Comment on the asymmetry in the cusp of the differential cross section for charge capture to the continuum

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For electron capture from noble-gas atoms to the continuum of energetic bare projectiles an asymmetry has recently been observed in the cusp of the differential cross section at $v_e = v$ (where v_e and v are electron and projectile velocities, respectively). It is shown that the Oppenheimer-Brinkman-Kramers approximation can provide an asymmetry if terms linear in $\vec{k} = \vec{v}_e - \vec{v}$ are kept in the expansion of the outgoing Coulomb wave. In contrast to the predictions of the second Born approximation, the resulting shape is independent of the projectile charge.

Since the pioneering experimental work of Crooks and Rudd[1] and the associated theoretical work of Salin[2] and of Macek[3] on electron capture to projectile-centered continuum states of energetic bare ions, much attention has been devoted to the singly differential cross section of the ejected electrons. It was found that the electrons are predominantly emitted into a narrow forward cone (characterized by a semiangle θ_0) with a velocity $\vec{v}_e$ close to the projectile velocity $\vec{v}$ in the laboratory frame. Dettmann, Harrison, and Lucas[4] predicted that this differential cross section should have a cusp at $v_e = v$. Since the electron velocity $\vec{k} = \vec{v}_e - \vec{v}$ in the projectile frame should be small compared to the projectile velocity they argued that all terms except the zero-order term in k/v may be discarded in an expansion of the electron Coulomb wave function. Under this assumption, the shape of the cusp calculated in the Oppenheimer-Brinkman-Kramers (OBK) approximation turns out to be symmetric around $v_e = v$. Several measurements[5] of electron capture from Ne and Ar gases have revealed, however, that the actual cusp is strongly asymmetric. In a recent letter Shakeshaft and Spruch[6] have attributed the asymmetry (which they state as "heretofore unexplained") to a contribution from the second-order Born term. The purpose of this note is to show that by retaining terms linear in k/v in the outgoing Coulomb wave function, the OBK approximation can provide an asymmetric cusp in the singly differential cross section.

For simplicity, we consider the electron capture from a hydrogen atom to the continuum of an energetic bare projectile with charge Z_p and velocity v. According to Eqs. (2)–(4) of Ref. 6, the OBK singly differential cross section for capture to the continuum is given by

$$\frac{d\sigma_{cc}^{\mathrm{OBK}}}{dv_e} = (2\pi)^{-1} \int d\Omega_e |T^{\mathrm{OBK}}|^2 K\, dK \,, \tag{1}$$

where $d\Omega_e = \sin\theta_e\, d\theta_e\, d\phi_e$ is the element of the solid angle describing the direction of the emitted electron in the laboratory frame with $0 \leqslant \theta_e \leqslant \theta_0$ and $0 \leqslant \phi_e \leqslant 2\pi$. The momentum K ranges from $\frac{1}{2}v$ to ∞ (atomic units are used throughout the paper). In Eq. (1) the OBK T matrix is defined by

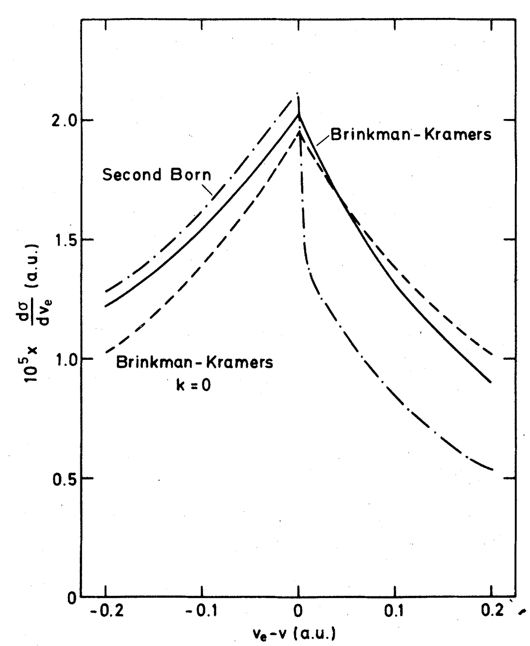

FIG. 1. Singly differential cross section for continuum electron capture from the $1s$ state of a hydrogen atom by a bare ion of $Z_p = 6$ incident with an energy of 2 MeV/nucleon in the laboratory frame. In the cross section electrons are accepted which emerge into a cone of semiangle $\theta_0 = 1.85°$. The solid curve, marked "Brinkman-Kramers" represents the OBK result of Eq. (6) and the dashed curve the OBK results with $k = 0$. The dash-dotted curve gives the contribution to the differential cross section from the sum of the first- and second-order Born terms (Ref. 6), however without inclusion of the $\vec{k}\cdot\vec{K}$ terms in the first Born (OBK) amplitude.

$$T^{\mathrm{OBK}} = -4\pi^3 K^2 \tilde{\psi}_{\vec{k}}^*(\vec{K})\tilde{\psi}_{1s}(\vec{T}) , \qquad (2)$$

where a tilde indicates a Fourier transform and $\vec{T}=\vec{v}+\vec{K}$. For $\tilde{\psi}_{\vec{k}}^*(\vec{K})$ we use the momentum wave function[7] for an electron with velocity $\vec{k}$ in the projectile-centered Coulomb field. If terms of the order k/K are not to be discarded one has to keep the scalar product

$$\vec{k}\cdot\vec{K} = kK(\cos\theta_K \cos\theta_k + \sin\theta_K \sin\theta_k \sin\phi_e) , \qquad (3)$$

where[6] the angles

$$\theta_K = \cos^{-1}(-v/2K) \qquad (4)$$

and

$$\theta_k = \tan^{-1}[v\theta_e/(v_e - v)] \qquad (5)$$

are directly related to the electron velocity v_e and emission angle θ_e in the laboratory system. Consistently keeping the terms linear in k/K and inserting Eqs. (2)–(5) into Eq. (1) we obtain the result

$$\frac{d\sigma_{cc}^{\mathrm{OBK}}}{dv_e} = 2^7 Z_p^3 \int d\Omega_e\, |\vec{v}_e - \vec{v}|^{-1} \times \int_{v/2}^{\infty} \frac{1}{(1-2\vec{k}\cdot\vec{K}/K^2)^2}\,\frac{dK}{K^{11}} . \qquad (6)$$

If the scalar product $\vec{k}\cdot\vec{K}/K^2$ is disregarded in Eq. (6) one recovers the original result of Dettmann et al.[4] [quoted as Eq. (1) in Ref. 6]. As is obvious from Eq. (6), this result predicts a symmetric cusp. If, however, the scalar product is retained Eq. (6) yields cross sections asymmetric with respect to $v_e = v$ because θ_k occurring in Eq. (3) is changed into $\pi - \theta_k$ [cf. Eq. (5)] if $v_e - v$ is changed into its negative. Of course, for $v\to\infty$ the asymmetry approaches zero.

For comparison with the results of Ref. 6 we have calculated $d\sigma_{cc}^{\mathrm{OBK}}/dv_e$ for $Z_p=6$, a cutoff angle $\theta_0 = 1.85°$, and a projectile energy of 2 MeV/nucleon. The results are shown in Fig. 1. In contrast to the predictions of the second Born approximation[6] the projectile charge Z_p only changes the scale but not the shape of the asymmetry in the differential cross section. Thus for $Z_p = 1$ when the second Born approximation yields an asymmetry which is "barely noticeable"[6] the $\vec{k}\cdot\vec{K}$ term in Eq. (6) should be the main source of any observed asymmetry. On the other hand, for higher projectile charges (see Fig. 1), *both* the $\vec{k}\cdot\vec{K}$ term in the OBK T matrix [Eq. (2)] *and* the second Born term[6] should contribute to the cusp asymmetry and for $v\to\infty$ the second Born term should dominate. We conclude that more experimental data on charge capture from hydrogen atoms to the continuum of bare projectiles with various charge numbers Z_p is needed to clarify which mechanism is responsible for the asymmetry in the cusp.

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[1]G. B. Crooks and M. E . Rudd, Phys. Rev. Lett. 25, 1599 (1970).

[2]A. Salin, J. Phys. B 2, 631 (1969); 2, 1255 (1969).

[3]J. Macek, Phys. Rev. A 1, 235 (1970).

[4]K. Dettmann, K. G. Harrison, and M. W. Lucas, J. Phys. B 7, 269 (1974).

[5]N. Meckbuch, K. C. R. Chiu, H. H. Brongersma, and J. Wm. McGowan, J. Phys. B 10, 3255 (1977); C. R. Vane, J. A. Sellin, M. Suter, G. D. Alton, S. B. Elston, P. M. Griffin, and R. S. Thoe, Phys. Rev. Lett. 40, 1020 (1978); M. Suter, C. R. Vano, J. A. Sellin, S. B. Elston, G. D. Alton, and R. S. Thoe, *ibid.* 41, 399 (1978).

[6]R. Shakeshaft and L. Spruch, Phys. Rev. Lett. 41, 1037 (1978); our notation follows this reference.

[7]H. A. Bethe and E. E. Salpeter, *Quantum Mechanics of One- and Two-Electron Atoms* (Academic, New York, 1957), see Eq. (9.12).