $C \to \infty$. These steady-state solutions to the master equation are unique for a fixed sum of m and n and are independent of the ratio of m to n or, equivalently, of the ratio of V_c to V_r. This can be seen by integrating the performance equation using the steady-state distribution given by Eq. (4) over all values of N.

It is also incorrect as stated in Ref. 1 that correct results can, in general, be obtained using the macroscopic diode equations to derive the microscopic performance. The master equation assumes discrete electron barrier crossings producing voltage increments of e/C. The macroscopic equation for the limit of $C \to \infty$ assumes continuous voltage increments so that the fluctuation voltage levels can be in error up to a value given by $e/2C$ if the macroscopic diode equations are used *ab initio* to derive the voltage levels in the master equation for microscopic diodes. This error is not significant when $e/2C \ll 1$.

It is also not correct as stated in Ref. 1 that the operation voltage for peak power is, in general, given by $V = -kT_c/q$. This decisive error in the theory and the physics of the power-conversion circuit is fundamental to this basic approach, and will be discussed in more detail in the Appendix. This error results in a lower-output voltage being used in the extensive computations of Ref. 1 for the power output than required to achieve maximum power.

The reason given in Ref. 1 for the selection of this low-output voltage is that the number of electrons in the cold rectifying diode crossing the diode barrier is determined only by the thermal energy of the electrons in the cold diode. This is shown in the Appendix to be incorrect. These barrier crossings are also functions of the thermal energy of electrons in the hot diode.

It is also not correct as stated in Ref. 1 that high-impedance mismatches that significantly reduce the available output power are required to achieve maximum efficiency. The impedance requirements are broad functions of circuit design, temperature ranges, and circuit parameters. This will be illustrated later by results computed for a specific circuit. These results show that high efficiency can be obtained for maximum power over a wide range of values of K.

It is not possible in this short comment to show the magnitude of each separate error in the theory and the physics of the power-conversion circuit made in Ref. 1. This comment will only indicate how large the total error is in the results of Ref. 1 by giving results computed from the theory and physics given in Ref. 2. Then the effect of these errors in theory and physics on the conclusions reached in Ref. 1 will be discussed. These conclusions will be shown to be invalid as being based not only on the incorrect computed results but also on fundamental errors in the physics of this basic approach.

In the results of the extensive computation reported in Ref. 1, it was concluded the maximum efficiency for the maximum power output is 32%. This maximum efficiency is obtained at 3.5×10^{-19} F for an operating voltage of 0.075 V, $T_c = 300$ K, $T_r = 1000$ K, and $K = 10^2$. For higher or lower values of K and C, it is stated in Ref. 1 that less efficiency is obtained for the maximum power output.

As an example, to show much higher efficiencies for any value of m to n, and a much broader range of values of K, the performance was computed for a value of C of 10^{-19} F and also for $T_c = 300$ K and $T_r = 1000$ K. The result were computed using Eqs. (15) and (19) of Ref. 2 to give results valid for a wide range of values of K.

Results are plotted in Fig. 1 for the output current I as a function of x (curve A) for $K=1$. For a constant ratio of m to n, x is proportional to the output voltage V and given by $x=-2\ (\beta m+\alpha n)/(\beta-\alpha)$. This general result shows the familiar shape of the curves of the output current as a function of the output voltage between the open-circuit voltage V_{oc} at $x=1$ and the short-circuit current I_{sc} at $x=0$.

Also plotted in curve B is the curve for $K=0$. These two curves show the statements in Ref. 1 that a high-impedance mismatch is required is not correct for this basic approach. The curves show that a broad range of impedance ratios can be used. Curves A and B show that equivalent output can be achieved with a maximum 3% change in output voltage.

These curves are independent of the value of the ratio of m to n for this range of diode nonlinearities. Therefore, the question of the correct ratio of m to n has no bearing on the shape of these curves. The values I_m and V_m for maximum power output can also be determined from these curves and the values for curve B are shown.

Although the ratio of m to n has no bearing on the shape of these curves or the point of maximum power, this ratio is important in determining the value of the voltage of maximum power for this performance curve. Using Eq. (25) of Ref. 2 and the value of $x=1$, the values of V_{oc} can be computed for the power-conversion circuit as a function of the ratio of m to n. The ratio of V_{oc} for $m/n=0$ to V_{oc} for $n/m=0$ is given by the ratio of T_r to T_c. This relation was not used in the selection of the low operating voltages in the results reported by EerNisse.

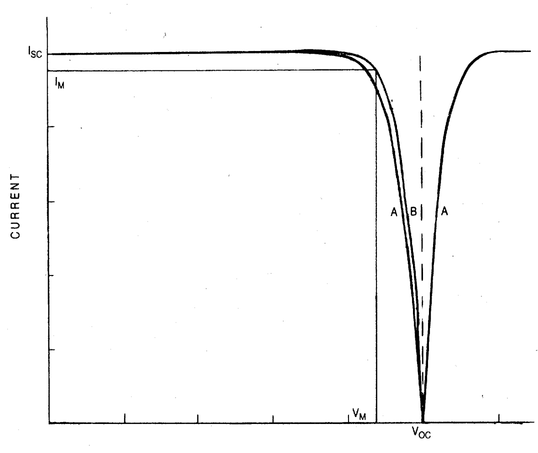

FIG. 1. Curves A and B give the output (or input) current for $K=1$ and $K=0$, respectively, as a function of output (or input) voltage for $\beta=64$ and $\alpha=19.2$. V_m and I_m are the output voltage and output current, respectively, for maximum power output given by curve A. V_{oc} and I_{sc} are the open-circuit voltage and short-circuit current, respectively, for the power-conversion circuit.

The efficiency of power conversions for the circuits of curve B was computed using Eqs. (20)–(22) of Ref. 2. The result show that efficiency at maximum power output is 64% for $m=n$ to 67% for $m=0$. These efficiencies are 91% and 96% of the Carnot efficiency, respectively, for this maximum power output and are approximately 200% larger efficiencies than that given by EerNisse for this range of capacities.

Curves A and B can also be used to show the performance for other temperature ranges for which the value of $\beta-\alpha$ remains constant. For example, if T_c/T_r is decreased from 0.3 to 0.1 the value of $\beta-\alpha$ can be held constant by either increasing the value of C or by increasing both temperature levels. For the ratio of T_c/T_r of 0.1, the efficiency at maximum power from curve B is 84% for $m=n$ and 89% for $m=0$. These efficiencies for maximum power output are 93% and 99%, respectively, of the Carnot efficiencies.

As C increases, the efficiency for maximum power decreases. For example, an efficiency approaching 80% for maximum power can be obtained for an increase in C in the range of two orders of magnitude for $T_c/T_r=0.1$. This is much larger than the values of maximum efficiency given by Ref. 1 for smaller values of C.

The above results answer the questions raised in Ref. 1 concerning the theory of the basic approach and show, using computations based on the theory of Ref. 2, that high power at maximum efficiency can be obtained. The computations in Ref. 1 are based on separate errors in theory. In addition, the general conclusions stated in Ref. 1 are based not only on these incorrect computations but also on fundamental errors in the physics of this basic approach to power conversion.

One fundamental error in the physics of this basic approach is in the conclusion that the available fluctuation power is so limited that the efficiency must be sacrificed to ensure that the maximum power available from each energy fluctuation is converted to useful power. In fact, the opposite is true. The argument put forth in Ref. 1 concerning the requirement for operating at maximum power output is not valid because it is not consistent with two inherent and important advantages of the power conversion of energy fluctuations.

One of these advantages is the inherent ability to transfer the fluctuation energy across the thermal barrier in the near field of the charges at a power rate that can be orders of magnitude larger than either the maximum power rate of radiated energy or the maximum power rate required for any practical applications at present. The analysis of the available power and designs to transfer with high efficiency this extremely large power will be given in a later paper.

The other advantage is the inherent ability to achieve the maximum efficiency by controlling the power rate and the voltage range in which fluctuation energy is transferred across the thermal barrier. This is distinct from solar cells where each crystal of a polycrystalline device must convert each incoming photon above a fixed energy value to useful output for maximum efficiency. As a result of these two inherent advantages the individual conversion circuits or conversion areas can be operated efficiently for useful application at orders-of-magnitude-lower power-output rates than the maximum power-output rate of the individual conversion circuit or conversion area.

The physics of this basic approach does not support the conclusion that no net power can be obtained for practical circuit capacitance. The theory and physics for the basic approach is valid for the fluctuation power from regions containing large numbers of atoms. Furthermore there is no known inherent physical factor that can prevent the fabrication of a practical cold rectifying structure composed of a comparable large number of atoms.[8] It is reasonable to assume that it is possible to achieve practical designs for the basic approach, but to show in detail the many promising structures and design concepts that have been discovered and analyzed is beyond the scope of this reply. A paper will be published for this purpose that will include the following designs that were not noted or considered in Ref. 1.[9]

Quantum-effect diodes: The inherent band structure in bulk material and barrier layers of diodes provides a design option to limit barrier crossing to within a bandwidth of energy so as to provide a higher efficiency for a given circuit capacity than shown by the curves in Fig. 1. Computations will be given to show over 80% efficiency for larger circuit capacities.

Thin-film approach: Although not inherently different from discrete circuit components, the fabrication of thin films is a more familiar and readily available fabrication process. The physics of the design is to limit the thickness of the films so that the effective region of each energy fluctuation on the hot side of the thermal barrier is confined to a similar small region of the cold rectifying diode barrier. The many diode structures of conventional solar cells as well as tunnel diodes can be used in this structure for this approach as will be shown in the later paper.

Heat pump, refrigeration, and air conditioning: Much larger capacities than analyzed in Ref. 1 can be efficiently used in refrigeration, heat pump, and air conditioning cycles. The reversible nature of this basic approach is not shown in the results in Ref. 1. The performance for the reversible thermoelectric cycle is given for curve A as the operating voltage is increased beyond the point of current reversal. At that point, the negative current given by curve A for voltages greater than V_{oc} becomes a power input to the circuit. The potentially important application in this mode as a heat pump or a refrigerator is not shown in the results plotted by EerNisse as, again, the output voltage selected was too low.

The requirements for practical circuits for applications as heat pump or refrigeration meet different requirements that must be evaluated separately for each application. For example, for a home heat pump operating from a temperature of 32 °F outside to 75 °F inside, the performance computed for circuit capacities larger than the range computed in Ref. 1 shows that the heat pumped inside the home is larger by a factor of 10 than the heat energy of the input power to the pump. The result of this capability is to reduce by an order of magnitude the amount of energy used for this important application.

Temperature effect on diodes: By lowering diode temperature, equivalent performance can be maintained for larger diodes. Multistage circuits have been analyzed to show how this alternate design approach can enable high efficiency to be achieved from larger diodes.[10]

The work of analyzing the alternate designs capable of implementing this basic approach represents the results of work over a period of years directed to determining if the potential promise that was shown to exist in Ref. 1 can be realized. To summarize these results, it has been shown for these alternative designs that the capability does exist to obtain effectively the efficiencies as a function of voltage given by curves A and B for much larger circuit capacities than the range used in Ref. 1. This capability does eliminate the question of a physical barrier that can prevent achieving high-efficiency performance for high power outputs.

Finally, in summary, it has been shown that the comments in Ref. 1 are based on fundamental errors in the theory of the reversible thermoelectric converter of energy fluctuations and that there is no theoretical limit on the size of physical components to prevent obtaining high output power at high efficiencies from practical circuits.

APPENDIX

One conclusion in Ref. 1 is that the power output occurs at output voltages so low for this basic approach that low efficiency results. This conclusion is based on a fundamental error in the physics of the power-conversion circuit. This error in physics is the assumption that, for the general case, the output voltage for maximum power is $V = -2kT_c/q$, where T_c is the temperature of the cold diode. This general result for the value of V for maximum power is inconsistent with the performance equations derived in Ref. 2.

To enable this fundamental error to be easily understood, let us consider the case where the temperature T_c of the cold diode approaches zero. For this case the nonlinearity factor for the cold-diode current $e/kT_c \to \infty$ so that the rectifying-thermal-emission diode approaches the properties of a perfect rectifier of the fluctuation energy coupled from the hot diode. Then, for this case, the above general equation for V indicates that the output voltage for maximum power also approaches zero. This surprising value for V for maximum power means the maximum conversion output power goes to zero as the efficiency of the rectifier increases. This, of course, cannot be correct. The behavior of the power-conversion circuit as $T \to 0$ is discussed in the Ref. 1 as a limiting case both for the Alkemade-diode model and for the continuous-voltage models using heated linear resistors. The results are in agreement with the expected properties of a perfect rectifier.

It is clear from the physics of this basic approach to power conversion that as we increase the output voltage from an initial short circuit of a perfect rectifier the power-conversion output power will increase. This increase will continue until the relative increase in output voltage is less than the relative fluctuations of the heated diode. This probability is given by the fluctuation voltage distribution of the heated diode. From the physics, then, it is evident that for this limiting case of a perfect rectifier circuit, the temperature of the heated diode is the primary factor in determining the voltage at which maximum power occurs.

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