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Proposed neutron interferometer test of some nonlinear variants of wave mechanics

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A family of nonlinear variants of the Schrödinger equation was defined by Bialynicki-Birula and Mycielski by adding terms of the form $F(|\psi|^2)$ to the Hamiltonian. It is proposed that this family be tested by observing whether a phase shift occurs when an absorber is moved from one point to another along the path of one of the coherent split beams in a neutron interferometer. If F is b times a logarithmic function, which is the most important case, a null result with apparatus now available would impose an upper bound on b of 1.5×10^{-12} eV, more than two orders of magnitude smaller than the bound estimated by the above authors on the basis of the Lamb-shift measurement.

Various authors have suggested that the Schrödinger equation is only an approximation of the true nonlinear wave equation.[1-4] These suggestions are largely motivated by the fact that nonlinear equations can have solutions which are qualitatively different from those of the standard linear equation—e.g., nonspreading wave packets in the absence of a potential (Ref. 4), and superpositions in which all terms but one die away asymptotically in time (Ref. 3).

The important family of nonlinear wave equations investigated by Bialynicki-Birula and Mycielski (Ref. 4) consists of those of the form

$$i\hbar\frac{\partial\psi(\vec{r},t)}{\partial t}=\left(-\frac{\hbar^2}{2m}\Delta+U(\vec{r},t)\right)\psi+F(|\psi|^2)\psi, \quad (1)$$

where $\vec{r}$ is the position vector in an n-dimensional configuration space and F is real valued. Any square-integrable solution $\psi(r,t)$ of an equation of the form (1) shares a number of properties with solutions of the linear Schrödinger equation, e.g., (i) the norm $(\langle\psi|\psi\rangle)^{1/2}$, defined in the standard way, is preserved in time; (ii) in the absence of the potential, invariance under the full Galilei group holds, with ψ transforming as in the linear theory; (iii) an equation of continuity, $\partial\rho/\partial t=-\nabla\cdot\vec{j}$, holds, where the density ρ and the current density $\vec{j}$ are defined as in the linear theory. On the other hand, most of the nonlinear equations have the undesirable feature of generating correlations between two particles even when there is no interaction potential between them. In order to eliminate this feature, the authors postulate that if a system consists of noninteracting subsystems, then the solution of an admissible equation can be constituted for the system by taking the product of arbitrary solutions of this equation for separate subsystems. Using the elementary properties of the logarithm of a product, they prove that the only equation of form (1) which satisfies this "separability condition" is

$$i\hbar\frac{\partial\psi(\vec{r},t)}{\partial t}=\left(-\frac{\hbar^2}{2m}\Delta+U(\vec{r},t)\right)\psi+b\ln(a^n|\psi|^2)\psi. \quad (2)$$

Of the two constants a and b in Eq. (2), the latter is by far the more interesting physically. In order to preserve the norm of solutions, b must be real; and it must be universal if the "separability condition" is to hold for any pair of systems with their interaction switched off. The dimension of a must be length, but its value is not physically significant, since changing it can be compensated by multiplying the wave function by a phase harmonically dependent on time, or, equivalently, by adding a constant to U. Furthermore, there is no need for the conventionally chosen value of a to be universal any more than a constant added to the potential in the linear theory would have to be, since the value of a does not enter into the nonlinearity. The physically interesting values of b are positive, since only these permit the construction of free-particle wave packets which do not spread. In view of the great success of standard quantum mechanics, the value of b must be small, and Bialynicki-Birula and Mycielski estimate that the Lamb-shift measurement sets an upper limit on b:

$$b<4\times 10^{-10}\ \text{eV}. \quad (3)$$

The main purpose of this paper is to propose an experiment which would either exhibit the presence of a nonlinear correction to the Schrödinger equation or (the more likely outcome) set an upper bound on b several orders of magnitude lower than that of (3).

Although this paper is primarily concerned with the logarithmic nonlinear equation (2), there are two reasons for giving some consideration to more general nonlinearities. In the first place, one could postulate equations of the form (1) for one-particle systems only and leave open the question of the form of a nonlinear Schrödinger equation for more complex systems. With this restriction the question of separability does not even arise. In the second place, the test which will be proposed for Eq. (2) can be adapted to the general equation (1) (restricted to a single particle), provided that some additional experimental precautions are taken.

The experiment is to be performed using a neutron interferometer,[5-12] in which the wave packet of a single neutron is split into two coherent beams I and II of negligible overlap which propagate several centimeters before recombination. The most suitable apparatus for the experiment is the two-crystal interferometer of Ref. 7, because the space it provides between splitting and recombination is uninterrupted by a crystal.

Essentially the experiment consists of observing the phase shift Δ due to moving a partial absorber from one position P to another position P' in the path of beam I (see Fig. 1). Since the beams are well collimated and nearly monochromatic, quantum mechanics predicts that Δ is negligible. The nonlinear theories, however, imply that the translation of the absorber causes non-negligible phase shift in each ray of beam I. In order to test Eq. (1), where the function F is general, beam I must be prepared (by initial stops) so that its amplitude is nearly uniform over a transverse cross section, thus ensuring approximately the same phase shift in each ray. It will be seen that the condition of uniform amplitude is unnecessary when F is logarithmic, as in Eq. (2). For the present, uniformity is assumed, so that $F(|\psi|^2)$ serves as an effective potential depending only upon the distance s along the path of beam I. In fact, since the beam is well collimated, $|\psi|$ has a constant value between P and P': it is the initial amplitude $|\psi_I|$ if the absorber is inserted at P', and $|\alpha\psi_I|$ if the absorber (with amplitude attenuation factor α) is inserted at P. Elsewhere the contribution of the nonlinear term is the same whether the absorber is inserted at P or P'. Since the nonlinear term is presumably small compared to the total energy E of the neutron, and since U is zero in the region of interest, the WKB approximation immediately yields a very good expression for the phase shift:

$$\begin{aligned}\Delta &\simeq \int_P^{P'} ds \left(\frac{2m}{\hbar^2}\,[E - F(|\alpha\psi_I|^2)]^{1/2} - \frac{2m}{\hbar^2}\,[E - F(|\psi_I|^2)]^{1/2}\right)\\ &\simeq (d/\hbar)(m/2E)^{1/2}[F(|\psi_I|^2) - F(|\alpha\psi_I|^2)]\\ &\simeq (\tau/\hbar)[F(|\psi_I|^2) - F(|\alpha\psi_I|^2)]\,, \end{aligned} \tag{4}$$

where d is the distance from P to P' and τ is the time for the center of the wave packet to travel this distance.

If $F(x)$ is taken to be $b\ln(a^3x)$, as in Eq. (2), then

$$\Delta \simeq (2/\hbar)\tau b \ln|\alpha|\,. \tag{5}$$

Because Eq. (5) is obtained by taking the logarithm of the ratio of $a^3|\psi_I|^2$ to $a^3|\alpha|^2|\psi_I|^2$, the constant a cancels out, and, furthermore, the assumption made earlier that the amplitude $|\psi_I|$ is approximately uniform throughout most of the transverse cross section of beam I is not needed. Equation (5) is also immediately derived by using the fact noted in Ref. 4 that if $\psi(\vec{r},t)$ is a solution to Eq. (2), so also is

$$\alpha\psi(\vec{r},t)\exp[(i/\hbar)bt\ln|\alpha|^2]\,.$$

Although Eqs. (4) and (5) express the phase shift Δ implied by the nonlinear equations (1) and (2), no investigation has been made of the implications of the latter equations for the diffraction of neutrons by crystals and for other matters relevant to the performance of the neutron interferometer. The quantum-mechanical theory of this performance is intricate (Ref. 7 and further references given there), and the introduction of a nonlinear term would cause additional complications. It is reasonable to suppose, however, that when the nonlinear term is very small, as has been assumed, then nonlinear wave mechanics predicts the same general features of interferometer performance as the standard linear theory, despite their differences with respect to the magnitudes

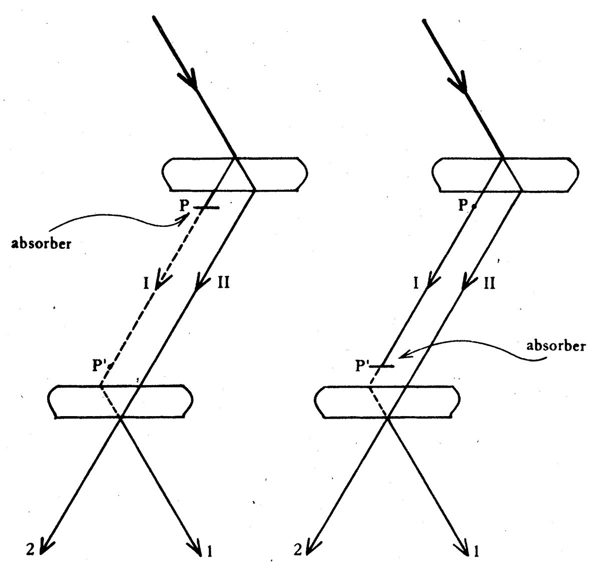

FIG. 1. Dashed line indicates the attenuation of the amplitude of a beam by a factor α. Output beams I and II are directed towards the neutron counters.

of phase shifts. If this supposition is not true, then the large body of data concerning coherent recombination of split neutron beams would *ipso facto* constitute evidence against the nonlinear theories.

We anticipate that within experimental error the phase shift Δ will be zero, and we shall now calculate the upper bound thereby imposed upon b, using the specifications of the MIT two-crystal neutron interferometer. The path length between the two crystals is 4.5 cm for 1.5-Å wavelength neutrons. To obtain an attenuation of the amplitude by $|\alpha| = e^{-1/2}$, which is optimum for the experiment, there exist convenient absorbers of thickness 2 mm or less. Hence the distance between the two loci P and P' of insertion of the absorber can be 4 cm, so that $\tau = 1.5\times 10^{-5}$ sec. The fringe contrast and count rate exhibited in Ref. 7 indicate that an uncertainty in Δ of 2° can be achieved in an experiment of reasonable duration. Therefore, a null result would imply

$$b = \hbar\Delta/\tau < 1.5\times 10^{-12}\ \text{eV}. \tag{6}$$

This upper limit could be lowered by increasing the neutron count, which requires the expenditure of more time or the achievement of a higher incident flux,[13] or by building a longer interferometer.

Equation (1) is more difficult to test than Eq. (2), not only because of the need to assume the uniformity of $|\psi_I|$, but also because of the generality of the function F, which need not even be monotonic. However, it would be shown that F varies little over a wide range of its argument—and hence differs little from a constant that could be incorporated into U—if null phase shifts were found with each of a fairly large number of absorbers having different attenuation factors α. Previous instances of coherent recombination of split neutron beams already constitute strong evidence against Eq. (1) with any F which is neither extremely small nor logarithmic, since the profile of intensity in the transverse cross section of a diffracted neutron beam is known to be, in general, highly nonuniform.[14]

If the phase shift Δ is (unexpectedly) found to be nonzero, then the path distance from P to P' should be varied, since both Eqs. (4) and (5) imply that Δ is proportional to this length.

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[13] It should be noted that in present practice the flux of neutrons through the interferometer is only a few hundred per minute. Since the mean transit time is of the order of 10^{-5} sec, one can reasonably assume that the wave packets of neutrons passing through the interferometer are nonoverlapping, and thus one has no complications from interactions among them. Clearly, the flux could be considerably increased without losing this benefit.

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