

## Necessary conditions for the initiation and propagation of nuclear-detonation waves in plane atmospheres

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The basic conditions for the initiation of a nuclear-detonation wave in an atmosphere having plane symmetry (e.g., a thin, layered fluid envelope on a planet or star) are developed. Two classes of such a detonation are identified: those in which the temperature of the plasma is comparable to that of the electromagnetic radiation permeating it, and those in which the temperature of the plasma is much higher. Necessary conditions are developed for the propagation of such detonation waves for an arbitrarily great distance. The contribution of fusion chain reactions to these processes is evaluated. By means of these considerations, it is shown that neither the atmosphere nor oceans of the Earth may be made to undergo propagating nuclear detonation under any circumstances.

### I. INTRODUCTION

The possibility that a nuclear explosion might trigger the nuclear detonation of the atmosphere or oceans of the Earth has been seriously investigated on several occasions since 1943.<sup>1-3</sup> Despite resolving all physical uncertainties in such a way as to favor a detonation, these investigations all reached the conclusion that such a detonation is impossible.

The present paper reviews and extends these previous studies by deriving the general necessary conditions for the initiation and propagation of nuclear-detonation waves in plane atmospheres or layered fluids, taking into account the great advances in theoretical, experimental, and computational physics that have been made in the interim. In particular, accurate experimental measurements of cross sections and branching ratios have recently become available for reactions of interest, new reaction mechanisms have been proposed, more precise theoretical reaction-rate formalisms have been worked out, and finally, numerical models have been constructed which allow the detailed exploration of the effects of nonthermal nuclear reactions. As we shall see, the effect of these advances is to reduce the physical uncertainties in such a way as to preclude the detonation of the atmosphere or oceans by an even greater margin.

Section II deals with the general characteristics and types of nuclear-detonation waves, while Sec. III treats the needed nuclear cross sections. Sections IV-VI give detailed conditions for the existence of nonequilibrium, equilibrium, and fusion-chain-mediated nuclear-detonation waves, respectively, and further show that such conditions cannot be met in the Earth's atmosphere. Section VII describes detailed computer calcula-

tions which lead to the same conclusion. Section VIII treats nuclear-detonation waves in an oceanic environment, and demonstrates their impossibility for the case of the Earth. Finally, Section IX summarizes our principal conclusions.

### II. GENERAL CHARACTERISTICS OF NUCLEAR-DETONATION WAVES

A nuclear-detonation wave is basically a shock wave which has its energy and propagation velocity maintained against hydrodynamic and heat-conduction losses by nuclear reactions occurring within the wave. Most dominant features of the power flow in a nuclear-detonation wave bear a direct analogy to those encountered in chemical-detonation wave theory.<sup>4,5</sup> Such a wave rapidly reaches a steady-state configuration in its co-moving frame in which the locally deposited reaction energy flows in a time-stationary fashion into doing hydrodynamic work on the material both ahead and behind the wave (thereby shock-heating and compressing the material ahead of the wave, owing to its relatively much lower temperature and pressure), and into internal energy of the immediately post-shock material, from which it flows by radiative and electronic thermal conduction into all cooler portions of the material, both pre-shock and further post-shock. Nuclear-detonation waves have been studied previously in connection with supernova explosions<sup>6-8</sup> and laser-induced fusion.<sup>9-13</sup>

Sufficient conditions for the propagation of a self-sustaining detonation depend on the precise nature of the interactions between the plasma components, and are discussed in a general way by Zel'dovich and Kompaneets.<sup>5</sup> Two *necessary* conditions that follow from conservation of energy and momentum are

$$\dot{E}_N > \dot{E}_{\text{rad}} \quad (\text{ignition condition}) \quad (1)$$

over some portion of the wave, and that we have

$$\rho \mathcal{E}_{\text{NI}} > E_{\text{int}} \quad (\text{breakeven condition}), \quad (2)$$

where  $\dot{E}_N$  is the rate of nuclear energy generation per unit volume,  $\dot{E}_{\text{rad}}$  is the rate of radiative energy loss per unit volume,  $\mathcal{E}_{\text{NI}}$  is the total nuclear energy generated per gram of material, and  $E_{\text{int}}$  and  $\rho$  are the internal energy density and matter density, respectively, measured at a common point in the detonation. (Here we have assumed the internal energy of the wave is large compared to the ambient internal energy of the unshocked fuel.)

The most striking difference between chemical- and nuclear-detonation waves is that the latter generate  $\sim 10^7$  times more energy per gram of fuel, resulting in typical nuclear-detonation wave velocities of several thousand km/sec, compared with a few km/sec for the chemical case.

If no radiation were present, a nuclear-detonation wave would also be  $\sim 10^7$  times hotter than a chemical wave (i.e.,  $kT \sim 3$  MeV vs  $\frac{1}{3}$  eV). In general, however, the enormous heat capacity of the radiation field at these high temperatures serves to limit greatly the temperatures actually attained in nuclear-detonation waves. In particular, if the radiation field is in thermal equilibrium with the detonating material, the temperature  $T$  and the internal energy  $E_{\text{int}}$  are related by (neglected relativistic- electron effects)

$$aT^4 + \frac{3}{2}\rho N_A kT/\bar{A} = E_{\text{int}}, \quad (3)$$

where  $a = 7.56 \times 10^{-15}$  erg cm $^{-3}$ °K $^{-4}$  is the radiation-energy-density constant,  $N_A$  is Avogadro's number,  $\bar{A}$  is the mean atomic number for all plasma components including electrons, and  $k$  is Boltzmann's constant. For a typical internal energy corresponding to 1 MeV/nucleon, we find  $kT = 1.6$  keV for  $\rho = 10^{-3}$  g/cm $^3$ , and  $kT = 9.2$  keV for  $\rho = 1$  g/cm $^3$ .

The degree of radiative equilibration, and thus the temperature of the detonation is determined by the optical thickness of the detonating region and the time available for radiation to be emitted and thermalized as the wave sweeps over a given element of matter. The principal radiative processes involved are bremsstrahlung,<sup>14</sup> which provides direct radiative cooling, and inverse Compton scattering,<sup>15</sup> which for sufficiently great optical thicknesses can greatly magnify the effect of bremsstrahlung cooling by up-scattering the low-energy photons it produces at the expense of electron thermal energy. The question of radiative equilibration within a shock wave has been extensively studied in astrophysics with re-

gard to supernova shock waves<sup>16,17</sup> and neutron-star accretion,<sup>18,19</sup> as well as by Konopinski *et al.*<sup>1</sup> It is found in these studies that for phenomena whose specific energy is  $\sim 1$  MeV/nucleon, 10–100 (typically 30) Compton-scattering events (per low-energy bremsstrahlung photon) are required to bring the radiation field into approximate energy equilibrium with the electrons (see also Ref. 10).

Most of the detonation waves studied to date in astrophysics<sup>6-8</sup> have involved very optically thick systems which, consequently, attain full radiative equilibrium, and experience essentially no radiative losses in the sense of Eq. (1). Detonation waves studied in laser-induced micro-explosions,<sup>9-13</sup> on the other hand, have generally involved relatively optically thin pellets in which the radiation field is so far from equilibrium that only a small fraction of the pellet's internal energy resides in radiation. For detonations to propagate in such (nonequilibrium) circumstances, the rate of nuclear energy generation must exceed the *total* rate at which energy is being radiated by the plasma by an amount sufficient to make up hydrodynamic and particle heat-conduction losses.

For cases of intermediate optical thickness, such as the planetary and stellar atmospheres of present interest, one must consider necessary conditions for the existence of detonation waves in both equilibrium and nonequilibrium cases. This is undertaken in the following sections.

### III. NUCLEAR CROSS SECTIONS

On the basis of the large number of quantitative nuclear cross section measurements that have been made over the past four decades, theoretical and empirical models<sup>20-22</sup> have been developed that are capable of accurately predicting and/or fitting such cross sections over a wide range of nuclei and reaction types. In particular, these models are directly applicable to the nuclear reactions that have been implicated in the possible detonation of the Earth's atmosphere or ocean by nuclear bomb explosions. Using these models, the total cross section  $\sigma_{12}(E)$  for the reaction of nuclei of types 1 and 2 can be expressed in the form<sup>22</sup>

$$\sigma_{12}(E) = S e^{-2\pi\eta} / E \text{ b}, \quad (4)$$

where  $E$  is the center-of-mass energy of the two colliding particles in MeV,  $S$  is the reaction strength factor in MeV b, which is approximately independent of energy and is given by

$$S = \kappa(Z_1 Z_2 / \sqrt{A}) \exp(2X - \alpha E), \quad (5)$$

and  $e^{-2\pi\eta}$  is the factor by which the reaction strength is reduced by the necessity for the nuclei to penetrate their mutual Coulomb barriers. The terms in these equations are defined by

$$\alpha = 0.1215(AR^3/Z_1Z_2)^{1/2} \text{ MeV}^{-1}, \quad (6)$$

$$X = 0.52495(AZ_1Z_2R)^{1/2}, \quad (7)$$

$$\eta = 0.15748Z_1Z_2(A/E)^{1/2}, \quad (8)$$

$$A = A_1A_2/(A_1 + A_2). \quad (9)$$

Here  $A_1$  and  $A_2$ , and  $Z_1$  and  $Z_2$  are the atomic weight and the atomic number of nuclei 1 and 2, respectively,  $R$  is the nuclear interaction radius in fermi, and  $\kappa$  is the reflectivity factor. The advantage of expressing  $\sigma_{12}$  in this form is that the unspecified parameters are either explicitly energy independent (in the case of the interaction radius) or become so when averaged over reaction resonances (in the case of the reflectivity factor). For reactions in which the intermediate compound nucleus formed has  $(A_1 + A_2) > 20$  and an excitation energy of  $\geq 3$  MeV, so many relatively closely spaced resonances exist that they can be successfully treated statistically.<sup>22</sup> For the case when  $\sigma_{12}$  is taken to be the resonance-averaged total cross section for *all* nuclear reactions involving compound nucleus formation (including nuclear elastic scattering), such a statistical treatment gives<sup>22</sup>

$$\kappa = \begin{cases} 0.20 & \text{for proton-induced reactions} \\ 0.32 & \text{for } \alpha\text{-induced reactions} \\ 0.16 & \text{for neutron-induced reactions,} \end{cases} \quad (10)$$

$$R = \begin{cases} 1.25A_1^{1/3} + 0.1 & \text{for } n\text{- or } p\text{-induced reactions} \\ 1.09A_1^{1/3} + 2.3 & \text{for } \alpha\text{-induced reactions.} \end{cases}$$

The resulting total cross sections [calculated by use of Eq. (4)] are found to be accurate to within about a factor of 2 when compared with experimental measurements.

The assumptions involved for Eqs. (4)–(10) become invalid for non-neutron-induced reactions when

$$E \geq E_c \equiv 1.44Z_1Z_2/R \text{ MeV}, \quad (11)$$

where  $E_c$  is the Coulomb barrier energy for the reaction. In such cases, an upper limit on  $\sigma_{12}(E)$  is the larger of  $\pi R^2$ , the geometric cross section of the interacting nuclei, and  $\pi \lambda^2$ , the maximum  $s$ -wave resonant cross section, where  $\lambda$  is the de Broglie wavelength of the interacting system ( $\pi \lambda^2 = 0.6566/AE \text{ b}$ ).

In discussing the prospects for atmospheric ignition, the nuclear reactions for which detailed nuclear cross sections will be required are

$^{14}\text{N} + ^{14}\text{N}$  reactions yielding charged particles,  $^{14}\text{N}(\alpha, p)^{17}\text{O}$ , and  $^{11}\text{B}(p, 2\alpha)^4\text{He}$ . The existing experimental cross section measurements<sup>23</sup> for  $^{14}\text{N} + ^{14}\text{N}$  fusion reactions cover the lab energy range of 9.4–22 MeV and are plotted in Fig. 1. It is apparent the cross section is insignificant for lab energies below 10 MeV, owing to Coulomb repulsion, but rises rapidly to a near-geometric cross section of  $\sim 1$  b above 20 MeV. No resonances in the fusion cross section are seen to occur, as expected from the large number of closely spaced, overlapping energy levels in the intermediate  $^{28}\text{Si}^*$  compound nucleus. The  $^{14}\text{N} + ^{14}\text{N}$  fusion reactions were found to be dominated by the three-product reactions  $^{14}\text{N}(^{14}\text{N}, 2\alpha)^{20}\text{Ne}$ ,  $^{14}\text{N}(^{14}\text{N}, 2p)^{26}\text{Mg}$ ,  $^{14}\text{N}(^{14}\text{N}, \alpha p)^{23}\text{Na}$ , and  $^{14}\text{N}(^{14}\text{N}, pn)^{26}\text{Al}$  which are exothermic by 7.92, 7.36, 5.54, and 2.58 MeV, respectively. No significant number of two-product reactions [e.g.,  $^{14}\text{N}(^{14}\text{N}, \alpha)^{24}\text{Mg}$ ] were observed; indeed, such reactions are believed to constitute  $\leq 10\%$  of the  $^{14}\text{N}$  fusion reactions, due to their small statistical weights relative to three-product reactions when the very highly excited nature of the intermediate  $^{28}\text{Si}^*$  nucleus is taken into account.

Owing to its lack of resonant structure, the  $^{14}\text{N} + ^{14}\text{N}$  fusion cross section can be well fitted by the statistical formalism outlined above. In particular, the low-energy behavior of the cross

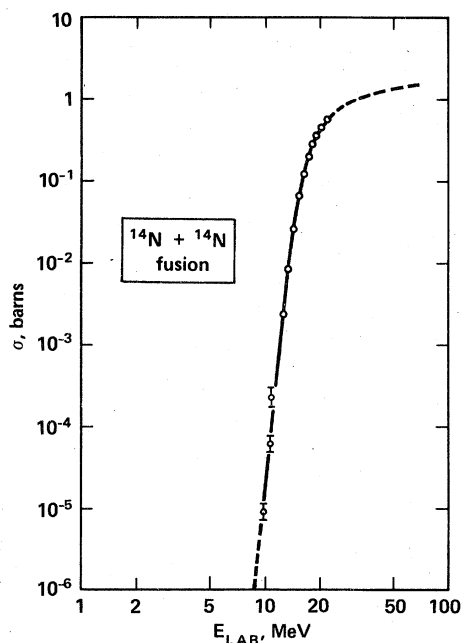


FIG. 1. Experimental (solid line and points) and extrapolated (dashed lines) cross sections for  $^{14}\text{N} + ^{14}\text{N}$  fusion as a function of the laboratory energy of the bombarding  $^{14}\text{N}$  nucleus.

section can be determined from Eqs. (4)–(9) by fitting  $\kappa$  and  $R$  to the existing data yielding  $\kappa \approx 0.03$  and  $R \approx 8.01$  F. The high-energy behavior can be extrapolated using the parametrization of Wong<sup>24</sup> in the form

$$\sigma(E) = (R^2 \hbar \omega_0 / 2E) \ln \{ 1 + \exp[2\pi(E - E_0) / \hbar \omega_0] \}, \quad (12)$$

with  $\hbar \omega_0 = 2.96$  MeV and  $E_0 = 8.25$  MeV. Equation (12) is believed accurate to  $\pm 30\%$ ,<sup>24</sup> up to a center-of-mass energy of 40 MeV, where direct spallation reactions instead of compound nuclear reactions become dominant.

The cross section for the  $^{14}\text{N}(\alpha, p)^{17}\text{O}$  reaction has been measured by several groups of experimenters,<sup>25–27</sup> and a compilation of the results is given in Fig. 2, labeled  $\sigma_{\text{EXPT}}$ . The results of the statistical theory for  $\alpha + ^{14}\text{N}$  compound nucleus formation with  $R = 3.33$  F,  $\kappa = 0.32$  are found to give a reasonable fit to the experimental cross section, when an average over the many narrow resonances is taken. This statistical cross section is thus adopted for use below, with the geometric cutoff at  $\pi R^2 = 0.35$  b, and is plotted as  $\sigma_{\text{ADPT}}$  in Fig. 3.

The cross section for the  $^{11}\text{B}(p, 2\alpha)^4\text{He}$  reaction is now well known<sup>28</sup> and is plotted in Fig. 5 (see Sec. VI).

The cross sections for the nuclear reactions

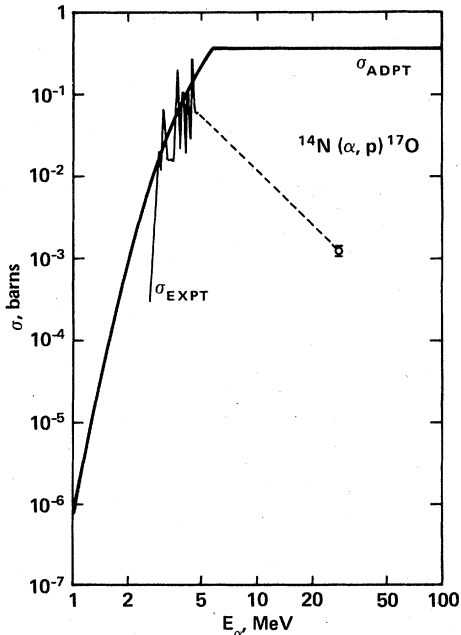


FIG. 2. Experimental ( $\sigma_{\text{expt}}$ ) and adopted ( $\sigma_{\text{adpt}}$ ) cross sections for the  $^{14}\text{N}(\alpha, p)^{17}\text{O}$  reaction as a function of laboratory energy of the bombarding  $\alpha$  particle.

potentially involved in ocean burning (i.e., the various reactions of hydrogen and oxygen isotopes) are well known because of their role in hydrogen burning in stars. Convenient representations of their thermal distribution-averaged cross sections  $\langle \sigma V \rangle$ 's are given in Ref. 21.

#### IV. RADIATIVE CONSTRAINTS ON NONEQUILIBRIUM NUCLEAR DETONATIONS

Since the temperature of the ions in a non-equilibrium nuclear detonation greatly exceeds that of the radiation field, energy transferred to radiation is very unlikely to be transferred back to the ions (via the electrons), and can therefore be regarded as lost. The ignition criterion [Eq. (1)] can then be recast to give a lower limit on the nuclear energy generation rate required for ignition at a given ion temperature in terms of the total radiative energy emission rate. Because the relative importance of the inverse Compton effect depends critically on the optical depth of the igniting region, it is convenient to first consider the less-stringent lower limit obtained by ignoring photon up-scattering by the hotter electrons and including only the energy loss due directly to bremsstrahlung radiation.<sup>14</sup>

The nuclear energy generation rate  $\dot{E}_N$  may be

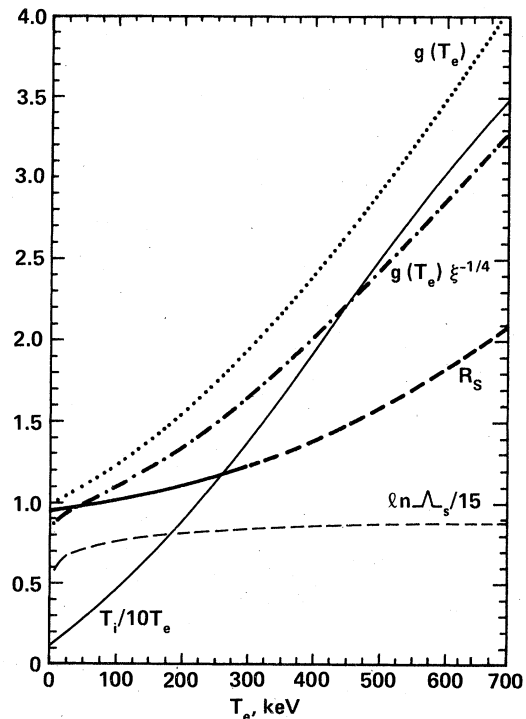


FIG. 3. Relativistic correction factors and other parameters related to electron-ion temperature balance for the case of the Earth's atmosphere.

written (cf. Ref. 29)

$$\dot{E}_N = [n_1 n_2 / (1 + \delta_{12})] \langle \sigma v \rangle Q, \quad (13)$$

where, as usual,  $n_1$  and  $n_2$  are the number densities of the reacting species;  $\delta_{12}$  is 1 if the reactants are identical and 0 otherwise;  $\langle \sigma v \rangle$  is the velocity distribution average of the product of the relative thermal velocity  $v$  of two potentially reacting ions and the reaction cross section at that velocity,  $\sigma$ ; and  $Q$  is the reaction energy. To further favor ignition, we will assume that all reaction products deposit their energy in the ions. We shall also assume here that this deposition takes place locally and instantaneously and that the ion distribution can be characterized by a temperature  $T_i$ . The possible effects of non-thermal ions are considered in Sec. VI. The bremsstrahlung radiation rate  $\dot{E}_{\text{brem}}$  may be written<sup>30</sup>

$$\dot{E}_{\text{brem}} = 3.32 \times 10^{-15} g(T_e) T_e^{1/2} n_e n_i Z_1^2, \quad (14)$$

where  $\dot{E}_{\text{brem}}$  is measured in units of  $\text{keV sec}^{-1} \text{cm}^{-3}$ . Here  $n_e$  and  $n_i$  are the electron and total ion number densities, respectively, in  $\text{cm}^{-3}$ ;  $T_e$  is the electron temperature in keV;  $g(T_e)$  is a monotone, slowly increasing function, lowered-bounded by unity, which accounts for the relativistic increase of radiation emission<sup>14,17</sup> as the average thermal electron energy becomes non-negligible compared to the rest energy of an electron; and

$$Z_1^2 = \frac{1}{n_i} \sum_{\text{all ionic species}} n_j Z_j^2, \quad (15)$$

where  $n_j$  and  $Z_j$  are the number density and atomic number of ion species  $j$ .

To relate  $T_i$  and  $T_e$ , we note that the energy flow from the ions to the electrons is given by<sup>30</sup>

$$\begin{aligned} \dot{E}_{i \rightarrow e} &= 1.51 \times 10^{-13} (T_i - T_e) (Z_0^2 / A_0) n_i n_e \\ &\quad \times R_s (\ln \Lambda_s - \frac{1}{2}) T_e^{-3/2} \text{ keV sec}^{-1} \text{ cm}^{-3}, \end{aligned} \quad (16)$$

$$R_s \approx 0.96 [1 + 0.0004 T_e (1 + 0.0045 T_e)] \quad \text{for } T_e \lesssim 300 \text{ keV, } n_e \sim 10^{20} \text{ cm}^{-3}, \quad (17)$$

where  $T_i$  and  $T_e$  are in keV,  $\ln \Lambda_s$  is the usual Spitzer-Coulomb logarithm including the quantum-mechanical correction,<sup>30</sup>  $R_s$  is a correction factor for ion screening and relativistic electron effects,<sup>31</sup> and

$$\frac{Z_0^2}{A_0} \equiv \frac{1}{n_i} \sum_{\text{all ionic species}} \frac{n_j Z_j^2}{A_j}, \quad (18)$$

where  $A_j$  is the atomic weight of ion species  $j$ . The electron and ion temperatures are initially the same and diverge under the influence of

nuclear heating and radiative cooling until the ion-electron energy transfer rate just balances bremsstrahlung losses, yielding the steady-state relation

$$T_i = T_e + (T_e / 6.74)^2 \xi(T_e) \text{ keV}, \quad (19)$$

where, for convenience, we have defined

$$\xi(T_e) = \frac{g(T_e) Z_1^2}{R_s (\ln \Lambda_s - \frac{1}{2})} \left( \frac{A_0}{Z_0^2} \right). \quad (20)$$

Solving for  $T_e$ , we find

$$T_e = (-1 + \sqrt{1 + 4CT_i}) / 2C, \quad (21)$$

where  $C = 2.20 \times 10^{-2} \xi(T_e) \text{ keV}^{-1}$ . In the physically interesting limit of  $4CT_i \gg 1$ , we have

$$T_e \approx (T_i / C)^{1/2} = [45.5 T_i / \xi(T_e)]^{1/2}. \quad (22)$$

Thus, from (14) we find

$$\begin{aligned} \dot{E}_{\text{brem}} &\approx 8.62 \times 10^{-15} n_e n_i T_i^{1/4} \\ &\quad \times g(T_e) \xi^{-1/4} Z_1^2 \text{ keV sec}^{-1} \text{ cm}^{-3}. \end{aligned} \quad (23)$$

Then, in order that  $\dot{E}_N > \dot{E}_{\text{brem}}$  (the *minimal* necessary requirement for the detonation to generate more thermonuclear power than it loses to radiation), we must have

$$\begin{aligned} Q^* \langle \sigma v \rangle &> 4.85 \times 10^{-17} \left( \frac{1 + \delta_{12}}{n_1 n_2} n_i n_e \right) \\ &\quad \times g(T_e) \xi^{-1/4} T_i^{*1/4} Z_1^2, \end{aligned} \quad (24)$$

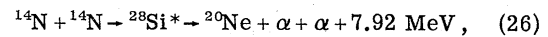
where  $Q^*$  and  $T_i^*$  are  $Q$  and  $T_i$  in MeV,  $\langle \sigma v \rangle$  is in  $\text{cm}^3/\text{sec}$ , and as before, number densities are in  $\text{cm}^{-3}$ .

For the sea level atmosphere with normal composition and standard temperature (STP),  $Z_0^2 / A_0 \approx 3.6$ ,  $Z_1^2 \approx 53$ ,  $n_i = 2.69 \times 10^{19} \text{ cm}^{-3}$ ,  $n_e / n_i \approx 7.2$ , and  $n_{14\text{N}} / n_i \approx 0.8$ . Under these conditions, criterion (24) for  $^{14}\text{N} + ^{14}\text{N}$  reactions becomes

$$Q^* \langle \sigma v \rangle > 5.8 \times 10^{-14} g(T_e) \xi^{-1/4} T_i^{*1/4}, \quad (25)$$

where  $g(T_e) \xi^{-1/4}$  is a factor of order unity which is plotted in Fig. 3 along with  $g(T_e)$ ,  $R_s$ ,  $\ln \Lambda_s / 15$ , and  $T_i / 10 T_e$  [for the exact solution (21)].

From Sec. III, we note that the most energetic of the dominant three-product  $^{14}\text{N} + ^{14}\text{N}$  fusion reactions is



and thus we can take 7.92 MeV as an upper limit on the  $Q$  value for  $^{14}\text{N}$  fusion. This upper limit becomes a substantial overestimate for  $^{14}\text{N} + ^{14}\text{N}$  center-of-mass energies above 10 MeV, due to the increasing fraction of endothermic fusion and spallation reactions. In addition, many of the

products are formed in excited states that emit  $\gamma$  rays and thus represent a source of radiative energy loss.<sup>23</sup> Subsequent reactions between  $^{14}\text{N}$  nuclei and the  $\alpha$ ,  $p$ , and  $n$  fusion reaction products may slightly increase the net  $Q$  value for  $^{14}\text{N}$  fusion. However, over the time scale involved in radiative cooling at atmospheric density ( $<10^{-5}$  sec), such reactions are either rare, endothermic, or both. (The effect of fusion chains is treated in Secs. VI and VII and is also found to be insignificant). Reactions involving minor atmospheric constituents (e.g., oxygen and argon) are required by criterion (25) to have cross sections much greater than 10 b in order to cause a detonation, owing to the large ratio of radiating to reacting particles. Such large cross sections have never been observed for *any* nuclear reaction involving charged nuclei, the largest being 5 b for the peak of the 107-keV resonance of the ( $\text{DT} \rightarrow n\alpha$ ) reaction. Thus it suffices to consider whether reaction (26) can cause a nonequilibrium detonation of the atmosphere.

The energy generation rate due to reaction (26) based on the cross sections of Fig. 1 is plotted in Fig. 4 together with the energy losses due to bremsstrahlung. By criterion (2), the maximum temperature  $T_{i,\text{max}}$  which the ions can reach in steady state with the electrons if all the nitrogen is burned is 853 keV (corresponding to an electron temperature of 139 keV). At this temperature, radiation losses exceed nuclear energy generation by a factor of  $7 \times 10^4$ . At lower ion temperatures this factor becomes astronomically large. Thus, by criterion (1) a nonequilibrium nuclear-detonation wave is not possible in the terrestrial atmosphere.

It is interesting to note that electron-ion bremsstrahlung radiative energy losses, which scale as  $Z^3$  on a per ion basis, are so much greater for the atmosphere than for a DT plasma that even if nitrogen had the same effective  $Q\langle\sigma v\rangle$  as DT, the best nuclear fuel known, burning in an optically (and thus neutron-) thin configuration, it would still fail by more than a factor of 5 to satisfy the *minimal* detonation criterion (25).

Moreover, these optimistic considerations have ignored the huge hydrodynamic and thermal conduction losses inevitably associated with non-equilibrium thermonuclear detonations, as well as inverse Compton scattering losses, which greatly multiply the effect of bremsstrahlung losses in all but absolutely optically thin detonations. Moderate estimates of the combined effects of these factors indicate that the *minimum* "safety factor" of  $7 \times 10^4$  precluding the non-equilibrium nuclear detonation of the atmosphere (noted above) should be increased to  $10^6$ – $10^7$ .

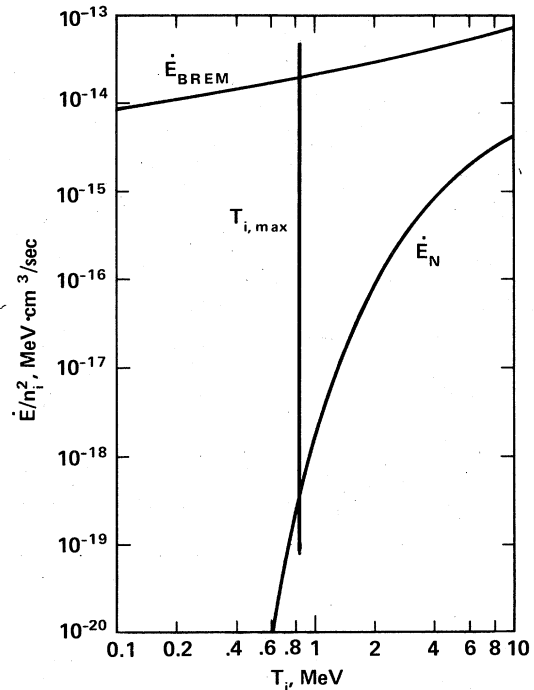


FIG. 4. Rate of bremsstrahlung energy loss  $\dot{E}_{\text{brem}}$  in a plasma of atmospheric density and composition, compared with an upper limit to its rate of nuclear energy generation  $\dot{E}_N$ , as a function of the ion temperature  $T_i$ .  $T_{i,\text{max}}$  is the largest steady-state ion temperature that could be reached if all the nitrogen in the plasma were burned to  $^{20}\text{Ne}$  via reaction (26).

## V. EQUILIBRIUM NUCLEAR DETONATIONS

As was discussed in Sec. II, the radiation field in an optically thick nuclear-detonation wave will typically be in near equilibrium with the electrons after an average photon has undergone about 30 Compton scatterings within the hot, burning region. An average photon will diffuse out of the wave during this number of scatterings unless the half-width  $l$  of the wave exceeds  $\sim\sqrt{30} \approx 5$ –6 Compton scattering lengths ( $\equiv 1/n_e\sigma_c$ ). This consideration sets a lower limit on the half-width of an equilibrium nuclear-detonation wave. Since  $n_e/\rho$  is  $\sim 3 \times 10^{23}$  electrons/g for all light elements except  $^1\text{H}$ ,  $^3\text{H}$ , and  $^3\text{He}$ , this half-width condition can be reexpressed as

$$\rho l \approx \frac{1}{2} \int_{\text{WF}} \rho(s) ds \approx 25\text{--}30 \text{ g/cm}^2, \quad (27)$$

where  $\int_{\text{WF}} ds$  is a line integral through the detonation wave front, and  $\rho$  is the characteristic density of the wave front.

The temperature of the ionic component of the plasma in an equilibrium detonation is neces-

sarily not greatly different from that of the electron component, except at extremely high matter densities ( $>10^4$  g/cm<sup>3</sup>) unlikely to be ever attainable outside of laser-induced fusion micro-explosions<sup>32</sup> or stellar cores. Besides being coupled far more strongly to the electron component of the plasma by ion-electron coupling [see Eq. (16)] at the much lower electron temperatures ( $\lesssim 10$  keV) which we have seen are characteristic of equilibrium detonations, the ions receive much less of the thermonuclear power arising from fusion reactions, since all the charged debris of such reactions also couple far more strongly to the much cooler electrons than in the nonequilibrium case, and deposit correspondingly less of their energy in the ion component of the plasma [see Eqs. (34) and (35) in Sec. VI]. Taken together these factors ensure that  $T_i \approx T_e$  for all equilibrium nuclear detonations of current interest.

Since the reaction rate of essentially all fusion reactions (in particular, see Figs. 1 and 2) drops sharply with decreasing ion temperature (owing to the relatively much stronger Coulomb repulsion between nuclei), the  $Q\langle\sigma v\rangle$  product is very much lower in an equilibrium detonation than in a nonequilibrium detonation. This is more than balanced, however, by the elimination of radiation losses in radiative equilibrium. Thus, any nuclear fuel may be burned in radiative equilibrium, with an efficiency limited only by hydrodynamic losses associated with explosive disassembly of the burning fuel. Thus the sun is able to efficiently burn protons, despite their being  $\approx 25$  orders of magnitude less reactive than DT, although it could not do so without the hydrodynamic confinement supplied by gravity and the radiative confinement supplied by its huge optical thickness.

The condition for negligible radiative energy loss is that the energy-weighted diffusion velocity  $v_\gamma$  of a photon across the wave's half-width be much less than the velocity of the detonation wave with respect to the detonating material, which by the Chapman-Jouget relation<sup>5</sup> (in the absence of losses) is just equal to the final sound speed in the detonation products,  $c_s$ . This condition can thus be expressed in the form

$$v_\gamma \sim \frac{4}{3} c / \kappa \rho l \ll c_s, \quad (28)$$

where  $c$  is the speed of light, and  $\kappa$  is the total radiative opacity of the detonating material. The portion of  $\kappa$  due to Compton scattering is  $\kappa_c \equiv \sigma_c \rho / n_e \approx 0.2$  cm<sup>2</sup>/g for the usual case of  $n_e / \rho \approx 3 \times 10^{23}$  electrons/g discussed above. For low- $Z$  plasmas and  $T_e \gtrsim 1$  keV, Compton scattering

is the dominant source of radiative opacity. Since the sound speed in an equilibrium detonation wave is typically  $(2-3) \times 10^8$  cm/sec (see Sec. II), the condition for radiation being trapped within detonation waves in low- $Z$  plasmas becomes

$$\rho l \gg 650 \text{ g/cm}^2. \quad (29)$$

Equation (29) thus represents a necessary condition for the attainment of an equilibrium detonation, except for the unique case of DT, which burns sufficiently well at the low temperatures characteristic of equilibrium detonation waves to tolerate substantial radiation losses. The general ignition criteria for equilibrium nuclear-detonation waves is still given by Eq. (1) with  $\dot{E}_{\text{rad}} = \bar{V}(aT^4 \bar{v}_\gamma) \sim acT^4 / \kappa \rho l^2$ , where  $T$  is the common plasma temperature and  $a$  is the equilibrium radiation density constant given in Sec. II.

The condition for hydrodynamic losses not to quench the detonation is that the characteristic nuclear burn time  $t_{\text{burn}}$  not be much greater than the hydrodynamic disassembly time  $t_{\text{hydro}}$ . This can be written

$$t_{\text{burn}} \approx \hat{A} / \rho \langle \sigma v \rangle N_A \lesssim l / c_s \approx t_{\text{hydro}} \quad (30)$$

or

$$c_s \hat{A} / \rho l \langle \sigma v \rangle N_A \lesssim 1, \quad (31)$$

where  $\hat{A}$  is the plasma mass associated with one nucleus of the most abundant reactant species in the reaction of interest, in atomic mass units. An upper limit on the maximum temperature  $T_{\text{max}}$  and thus the maximum value of  $\langle \sigma v \rangle / c_s$ , can be found from Eqs. (2) and (3) by optimistically assuming that all the nuclear fuel burns as it passes through the detonation wave. We may then rewrite (3) above as

$$T_{\text{max}} < (\rho Q' / a)^{1/4}, \quad (32)$$

where  $Q'$  is the nuclear reaction energy available per unit mass of nuclear fuel. These results yield a minimum necessary value of  $l$  that material of given density  $\rho$  must have to sustain an equilibrium nuclear-detonation wave.

For the case of the Earth's atmosphere ( $Q' \approx 3 \times 10^{17}$  erg/g and  $\rho \approx 2 \times 10^{-3}$  g/cm<sup>3</sup>, about twice the ambient atmosphere density<sup>5</sup>), we find  $T_{\text{max}} < 1.4$  keV. The nuclear reaction rate  $\langle \sigma v \rangle$  for nitrogen-nitrogen reactions is so low at this temperature ( $\sim 10^{-180}$  cm<sup>3</sup>/sec) that even if all the nitrogen and oxygen in the universe were somehow to be assembled so that their density was that of the Earth's atmosphere, and the entire mixture heated to 1.4 keV and maintained in this condition, not one single nitrogen-nitrogen fusion reaction would take place in the lifetime of the universe. Consideration of minor atmospheric constituents

does not appreciably improve the prospects for detonation. The Earth's atmosphere thus fails to support an equilibrium nuclear detonation by a literally astronomical margin at the nuclear reaction rate corresponding to the maximum temperature that could be attained if the atmosphere were to burn to completion.

## VI. FUSION CHAIN REACTIONS

In discussing nonequilibrium detonation waves, we assumed that the ions had a thermal distribution. Jetter<sup>33</sup> and McNally<sup>34,35</sup> have suggested however, that the fusion products, which are generally produced with a much higher energy and nuclear reactivity than the rest of the plasma, may induce a significant number of nuclear reactions before they slow down, perhaps enough to lead to a diverging, nonthermal fusion chain reaction.

The principal constraints on such a chaining process are that the potentially reactive fusion products, termed "chain centers," will be so rapidly slowed by Coulomb friction with the ions or electrons that they will not have a significant chance to react, or that they will be absorbed by reactions which produce no new chain centers. These constraints can be expressed by requiring that

$$\nu f_E \prod_j \frac{\sigma_{Nj} n_{Nj}}{\sigma_{sj} n_{sj} + \sigma_{Aj} n_{Aj} + \sigma_{Nj} n_{Nj}} > 1 \quad (33)$$

for a chain reaction to occur. Here  $\sigma_{Nj}$  is the characteristic cross section for a chain-center producing reaction to occur,  $\sigma_{sj}$  and  $\sigma_{Aj}$  are the cross sections for a chain center to be stopped or absorbed, respectively,  $n_{Nj}$  is the number density of ions with which the chain centers may react to make new chain centers,  $n_{sj}$  and  $n_{Aj}$  are the effective number density of particles contributing to stopping or absorbing chain centers, respectively,  $f_E$  is the average fraction of the chain centers that escape from the reacting region during a chain cycle, the subscript  $j$  refers to reaction step  $j$  of the chain, and  $\nu$  is the factor by which the number of chain centers would be increased in the absence of losses per completed chain cycle. For the physically interesting case when the chain centers move much faster than the background ions, but still much slower than the electrons, we have<sup>30</sup>

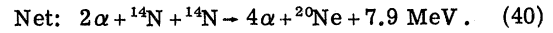
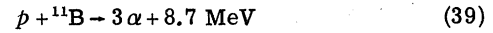
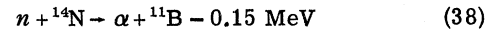
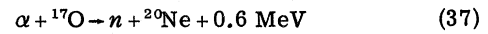
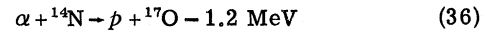
$$\sigma_{se} \approx 547 \frac{1}{\sqrt{A_c}} \frac{Z_c^2}{E_c^{1/2} (\text{MeV}) T_e^{3/2}} \left( \frac{\ln \Lambda}{15} \right) \text{b}, \quad (34)$$

$$\sigma_{si}^c \approx 1.95 \frac{A_c}{A_i} \frac{Z_c^2 Z_i^2}{E_c^2 (\text{MeV})} \left( \frac{\ln \Lambda}{15} \right) \text{b}, \quad (35)$$

where  $\sigma_{se}$  is the stopping cross section due to

electron Coulomb drag and  $\sigma_{si}^c$  that due to ion Coulomb drag;  $A_c$  and  $A_i$ , and  $Z_c$  and  $Z_i$  are the atomic weights and atomic numbers of the chain centers and background ions, respectively,  $E_c$  (MeV) is the energy of a chain center in MeV, and (as usual)  $T_e$  is in keV. In addition, there is a nuclear scattering contribution to the ion stopping cross section  $\sigma_{si}^N$  which, though quite variable, is typically  $\sim 1$  b. Since all measured cross sections for nuclear reactions between any two charged nuclei are  $\lesssim 1$  b, with the exception of the 5-b resonance in the DT fusion reaction, Eqs. (33)–(35) place severe constraints on the conditions under which fusion chains may propagate.

As a specific example, consider the fusion chain which McNally<sup>35</sup> has suggested to be the "most dangerous" with respect to the ignition of the atmosphere:



The highest-energy  $\alpha$  produced in this chain has an energy of 3.9 MeV [plus about  $\frac{2}{3}$  of the center-of-mass energy involved in reaction (39)], while an average  $\alpha$  has an energy of  $\lesssim 2$  MeV. Conservatively taking  $E_c = 5$  MeV,  $\ln \Lambda = 15$ , and  $n_N/n_s = 1$  (where both reaction centers and stopping particles are considered to be nitrogen nuclei), and ignoring stopping effects other than ion Coulomb drag, we find from (33) and (35) that a *minimum* necessary condition for the net reaction (40) (considered exceedingly generously as a one-step fusion chain) to occur is  $\sigma_N > 4.4$  b which, as noted above, is considerably greater than the largest non-DT charged-particle nuclear reaction cross section.

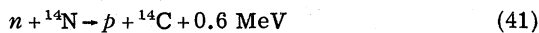
In addition, from (34) we see that unless  $T_e \gg 23$  keV, stopping effects due to electron Coulomb drag will require  $\sigma_N$  to be even larger. For example, at  $T_e = 1.4$  keV, the maximum temperature that could be involved in the equilibrium detonation of the atmosphere,  $\sigma_N$  would have to exceed 290 b for a fusion chain reaction to propagate.

The considerations of Sec. III, however, preclude values of  $\sigma_N$  greater than 0.4 b, and this is confirmed by existing experimental measurements (see Fig. 3).

The fusion chain (36)–(40) is also ruled out by other independent arguments. First, the  $p + {}^{11}\text{B}$



reaction (39) has been extensively studied because of its controlled-thermonuclear-reaction (CTR) interest,<sup>36,37</sup> and its cross section is now well-known.<sup>28</sup> Detailed computer-based simulation studies in which a beam of protons of optimal energy was injected into a very hot ( $T_e \sim 50$  keV)  $^{11}\text{B}$  plasma<sup>37,38</sup> resulted in the production of less than 15% as much energy by nonthermal nuclear reactions as was originally present in the proton beam. Thus reaction (39), the *only* appreciably exothermic reaction in the fusion chain, would in fact represent an energy *sink* for the non-thermal ions. This result is readily appreciated by comparing the  $p^{11}\text{B}$  nuclear cross section to the Coulomb stopping cross section for protons on nitrogen, as is done in Fig. 5. It is apparent from these cross sections that the factor in Eq. (33) corresponding to reaction step (39) (termed a "per-step-loss" factor) will be *at most* 0.3 (even if  $n_{11\text{B}} \sim n_{14\text{N}}$ ). Second, the well-known reaction



competes with reaction (38), as does the nuclear elastic scattering of neutrons on  $^{14}\text{N}$ . An upper limit on the per-step-loss factor for reaction (38) is  $\approx 0.5$  (cf Ref. 39). Similarly, the per-

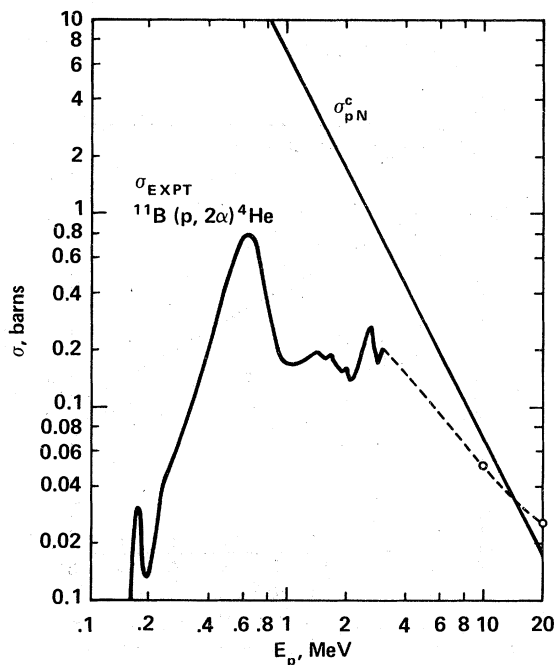


FIG. 5. Experimental cross section  $\sigma_{\text{EXPT}}$  for the  $^{11}\text{B}(p, 2\alpha)^4\text{He}$  reaction, compared with the proton stopping cross section due to Coulomb friction with a background nitrogen plasma,  $\sigma_{pN}^c$ , as a function of  $E_p$ , the laboratory energy of the bombarding proton.

step-loss factor for reaction (37) can be conservatively taken to less than 0.1 [cf. Ref. 40 and Eq. (35), even assuming  $^{17}\text{O}$  is as abundant as  $^{14}\text{N}$ ], while that for reaction (36) is at most 0.05 due to the large Coulomb drag cross sections for  $\alpha$  particles given above.

Combining these results, and noting  $\nu=2$  for fusion chain (36)–(40), we find that *at most*  $1.5 \times 10^{-3}$  of the effective chain centers present at the beginning of each fusion chain cycle will be present or have been replaced at its completion. Thus, this "chain" would die out exceedingly rapidly, even assuming it could be initiated by an external source of chain centers.

Likewise, no other fusion chain which has been proposed to be involved in atmospheric nuclear ignition comes at all close to diverging, even when very generous cross section estimates are used; rather all of them very rapidly "converge" to zero reaction rate.

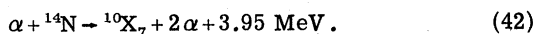
We further note that in addition to satisfying criterion (33), a fusion chain must either make up the radiation losses considered in Sec. IV, or face the exceedingly large electron stopping cross sections characteristic of relatively low temperature equilibrium detonations. Eq. (23) provides an approximate but, nonetheless, very severe criterion for a nonequilibrium fusion chain to exist if the thermally averaged  $\langle\sigma v\rangle$  is replaced with an appropriate nonthermal distribution average, and  $T_i$  is replaced with  $\frac{2}{3}E_e$ . Also, since the Compton-scattering  $\langle\sigma v\rangle$  is always much greater than fusion-chain  $\langle\sigma v\rangle$ 's, the radiation field in an optically thick medium will typically equilibrate before the chain has progressed more than a few generations, even with very optimistic assumptions about the nature of the chain.

Finally, we note that, as we have assumed above, the electrons should remain quite Maxwellian, since the energy-exchange coupling constant between electrons is  $A_i/A_e \geq 1836$  times that between ions and electrons,<sup>30</sup> while the ion-electron temperature ratios involved are generally much smaller than this factor.

#### VII. PROSPECTS FOR THE NUCLEAR DETONATION OF THE ATMOSPHERE: DETAILED NUMERICAL CALCULATIONS

While the preceding analytical considerations preclude the ignition of the Earth's atmosphere under any circumstances, it is of some interest to further confirm this conclusion by means of detailed numerical calculations. Such calculations were made using the FOKN nonthermal nuclear-burn computer code developed originally for laser-fusion calculations by Lee *et al.*,<sup>41</sup> and

later extended by Scharlemann, *et al.*<sup>42</sup> In its present form, this code follows the time evolution of the energy distributions of the reactants and products explicitly, utilizing the Fokker-Planck approximation for low-angle Coulomb scattering, and transfer matrices for high-angle Coulomb, nuclear, and radiative processes. The treatment of both the distribution functions and the radiative emission rates is relativistically correct, and an infinite isotropic, homogeneous plasma is assumed. In addition, the slight differences between the  $\ln\Lambda$  terms involved in the Coulomb interaction between different particle species are taken into account. The effects of an injected particle source are modeled by adding particles at a specified rate to a given energy group. Exponential *number* loss rates of one or more particle species can also be specified. This code is, thus, well suited to evaluate the possibility of nonequilibrium and fusion chain nuclear-detonation modes. Indeed, the major simplifications inherent in FOKN (i.e., the omission of hydrodynamic and inverse-Compton energy losses) greatly favor such detonation modes. As applied to the problem of atmospheric detonation, the code considers five particle species:  $^{14}\text{N}$ ,  $^4\text{He}$ ,  $^{20}\text{Ne}$ ,  $^{10}\text{X}_7$ , and electrons. Here  $^{10}\text{X}_7$  is a hypothetical nuclei with an atomic weight of 10 and a  $Z$  of 7, two of which mock up the effects of a  $^{20}\text{Ne}$  nucleus in terms of mass and radiative emission (which scales as  $Z^2$ ). In addition to Coulomb scattering between all species, the  $^{14}\text{N}$ - $^{14}\text{N}$  fusion reaction (26) is included, as well as an exceedingly generous representation of the fusion chain (36)–(40), given by



The cross sections assumed for these reactions are the measurements, upper limits, and/or fits for the  $^{14}\text{N} + ^{14}\text{N}$  fusion and  $^{14}\text{N}(\alpha, p)^{17}\text{O}$  reactions given in Figs. 2 and 3, except that the rate for reaction (42) is multiplied by a factor of  $1.5 \times 10^{-2}$  to take into account the upper limit per-step-loss factors for reactions (37)–(39) derived in Sec. VI.

Four cases were studied, in which the model atmosphere was subjected to conditions much more extreme than would result from any conceivable nuclear bomb explosion. In case I, an atmospheric-density nitrogen plasma ( $n_i = 2.55 \times 10^{19} \text{ cm}^{-3}$ ) with  $T_e = T_i = 10 \text{ keV}$  (initially) was bombarded with an equal number of 3.8-MeV  $\alpha$  particles injected into it in  $10^{-8} \text{ sec}$ . Case II was the same, except that the  $\alpha$  energy and electron temperature were more realistically taken to be 2.6 MeV and 100 keV, respectively. In case III, no  $\alpha$ 's were injected and the atmospheric-density nitrogen plasma initially had

$T_i = 853 \text{ keV}$  and  $T_e = 139 \text{ keV}$ , the maximum temperatures consistent with thermal steady state between electrons and ions even if all of the  $^{14}\text{N}$  were burned (see Sec. IV). To test the effects of an initial state *contrived* to be far from steady state, the extreme case of an atmospheric-density nitrogen plasma with  $T_i = 2.5 \text{ MeV}$  and  $T_e = 10 \text{ keV}$  (initially) was studied as case IV.

In all cases, the nitrogen plasma was very rapidly cooled (in times  $< 10^{-5} \text{ sec}$ ) by radiation losses, and no divergent fusion chain effects were observed. The temperature and energy generation time history of case IV is shown in Fig. 6. In no case did the nuclear reactions occurring in the cooling plasma come within a factor of 2900 of achieving breakeven, i.e., of equaling the energy originally present or injected into the plasma, as required by condition (2). Specifically, the energies produced in the four cases were  $2.4 \times 10^{-5}$ ,  $1.1 \times 10^{-5}$ ,  $8.9 \times 10^{-7}$ , and  $3.4 \times 10^{-4}$ , respectively, of that required for breakeven. The inclusion of the hydrodynamic and inverse-Compton effects omitted by FOKN, would, in most cases, lower these results by more than an order of magnitude. It would be nonphysical to run cases more energetic than these, because not enough nuclear energy is potentially available from the plasma to produce

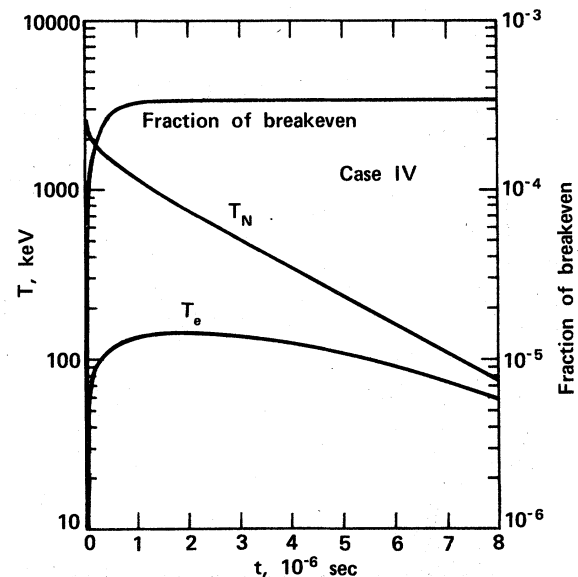


FIG. 6. Time evolution of the  $^{14}\text{N}$  and electron temperatures ( $T_N$  and  $T_e$ , respectively) for a  $2.55 \times 10^{19} \text{ cm}^{-3}$  nitrogen plasma, started at  $t=0$  with  $T_N=2.5 \text{ MeV}$  and  $T_e=10 \text{ keV}$  (case IV). Also shown is the fraction of breakeven represented by the nuclear energy generated (right-hand scale). The plasma is assumed to be perfectly confined, with bremsstrahlung radiation emission being the only energy loss.

any higher internal energies.

Finally, although the velocity spectra of the fusion-product ions did deviate significantly from a Maxwellian distribution in these runs, as expected, the  $^{14}\text{N}$  distribution remained very closely Maxwellian.

In general, the only deviation of the electron spectrum from Maxwellian was a very slight (2%–7%) depression of the low-energy tail due to their selective up-scattering by the more energetic ions, as expected from previous work.<sup>37,38,41</sup> The only exceptions to this behavior were transients occurring in the first  $10^{-8}$ – $10^{-7}$  sec of cases I and II, due to the co-injection of electrons with the  $\alpha$  particles to maintain charge neutrality, and case IV, due to huge initial mismatch between electron and ion temperatures. These deviations were too small and/or short lived to significantly affect the nuclear energy generated.

#### VIII. PROSPECTS FOR THE NUCLEAR DETONATION OF THE OCEANS

Nonequilibrium or fusion-chain-mediated detonations of the Earth's oceans can be immediately ruled out because there are no exothermic reactions between their principal components which have  $Q(\sigma v)$ 's at all comparable to those for radiative emission, even at the highest temperatures that could be reached if such reactions went

to completion. Specifically, (i)  $p+p$  reactions are mediated only by the weak interactions<sup>21</sup>; (ii) the  $^{16}\text{O}(p,\gamma)^{17}\text{F}$  reaction has a cross section of only  $\sim 15 \mu\text{b}$  above its Coulomb barrier<sup>43</sup> and  $^{16}\text{O}+p$  reactions producing charged particles are endothermic by several MeV<sup>44</sup>; and (iii)  $^{16}\text{O}+^{16}\text{O}$  reactions have reaction rates and  $Q$  values<sup>21</sup> considerably smaller than those for  $^{14}\text{N}+^{14}\text{N}$ . Thus, despite the fact that the effective  $Z$  of the ocean is a factor of 1.55 below that of the atmosphere, bremsstrahlung radiation losses still dominate nuclear energy generation by a very large margin, even when isotopic and other impurities are considered.

The prospects for initiating a propagating equilibrium thermonuclear detonation are relatively much more promising for the ocean than for the atmosphere, *a priori*, as the Coulomb barrier for proton-oxygen reactions is much lower than for nitrogen-nitrogen or helium-nitrogen ones, and because the medium is much more dense, implying that higher equilibrium temperatures may be attained (see Secs. II and III). Also, the ocean contains 0.016% deuterium by number relative to hydrogen, as well as 0.03% and 0.2% of  $^{17}\text{O}$  and  $^{18}\text{O}$ , respectively, relative to  $^{16}\text{O}$ , all of which may undergo exothermic, charged-particle-producing reactions with protons.

As was discussed in Sec. V, the propagation of an equilibrium detonation wave requires  $t_{\text{burn}} \lesssim t_{\text{hydro}}$ . We can write  $t_{\text{hydro}}$  in the form (for a plasma in equilibrium at temperature  $T$ , in keV)

$$t_{\text{hydro}} \approx l/c_s = l(\rho/\gamma P)^{1/2} = \begin{cases} (l/4.0 \times 10^7)(\bar{A}/T)^{1/2} \text{ sec} & \text{if matter pressure dominates} \\ (l/7.8 \times 10^6)\rho^{1/2}/T^2 \text{ sec} & \text{if radiation pressure dominates,} \end{cases} \quad (43)$$

where  $l$  is the characteristic detonation dimension in cm,  $\bar{A}$  is the average atomic weight per particle (including electrons), and  $\rho$  is in  $\text{g}/\text{cm}^3$ .

From (3) we see that radiation pressure exceeds matter pressure when

$$T > (3N_A \rho k / \bar{A} a)^{1/3} = 2.76(\rho/\bar{A})^{1/3} \text{ keV}. \quad (44)$$

If all the  $^{16}\text{O}$  in the ocean burned with the hydrogen [via the reactions  $^{16}\text{O}(p,\gamma)^{17}\text{F}$  ( $Q=0.60$  MeV) and  $^{17}\text{F}(p,\gamma)^{18}\text{Ne}$  ( $Q=3.92$  MeV), where we have neglected the beta decay of the  $^{17}\text{F}$  ( $t_{1/2}=66$  sec) due to the short hydrodynamic times involved (see below)], then an equilibrium temperature of 7.7 keV would be reached (where we have taken the detonating region to be twofold compressed<sup>5</sup> so that  $\rho=2 \text{ g}/\text{cm}^3$ ). At this temperature, we find, using (30) and Ref. 21, that  $t_{\text{burn}}(^{16}\text{O}) \approx 1.2 \times 10^8 \text{ sec} \approx 4$  years. On the other hand, for

$l=10^6$  cm, corresponding to an ocean depth of 10 km, we find  $t_{\text{hydro}}=3 \times 10^{-3}$  sec. Thus, by criterion (30) the oceans would fail to detonate via  $^{16}\text{O}$  burning by a factor of  $4 \times 10^{10}$ !

Burning all of the  $^{18}\text{O}$  in the ocean [via the reactions  $^{18}\text{O}(p,\alpha)^{15}\text{N}$  ( $Q=3.98$  MeV) and  $^{15}\text{N}(p,\alpha)^{12}\text{C}$  ( $Q=4.966$  MeV)] would suffice to raise its temperature to  $\approx 0.87$  keV at  $\rho=2 \text{ g}/\text{cm}^3$ , corresponding to a nuclear-burn time of  $1.0 \times 10^{18}$  sec and a hydrodynamic time of  $3.1 \times 10^{-2}$  sec. Thus  $^{18}\text{O}$  burning fails to propagate by a factor of  $3 \times 10^{19}$ .

Similarly, burning all the deuterium in the ocean [via the reaction  $^2\text{H}(p,\gamma)^3\text{He}$  ( $Q=5.494$  MeV)] would raise its temperature to 0.094 keV at  $\rho=2 \text{ g}/\text{cm}^3$ , yielding  $t_{\text{burn}}=1.1 \times 10^{11}$  sec and  $t_{\text{hydro}}=9.4 \times 10^{-2}$  sec, for a safety margin of  $1.2 \times 10^{12}$ .

Similar calculations for other minor oceanic

constituents (such as  $^{12}\text{C}$ ), show that their nuclear-burn rates are too slow by at least as many orders of magnitude to maintain a nuclear detonation.

We, therefore, conclude that thermonuclear-detonation waves cannot propagate in the terrestrial ocean by any mechanism by an astronomically large margin.

It is worth noting, in conclusion, that the susceptibility to thermonuclear detonation of a large body of hydrogenous material is an exceedingly sensitive function of its isotopic composition, and, specifically, to the deuterium atom fraction, as is implicit in the discussion just preceding. If, for instance, the terrestrial oceans contained deuterium at any atom fraction greater than 1:300 (instead of the actual value of 1:6000), the ocean could propagate an equilibrium thermonuclear-detonation wave at a temperature  $\geq 2$  keV (although a fantastic  $10^{30}$  ergs— $2 \times 10^7$  MT, or the total amount of solar energy incident on the Earth for a two-week period—would be required to initiate such a detonation at a deuterium concentration of 1:300). Now a non-negligible fraction of the matter in our own galaxy exists at temperatures much less than  $300^\circ\text{K}$ , i.e., the gas-giant planets of our stellar system, nebulas, etc. Furthermore, it is well known that thermodynamically-governed isotopic fractionation ever more strongly favors higher relative concentration of deuterium as the temperature decreases, e.g., the D:H concentration ratio in the  $\sim 10^2$  °K Great Nebula in Orion is about 1:200.<sup>45</sup> Finally, orbital velocities of matter about the galactic center of mass are of the order of  $3 \times 10^7$  cm/sec at our distance from the galactic core.

It is thus quite conceivable that hydrogenous matter (e.g.,  $\text{CH}_4$ ,  $\text{NH}_3$ ,  $\text{H}_2\text{O}$ , or just  $\text{H}_2$ ) relatively rich in deuterium ( $\geq 1$  at. %) could accumulate at its normal, zero-pressure density in substantial thicknesses or planetary surfaces, and such layering might even be a fairly common feature of the colder, gas-giant planets. If thereby highly enriched in deuterium ( $\geq 10$  at. %), thermonuclear detonation of such layers could be initiated artificially with attainable nuclear explosives. Even with deuterium atom fractions approaching 0.3 at. % (less than that observed over multiparsec scales in Orion), however, such layers might be initiated into propagating thermonuclear detonation by the impact of large (dia  $\geq 10^2$  m), ultra-high velocity ( $v \geq 3 \times 10^7$  cm/sec) meteors or comets originating from nearer the galactic center. Such events, though exceedingly rare, would be spectacularly visible on distance scales of many parsecs.

## IX. SUMMARY AND CONCLUSIONS

We have analyzed the general conditions for the initiation and propagation of nuclear-detonation waves of both the equilibrium and nonequilibrium varieties in plane symmetry, e.g., layered media. We specifically find that neither the Earth's atmosphere nor its oceans can propagate any type of nuclear detonation, by very large margins. We have considered the possibility of fusion-chain reactions and other nonthermal plasma phenomena, and found them of negligible importance.

In particular, we have shown the following.

(i) Even if nitrogen were many times as reactive as DT, the most reactive known fuel, the thermonuclear energy generation rate of the terrestrial atmosphere at *any* temperature would still not suffice to overcome the energy losses due to bremsstrahlung radiation and the inverse Compton effect.

(ii) Such high nuclear reactivities for nitrogen are precluded by basic physical laws governing the electrostatic repulsion of charged nuclei and the level density and parameters of nuclear energy states as well as by experimental measurements.

(iii) Energy lost to radiation cannot be utilized to initiate further nuclear reactions, because the huge heat capacity of the radiation field at atmospheric density results in a sufficiently low equilibrium temperature ( $\leq 1.4$  keV) that the electrostatic repulsion between nuclei prevents *any*  $^{14}\text{N}$ - $^{14}\text{N}$  reactions at all from occurring by an astronomically large factor ( $\sim 10^{45}$ ).

(iv) The fusion-chain reactions proposed by McNally fail not only owing the rapid slowing of the suggested chain centers by Coulomb drag, but also because of side reactions which absorb such chain centers, thereby precluding any possibility of a chain reaction.

(v) Detailed nonthermal nuclear-burn calculations were made in which the reactants, products, and electrons were not assumed to have Maxwellian velocity distributions, the kinematics and radiative emission were treated in a relativistically correct fashion, and separate Coulomb logarithms were calculated for each pair of interacting particles. These calculations included both  $^{14}\text{N} + ^{14}\text{N}$  fusion reactions and the "most dangerous" fusion chain ( $2\alpha + 2^{14}\text{N} \rightarrow ^{20}\text{Ne} + 4\alpha + 7.9$  MeV), assuming the highest physically possible reaction rates. Even at multi-MeV temperatures, no divergent chain effects occurred, the total nuclear energy generated always fell far below the input energy, and the material was always rapidly cooled by radiation

losses in  $<10^{-5}$  sec.

(vi) Similar considerations preclude the detonation of oceans of terrestrial composition, while admitting the possibility of detonating layers of suitable isotopic composition, density, and depth on planetary (and possibly stellar) surfaces.

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