Ionization collisions between hydrogen and hydrogenlike atoms at high energies

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An improved impulse approximation is used to investigate the ionization collisions between two excited one-electron atoms at high energies. It is shown that the cross sections can easily be estimated by using the quantities calculated in the case of the collisions between two excited hydrogen atoms.

Within the framework of the improved impulse approximation[1] we have studied the energy and state dependence of the ionization cross sections for the collisions between two excited hydrogen atoms.[2-4] In this note the following processes involving hydrogenlike atoms are investigated theoretically at high energies for both cases of simultaneous excitation ($n_A < n'_A$) and deexcitation ($n_A > n'_A$) of atom A by using the improved impulse approximation:

$$A^{(Z_A-1)+}(n_A l_A) + B^{(Z_B-1)+}(n_B l_B) \to A^{(Z_A-1)+}(n'_A l'_A) + B^{Z_B+} + e\,, \qquad (1)$$

with

$$n_A/Z_A \le n_B/Z_B\,. \qquad (2)$$

Here n and l are the principal and azimuthal quantum numbers, respectively, and Z the nuclear charge. Condition (2) is the requirement from the utilization of the impulse approximation. Our primary purpose is to find a scaling rule which relates the cross sections averaged over l_B to those for the ionization collisions between hydrogen atoms. When both atoms A and B have net charges ($Z_A, Z_B \ge 2$), the improved impulse approximation can be employed only at very high energies, since the relative motion between atoms in this approximation is described not by a Coulomb wave, but by a plane wave. Therefore, we confine our discussion mainly to the process (1) in which one of the atoms is a hydrogen atom, i.e., to a special case of $Z_A = 1$ or $Z_B = 1$. Since we use in the present discussion the Born approximation to the electron-atom A scattering amplitude, the collision energy E in the case of $Z_A \ge 2$ and $Z_B = 1$ should be higher than that allowed for the utilization of the Coulomb-Born approximation.[5] Thus the present discussion will be restricted to the case of Eq. (2) and also to the high, though nonrelativistic, collision energies.

We present a brief review of the improved impulse formula for the cross section averaged over l_B in the high energy limit. The average cross section for process (1) is expressed as

$$\sigma(E) \simeq C(n_A l_A, n'_A l'_A, n_B : |E_B^i|)/E \quad (E \to \infty)\,, \qquad (3)$$

where $C(\cdots\cdots)$ is a constant defined as (in atomic units)

$$C(n_A l_A, n'_A l'_A, n_B : |E_B^i|) = \frac{8}{3}\mu \int_0^\infty dp\left[\frac{3}{4}\pi - J(y_B)\right] |\epsilon^A(M_{eA} p)|^2/p^3. \qquad (4)$$

The notations employed here are as follows: μ and μ_{eA} are the reduced masses of the atom A-atom B and electron-ion core A^+ systems, respectively; p is the magnitude of the momentum transferred to atom A; $\epsilon^A(x)$ is the form factor of atom A. The function $J(y)$ is defined as

$$J(y) = \frac{y}{(y^2+1)^2} + \frac{3y}{2(y^2+1)} + \frac{3}{2}\tan^{-1} y. \qquad (5)$$

The quantity y_B is given by

$$y_B = \tfrac{1}{2}(p_B/p - p/p_B) \qquad (6)$$

with

$$p_B = (2M|E_B^i|)^{1/2},$$

where $M = 1 + 1/M_B$ (M_B is the mass of ion core B^+), and $|E_B^i|$ is the ionization potential of atom B.

Substituting the transformations

$$x = p/Z_A, \quad |E_H^i| = |E_B^i|/Z_B^2, \quad \epsilon^H(M_{eA} x) = \epsilon^A(M_{eA} p) \qquad (7)$$

into Eqs. (4) and (6), we can easily obtain

$$C(n_A l_A, n'_A l'_A, n_B :\ |E_B^i|) = \frac{\mu}{\mu_H}\frac{1}{Z_A^2}\tilde{C}(n_A l_A, n'_A l'_A, n_B :\ |E_H^i|), \qquad (8)$$

where $|E_H^i|$ is the ionization potential of a hydrogen atom in the state n_B, μ_H is the reduced mass of two hydrogen atoms, and

$$\tilde{C}(n_A l_A, n'_A l'_A, n_B\colon\ |E^i_H|) = \frac{8}{3}\mu_H \int_0^\infty dx\, g_n(x)\, h(x), \tag{9}$$

$$\left.\begin{aligned} g_n(x) &= \tfrac{3}{4}\pi - J(y_H), \\ h(x) &= |\epsilon^H(x)|^2/x^3, \\ y_H &= \tfrac{1}{2}(1/nx - nx), \end{aligned}\right\} \tag{10}$$

and

$$n = (Z_A/Z_B)n_B. \tag{11}$$

In Eqs. (10) and (11) we have employed the mass-disparity approximation ($M_{eA} \simeq 1$, $M \simeq 1$) and $|E^i_H| = 1/2n_B^2$.

In the special case of $Z_A = Z_B$, $\tilde{C}$ is identical to that for the collision between two hydrogen atoms with the same quantum numbers, and the cross section is proportional to Z_A^{-2} (= Z_B^{-2}). When Z_A and Z_B make such a combination that n becomes an integer, the coefficient $\tilde{C}$ is equal to that for the case of hydrogen atoms with n instead of n_B. Therefore, the values of $\tilde{C}$ already evaluated for the various combinations of the quantum numbers of hydrogen atoms[3] can be readily used to estimate the cross sections.

In the general case of Z_A and Z_B, all we have to do is carry out the calculations of $\tilde{C}$ by using the form factor $\epsilon^H(x)$ of a hydrogen atom for the transition $n_A l_A \to n'_A l'_A$. The coefficients $\tilde{C}(n_A, n'_A, n_B)$, which denotes $\tilde{C}(n_A l_A, n'_A l'_A, n_B)$ averaged over l_A and summed over l'_A, are calculated for some combinations of the quantum numbers for the two cases of $Z_A = 1$ and $Z_B = 1$. The results are shown in Tables I and II.

In order to see the dependence of the cross sections on Z_B and Z_A for large Z_B or Z_A, we try to take the limits of Eq. (9) for $n \to 0$ ($Z_B \to \infty$) and $n \to \infty$ ($Z_A \to \infty$ or $n_B \to \infty$). Since we have

$$\left.\begin{aligned} h(x) &\to C_1/x \quad (x \to 0) \\ &\to C_2 x^{-2(l_A + l'_A) - 11} \quad (x \to \infty), \\ \text{and} \quad & \\ g_n(x) &\to C_3 n^5 x^5 \quad (x \to 0 \text{ or } n \to 0), \\ &\to \tfrac{3}{2}\pi \quad (x \to \infty \text{ or } n \to \infty), \end{aligned}\right\} \tag{12}$$

where C_j's are certain constants, the function $g_n(x)h(x)$ converges uniformly to 0 in the order of $C_3 n^5 x^5 h(x)$ as $n \to 0$. Thus we can easily show

$$\tilde{C}(n_A l_A, n'_A l'_A, n_B) \propto n^5 = [(Z_A/Z_B)n_B]^5 \quad (n \to 0), \tag{13}$$

which means

$$\sigma \propto Z_B^{-5} \quad (Z_B \to \infty). \tag{14}$$

This dependence can be confirmed by numerical calculations of $\tilde{C}$ for $n \lesssim 0.1$. It should be noted, however, that condition (2) gives $n_A \leq n$; hence the mathematical limit of $n \to 0$ does not have any significant physical meaning. The numerical results presented in Table I show roughly the Z_B^{-2} dependence for $1.0 \lesssim n \lesssim 2.0$. When $n \to \infty$, on the other hand, the function $g_n(x)\, h(x)$ does not converge uniformly to $\frac{3}{2}\pi\, h(x)$. Therefore, $\lim_{n\to\infty} \tilde{C}$ is not equal to $4\pi\mu_H \int_0^\infty dx\, h(x)$, which diverges logarith-

TABLE I. Values of $\tilde{C}(n_A, n'_A, n_B)$ in units of (keV) πa_0^2 for various Z_B ($Z_A = 1$) [or n (= $Z_A n_B/Z_B$)].

Z_B	(n_A, n'_A, n_B) (1, 2, 10)	(2, 1, 10)	n
1	9.344(1)[a]	2.336(1)	10
2	5.767(1)	1.442(1)	5
3	3.900(1)	9.749(0)	$\frac{10}{3}$
4	2.753(1)	6.883(0)	$\frac{5}{2}$
5	1.998(1)	4.995(0)	2
6	1.480(1)	⋯	$\frac{5}{3}$
7	1.115(1)	⋯	$\frac{10}{7}$
8	8.520(0)	⋯	$\frac{5}{4}$
9	6.591(0)	⋯	$\frac{10}{9}$
10	5.156(0)	⋯	1

[a]The number in the brackets indicates the power of 10 by which the entry must be multiplied. ⋯ denotes the process, not satisfying the condition given by Eq. (2).

TABLE II. Values of $\tilde{C}(n_A, n'_A, n_B)$ in units of (keV) πa_0^2 for various Z_A ($Z_B = 1$) [or n (= $Z_A n_B/Z_B$)].

Z_A	(n_A, n'_A, n_B) (1, 2, 1)	(2, 1, 1)	n
1	5.152(0)[a]	⋯	1
2	1.997(1)	4.993(0)	2
3	3.458(1)	8.646(0)	3
4	4.708(1)	1.177(1)	4
5	5.766(1)	1.442(1)	5
6	6.673(1)	1.668(1)	6
7	7.463(1)	1.866(1)	7
8	8.160(1)	2.040(1)	8
9	8.782(1)	2.196(1)	9
10	9.344(1)	2.336(1)	10

[a]The number in the brackets indicates the power of 10 by which the entry must be multiplied. ⋯ denotes the process, not satisfying the condition given by Eq. (2).

mically. However, we may consider that for a large but finite value of n $\tilde{C}$ is roughly equal to $4\pi\mu_H \int_{1/n}^{\infty} dx\, h(x)$, since the function $g_n(x)$ behaves like a step function when n is large. Thus we have

$$\tilde{C}(n_A l_A, n'_A l'_A, n_B) \propto \ln n \quad (n \to \infty),$$

and

$$\sigma \propto Z_A^{-2} \ln n = Z_A^{-2} \ln[(Z_A/Z_B)n_B] \quad (n \to \infty). \tag{15}$$

Actually, the numerical results presented in Table II show the proportionality of $\tilde{C}$ to ln Z_A for $Z_A \gtrsim 4$. The similar charge dependence was obtained for the total inelastic cross sections for collisions between atomic particles.[6]

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