Classical theory of scattering of an electron beam by a laser standing wave

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A classical theory of scattering of an electron beam by a laser standing wave (the Kapitza-Dirac effect) is presented. The prediction of the theory is in good agreement with all the experimental results which were reported independently by Bartell *et al.*, Schwarz *et al.*, Takeda *et al.*, and Pfeiffer. This shows that the interaction of a free electron with coherent radiation is basically a classical process.

I. INTRODUCTION

On the basis of the effect of "stimulated Compton scattering," Kapitza and Dirac suggested in 1933[1] that a beam of electrons of well-defined momentum $\vec{p}$ could be reflected from a standing light wave of wavelength λ, now known as "Kapitza-Dirac effect." They predicted that the reflection is required to obey the first-order Bragg condition with a lattice-spacing equivalent to $\frac{1}{2}\lambda$, and the probability of an electron being reflected is proportional to the square of the intensity of the light beam. They also suggested that the intensity of ordinary light source (green mercury light of $\lambda = 5460$ Å) is too low for any possible experimental observation.

The experimental investigation of this effect became possible after the advent of laser wave.[2] Observations of the scattering of electron beams by laser standing waves were first reported independently by Bartell-Thompson-Roskos[3] and Schwarz-Tourtellotte-Gaertner[4] in 1965. Subsequently, four independent and detailed experimental results of Schwarz,[5] Bartell-Roskos-Thompson,[6] Takeda-Matsui,[7] and Pfeiffer[8] were published. These results were not only different from each other but also inconsistent with the prediction of Kapitza and Dirac. In the experiments of Schwarz,[5] Bartell *et al.*,[6] and Takeda *et al.*,[7] the maximum deflection probabilities were reported not at the Bragg condition, but at smaller deflection angles. In Pfeiffer's experiment,[8] no deflected electrons were observed at the Bragg condition although the detection system used should have been capable of detecting deflection probabilities of less than 1%. Schwarz also reported that the deflection would be minimum if the alignment between the laser beam and the electron beam was set to satisfy the Bragg angle.[9] The confused situation of the experimental results has even caused some authors to argue, on the basis of the picture of the Kapitza-Dirac scattering, that Schwarz's results only represented noise.[8,10]

On the theoretical side, more-general quantum-mechanical treatments[11-17] as well as semi-quantum-mechanical treatments[18,19] were reported in the past decade by a number of authors. The quantum-mechanical approach has been developed rigorously so that it can be applied to experimental conditions. The semi-quantum-mechanical approach also provides another interpretation for the scattering process as diffraction by a grating. In spite of all these efforts, a satisfactory theoretical explanation of the experimental facts has not yet been obtained. Obviously, the basic question is whether the quantum-mechanical (stochastic) approach has been developed adequately to describe this effect or that the interaction of a free electron with coherent radiation is basically a classical (deterministic) process.[20] The latter implies that the quantum-mechanical picture of electron-radiation interaction may not be valid for coherent radiation. Since the fundamental property of a laser, in comparison with that of ordinary light, is characterized by its coherency and not by its intensity, it is reasonable to believe that the scattering of an electron beam by a laser standing wave may not be physically interpreted as the "stimulated Compton scattering" of the Kapitza-Dirac effect which was considered implicitly for incoherent light.

In this work, a classical theory of the scattering of an electron beam by a laser standing wave is given. It will be shown that the results obtained by this theory are in good agreement with all the recent experimental results, including those of Schwarz *et al.* A summary of numerical results obtained from the previous theories is given in the Appendix for comparison.

II. CLASSICAL THEORY

Our classical theory will be developed by considering the effect of an electromagnetic plane wave on the motion of a free electron[20,21] according to the following assumptions:

(i) The laser beam is a coherent and linearly polarized electromagnetic plane wave with neither angular divergence nor frequency spread. It forms an ideal standing wave with a totally reflecting mirror.

(ii) The incident electron beam is monochromatic and has no angular divergence.

(iii) In a very intense laser beam, the Lorentz force due to the field of the laser acting on each individual electron in the electron beam is so strong that the much smaller effects of the electron-electron interaction, the radiation reaction, and the influence of the electron beam on the laser beam shall not be considered.

(iv) Since the velocities of the electrons used in the experimental measurements are much smaller than the velocity of light, we shall assume the interaction being nonrelativistic.

A. Motion of an electron in a laser standing wave

According to assumptions (iii) and (iv), the equation of motion of an electron in an electromagnetic standing wave is given by the Lorentz force equation

$$mc\left(\frac{d\vec{\beta}}{dt}\right)=e(\vec{E}_s+\vec{\beta}\times\vec{H}_s), \quad (1)$$

where m, e, and $\vec{\beta}=\vec{v}/c$ are the inertial mass, electric charge, and velocity of the electron, respectively; c is the velocity of light; and $\vec{E}_s$ and $\vec{H}_s$ are the electric and magnetic fields of the standing wave. From assumption (i), the standing wave is formed by the superposition of two plane waves of equal intensity and the same frequency but traveling in opposite directions. Therefore, we have

$$\vec{E}_s=\vec{E}_+(\tau_+)+\vec{E}_-(\tau_-)$$

and

$$\vec{H}_s=\vec{H}_+(\tau_+)+\vec{H}_-(\tau_-) \quad (2a)$$

with,

$$\vec{H}_+(\tau_+)=\vec{n}\times\vec{E}_+(\tau_+)$$

and

$$\vec{H}_-(\tau_-)=-\vec{n}\times\vec{E}_-(\tau_-), \quad (2b)$$

where $\vec{E}_+$ and $\vec{H}_+$ are the electromagnetic field of the advanced plane wave with retarded time $\tau_+ = t-\vec{n}\cdot\vec{\rho}$, direction of wave propagation $\vec{n}$, and position vector $\vec{\rho}=\vec{r}/c$; $\vec{E}_-$ and $\vec{H}_-$ are those of the retarded wave with advanced time $\tau_-=t+\vec{n}\cdot\vec{\rho}$ and direction of wave propagation $-\vec{n}$; $\vec{r}$ is the position of the electron measured by the laboratory observer relative to whom the standing wave is defined. Substituting Eqs. (2a) and (2b) into Eq. (1), we obtain the basic equation of motion

$$\vec{F}=mc\left(\frac{d\vec{\beta}}{dt}\right)=e[(1-\beta_n)\vec{E}_+ + (1+\beta_n)\vec{E}_- + (\vec{\beta}\cdot\vec{E}_+ - \vec{\beta}\cdot\vec{E}_-)\vec{n}], \quad (3)$$

where $\beta_n=\vec{n}\cdot\vec{\beta}$.

The picture of interaction is schematically shown in Fig. 1 where there are four unit vectors $\vec{k}$, $\vec{l}$, $\vec{m}$, and $\vec{n}$. $\vec{l}$ is perpendicular to $\vec{n}$ and lies in the plane of incidence formed by $\vec{n}$ and $\vec{\beta}_0(=\vec{v}_0/c)$, the initial velocity of the electron. $\vec{k}$, $\vec{n}$, $\vec{l}$ forms a triad with $\vec{k}=\vec{n}\times\vec{l}$. $\vec{m}$, the direction of polarization of the laser beam, is perpendicular to $\vec{n}$ and makes an angle ξ with $\vec{l}$. The electron with velocity $\vec{\beta}_0$ enters the standing wave at a small incident angle θ_0. After passing through the interaction region, it leaves the standing wave with a final velocity $\vec{\beta}$ and an angle of deflection Φ. Now let the electric fields of the advanced and retarded waves be

$$\vec{E}_+=\vec{m}E_0\sin(\omega\tau_+ + \varphi)$$

and

$$\vec{E}_-=\vec{m}E_0\sin(\omega\tau_- + \varphi+\pi), \quad (4)$$

where ω is the angular frequency of the laser, φ is the phase factor specified by the initial time relation between the standing wave and the electron, and π is the phase change after reflecting from the mirror. Since $d\tau_+/dt=1-\beta_n$, $d\tau_-/dt=1+\beta_n$, $\vec{k}\cdot\vec{m}=\sin\xi$, and $\vec{l}\cdot\vec{m}=\cos\xi$, the three components of the equation of the electron motion (3) become

$$F_k dt = mcd\beta_k = eE_0[\sin(\omega\tau_+ + \varphi)d\tau_+ - \sin(\omega\tau_- + \varphi)d\tau_-]\sin\xi, \quad (5a)$$

$$F_l dt = mcd\beta_l = eE_0[\sin(\omega\tau_+ + \varphi)d\tau_+ - \sin(\omega\tau_- + \varphi)d\tau_-]\cos\xi, \quad (5b)$$

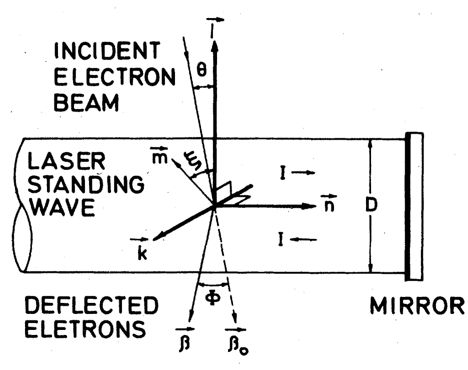

FIG. 1. Deflection of an electron by laser standing wave.

$$F_n = mc\left(\frac{d\beta_n}{dt}\right) = eE_0(\beta_k \sin\xi + \beta_l \cos\xi) \times[\sin(\omega\tau_+ + \varphi) + \sin(\omega\tau_- + \varphi)]\,. \tag{5c}$$

In the experimental investigations of the Kapitza-Dirac effect cited earlier, only the deflections of the electron beam along $\vec{n}$ were measured. Therefore, the following calculation on the electron motion is restricted to the $\vec{n}$ direction unless otherwise stated. Equation (5c) can be further simplified by eliminating β_k and β_l obtained from Eqs. (5a) and (5b). Let the initial conditions be $\vec{r} = \vec{r}_0$, $\vec{\beta} = \vec{\beta}_0$, and $\rho_n = \rho_{0n} = r_{0n}/c = -\tau_{+0} = \tau_{-0}$ at $t = 0$. The integration of Eqs. (5a) and (5b) is trivial and is given by

$$\beta_k = \beta_{0k} - 2(eE_0/mc\omega)[\sin(\omega t + \varphi)\sin\omega\rho_n - \sin\varphi\,\sin\omega\rho_{0n}]\sin\xi\,, \tag{6a}$$

$$\beta_l = \beta_{0l} - 2(eE_0/mc\omega)[\sin(\omega t + \varphi)\sin\omega\rho_n - \sin\varphi\,\sin\omega\rho_{0n}]\cos\xi\,, \tag{6b}$$

where $\beta_{0k} = 0$ because the electron beam is experimentally always aligned on the $\vec{n}$-$\vec{l}$ plane. With these results, we obtain from Eq. (5c),

$$\frac{d\beta_n}{dt} = 2\,\bar{\epsilon}^{1/2}\omega\beta_{0l}\cos\xi\,\cos\omega\rho_n \sin(\omega t + \varphi) - 2\,\bar{\epsilon}\omega[\sin(2\omega\rho_n)\sin^2(\omega t + \varphi) - 2\sin\varphi\,\sin\omega\rho_{0n}\cos\omega\rho_n \sin(\omega t + \varphi)]\,, \tag{6c}$$

where $\bar{\epsilon} = (eE_0/mc\omega)^2$. Note that each term on the right-hand side of this equation consists of a sinusoidal function of time. Since the velocity of the electron in the experiments is much smaller than that of light, the rapidly time-varying parts of the electron field interaction can be neglected. This assumption has been used in Refs. 12, 13, 15, and 16. As a matter of fact, these time-varying parts only cause a small broadening of the electron beam.[22] Thus we take the time average of Eq. (6c),

$$\frac{d\bar{\beta}_n}{dt} = 2\left(\frac{\bar{\epsilon}^{1/2}}{t}\right)\beta_{0l}\cos\xi\,\cos\omega\bar{\rho}_n[\cos\varphi - \cos(\omega t + \varphi)] - \bar{\epsilon}\omega\left[\sin(2\omega\bar{\rho}_n)\left(1 - \frac{1}{2\omega t}\sin 2(\omega t + \varphi) + \frac{1}{2\omega t}\sin 2\varphi\right) - 4\sin\varphi\,\sin\omega\rho_{0n}\cos\omega\bar{\rho}_n\,\frac{1}{\omega t} \times[\cos\varphi - \cos(\omega t + \varphi)]\right]\,, \tag{6d}$$

where $\bar{\rho}_n$ and $\bar{\beta}_n$ are the time-average position and velocity along the direction $\vec{n}$. With condition $\omega t \gg 1$, we obtain

$$\frac{d\bar{\beta}_n}{dt} = 2\left(\frac{\bar{\epsilon}^{1/2}}{t}\right)\beta_{0l}\cos\xi\,\cos\omega\bar{\rho}_n \times[\cos\varphi - \cos(\omega t + \varphi)] - \bar{\epsilon}\omega\sin(2\omega\bar{\rho}_n)\,. \tag{7}$$

For simplicity, let $\cos\xi = 0$; this means the polarization of the laser is adjusted perpendicular to the electron path.[7] The effect of the first term of Eq. (7) for $\cos\xi \neq 0$ shall be discussed later. Now we have

$$\frac{d\bar{\beta}_n}{dt} = \frac{d^2\bar{\rho}_n}{dt^2} = -\bar{\epsilon}\omega\sin(2\omega\bar{\rho}_n)\,, \tag{8}$$

which has exactly the same form as the equation of motion of a simple pendulum whose solution is well known.[23] From Eq. (8), we obtain the invariant relation

$$\epsilon_n \equiv \bar{\beta}_n^2 - \bar{\epsilon}\cos(2\omega\bar{\rho}_n) = \beta_{0n}^2 - \bar{\epsilon}\cos(2\omega\rho_{0n}) \equiv \epsilon_{0n}\,. \tag{9}$$

Further integration of Eq. (9) gives the interaction time t for the electron of velocity β_0 to pass through the laser beam of diameter D. Thus

$$t = \int_{\rho_{0n}}^{\bar{\rho}_n} \frac{d\bar{\rho}_n}{[\bar{\epsilon}\cos(2\omega\bar{\rho}_n) + \epsilon_{0n}]^{1/2}} = \frac{D}{c\beta_0}\,, \tag{10}$$

an incomplete elliptic integral of the first kind. For given values of ρ_{0n}, β_0, β_{0n}, and D, the value of $\bar{\rho}_n$ can be calculated from Eq. (10) by numerical method. Substituting this value into Eq. (9), we obtain the corresponding value of $\bar{\beta}_n$. Consequently, the angle of deflection of the electron Φ may be calculated:

$$\Phi = (\beta_{0n} - \bar{\beta}_n)/\beta_0 = \theta_0 - \bar{\beta}_n/\beta_0\,, \tag{11}$$

where θ_0, the incident angle, is very small in the experimental setup. Therefore, Φ is a function of $(\theta_0, \rho_{0n}, \beta_0, D)$. Furthermore, if $\omega\bar{\rho}_n = j\pi \pm \omega\rho_{0n}$ for $j = 0, \pm 1, \ldots$, we obtain from Eq. (9) that

$$\bar{\beta}_n = \pm\beta_{0n}\,. \tag{12}$$

This relation is of special interest because it is reduced to the condition of Bragg deflection[1] by taking $\bar{\beta}_n = -\beta_{0n}$ and $\beta_{0n} = \beta_{nB}$, where β_{nB} is the incident velocity satisfying the Bragg condition.

According to Eq. (9), the total energy of the electron in the interaction region is given by

$$\mathcal{E}_n = \tfrac{1}{2}m\bar{v}_n^2 + U_n(\bar{\rho}_n) = \tfrac{1}{2}mv_{0n}^2 + U_{0n}(\rho_{0n}) = \mathcal{E}_{0n}\,, \tag{13}$$

where $\mathcal{E}_n = \frac{1}{2}mc^2\epsilon_n$, $\mathcal{E}_{0n} = \frac{1}{2}mc^2\epsilon_{0n}$, and the potential energies $U_n = -\bar{\mathcal{E}}\cos(2\omega\bar{\rho}_n)$ and $U_{0n} = -\bar{\mathcal{E}}\cos(2\omega\rho_{0n})$, with $\bar{\mathcal{E}} = (e^2E_0^2/2m\omega^2)$. Obviously, $\mathcal{E}_n$ is conserved and depends on the ρ_{0n} and β_{0n} of the electron.

The solutions of Eq. (9) are standard[23] and can be described in a phase plane by curves of constant energy as shown in Fig. 2, where $\bar{\alpha}_n = \omega\bar{\rho}_n$ and $\epsilon_{0n}/\bar{\epsilon} = \mathcal{E}_{0n}/\bar{\mathcal{E}}$. For different values of ϵ_{0n}, the motion of the electron is described by different curves and is discussed as follows:

(i) Since $\beta_{0n}^2 \geq 0$, we always have $\epsilon_{0n}/\bar{\epsilon} > -1$ in Eq. (9).

(ii) For $1 > \epsilon_{0n}/\bar{\epsilon} > -1$, the curves are closed curves encircling the center points at $(\bar{\alpha}_n = \pm j\pi,\ \bar{\beta}_n = 0)$ for all integers j. The motion of the electron is periodic with $\bar{\alpha}_n$ and $\bar{\beta}_n$ oscillating about the center points with amplitudes $\alpha_a = \omega\rho_a = \frac{1}{2}\cos^{-1} \times(-\epsilon_{0n}/\bar{\epsilon})$ and $\beta_a = (\bar{\epsilon} + \epsilon_{0n})^{1/2} < (2\bar{\epsilon})^{1/2}$. The period T of the motion in each closed curve is equal to four times the integral of Eq. (10) integrated from $\bar{\rho}_n = 0$ to ρ_a. When the variable $\bar{\alpha}_n$ in Eq. (10) is changed to ψ by the relation $\sin\bar{\alpha}_n = \sin\alpha_a \sin\psi$, T is given by the complete elliptic integral of the first kind[24]

$$T = \frac{4}{\omega(2\bar{\epsilon})^{1/2}} \int_0^{\pi/2} \frac{d\psi}{(1 - k^2 \sin^2\psi)^{1/2}}$$
$$= \frac{\sqrt{2}\pi mc}{eE_0} \left[1 + (\tfrac{1}{2})^2 k^2 + \left(\frac{1\cdot 3}{2\cdot 4}\right)^2 k^4 + \cdots\right], \quad (14)$$

where $k^2 = \sin^2\alpha_a = \beta_{0n}^2/2\bar{\epsilon} + \sin^2\alpha_{0n}$ and $\alpha_{0n} = \omega\rho_{0n}$, the incident phase. It is evident from Eq. (14) that the period increases with the amplitude α_a. For a given β_{0n}, the motion of the electron with $\alpha_{0n} = 0$ or $\pm j\pi$ has the shortest period T_s with $k^2 = k^2{}_s = \beta_{0n}^2/2\bar{\epsilon}$ in Eq. (14).

(iii) For $\epsilon_{0n}/\bar{\epsilon} = 1$, Eq. (9) is reduced to $\bar{\beta}_n = \pm\beta_{nc} \times\cos\bar{\alpha}_n$, where

$$\beta_{nc} = (2\bar{\epsilon})^{1/2} = \sqrt{2}(eE_0/mc\omega) \quad (15)$$

is the critical velocity ($|\beta_{0n}| < \beta_{nc}$ holds for any electron moving along the closed curves in Fig. 2).

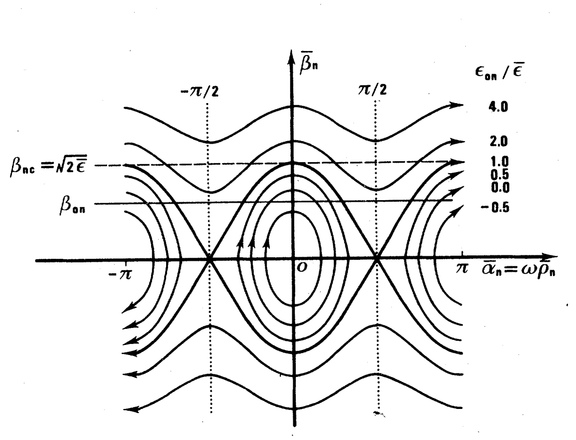

FIG. 2. Constant energy curves of the time average motion of the electron in a phase plane.

(iv) For $\epsilon_{0n}/\bar{\epsilon} > 1$, $\bar{\beta}_n$ will never reach zero and is always either positive or negative. The curves are open curves as shown in Fig. 2. This is always true for $|\beta_{0n}| > \beta_{nc}$.

B. Deflection of an electron beam by a laser standing wave

We have shown that the angle of deflection Φ for a single electron is a function of θ_0, β_0, t, E_0, ω, and ρ_{0n}. For an electron beam satisfying assumption (ii), the parameters $(\theta_0, \beta_0, t, E_0, \omega)$ for any electron of the beam are the same, but the parameter ρ_{0n} of the electrons spreads over many many wavelengths along $\bar{\rho}_n$ as the cross section of the electron beams used in the experiments are much larger than the wavelength λ of the laser. Therefore, the distribution of Φ of the electrons can be calculated if the ρ_{0n} of each electron is known. This is obviously impossible. However, since the interaction potential $U_n = -\bar{\mathcal{E}}\cos(2\omega\bar{\rho}_n)$ is periodic along $\bar{\rho}_n$ with identical unit cells of dimension $\frac{1}{2}\lambda$, the distribution of Φ is the same whether the interaction process is considered as taking place in a periodic array of a large number of identical cells or in a superposition of all of them. In the latter case, we have practically a uniform distribution of ρ_{0n} for a large number of electrons in one cell. Accordingly, it is theoretically sufficient if the electrons are considered to enter the laser beam at all positions within an unit cell between the incident phases $-\frac{1}{2}\pi \leq \alpha_{0n} \leq \frac{1}{2}\pi$ as defined by the dotted lines in Fig. 2.

For a given experimental condition, the corresponding distribution curve of $\Phi = \Phi(\alpha_{0n})$ for $\alpha_{0n} = -\frac{1}{2}\pi$ to $\frac{1}{2}\pi$ can be calculated by Eqs. (10), (9), and (11). Let δ be the resolving power of the detecting system. Since it is assumed that the electrons are uniformly distributed along α_{0n}, the fraction of electrons deflected into δ at Φ (the deflection probability) is equal to the total fractional contribution of α_{0n} from $\Phi - \frac{1}{2}\delta$ to $\Phi + \frac{1}{2}\delta$ along the distribution curve, namely,

$$P(\Phi)\delta = \frac{\Delta N}{N} = \frac{\Sigma[\Delta\alpha_{0n}]_{\Phi-\delta/2}^{\Phi+\delta/2}}{\pi}. \quad (16)$$

In the following, we shall briefly discuss the effects of β_{0n}, t, and E_0 on the distribution of Φ as a function of α_{0n}.

(i) β_{0n} Dependence: For a given value of β_{0n} $\times(=\beta_0\theta_0)$, we can see from Fig. 2 that the motions of electrons with different α_{0n} are described by different curves in the phase plane. Thus Φ is different for different electrons. When $|\beta_{0n}| > \beta_{nc}$, the motions of all the electrons are described by open curves in which the variations of $\bar{\beta}_n$ are small. Consequently, Φ is small for all incident phases from $-\frac{1}{2}\pi$ to $\frac{1}{2}\pi$ and $\Phi \to 0$ as $\beta_{0n} \to \pm 1$. When $|\beta_{0n}| < \beta_{nc}$, the motion of the electrons is divided into two groups: (a) Those electrons of incident phases between $\pm\frac{1}{2}\pi$ and $\pm\cos^{-1}(\beta_{0n}/\beta_{nc})$ are described by open curves with small Φ. (b) Those of incident phases between $\pm\cos^{-1}(\beta_{0n}/\beta_{nc})$ and 0 are described by closed curves. Since $\bar{\beta}_n$ in a closed curve is oscillating between $\pm\beta_a$, the latter group of electrons may be deflected in large angles depending upon the interaction time t. As $\beta_{0n} \to 0$, more and more electrons are described by the closed curves. Those electrons leaving the laser beam with $-\beta_{0n}$ are reflected. In this classical picture, any electron with $|\beta_{0n}| < \beta_{nc}$ may be reflected which is different from that of the quantum-mechanical picture in which the Bragg condition $\beta_{0n} = \beta_{nB} = \hbar\omega/mc^2$ must be strictly observed by any reflected electron.

(ii) t Dependence: Although the condition $\epsilon_{0n} < \bar{\epsilon}$ is satisfied, an electron may not be reflected if the interaction time is not suitable. Obviously the reflection of an electron occurs periodically with respect to t. When $t = (j + \frac{1}{2})T$ for $j = 0, 1, 2, \ldots$, reflection occurs. When $t = jT$, no reflection occurs. Similar dependence of the deflection probability on the interaction time has been discussed in quantum-mechanical approach by Gush and Gush.[16]

(iii) E_0 Dependence: According to Eqs. (14) and (15), T and β_{nc} are functions of E_0. By increasing E_0, β_{nc} will be increased and more electrons will satisfy $\epsilon_{0n} < \bar{\epsilon}$. On the other hand, an increase of E_0 will decrease T. In general, the probability of reflection increases with laser intensity.

III. THEORETICAL PREDICTION AND EXPERIMENTAL RESULTS

The application of this theory to the recent experiments is discussed in this section. Digital computer is used for the numerical calculation. The laser power and intensity are respectively given by $P = c\epsilon_0 a_0 E_0^2$ and $I = 2c\epsilon_0 E_0^2$, where ϵ_0 is the permittivity and a_0 is the beam area.

A. Bartell's experiment

The following is a description of Bartell's experiment[6]:

Laser beam: Peak power $P = 70$ MW, $D = 1.2$ cm, $E_0 = 1.5 \times 10^7$ V/m, and $\omega = 2.72 \times 10^{15}$ rad/sec.

Electron beam: Incident energy = 1640 eV, $\beta_0 = 8.0 \times 10^{-2}$, $\beta_{0n} = 3.5 \times 10^{-6}$, $\beta_{nc} = 4.6 \times 10^{-6}$, and $t = 5.0 \times 10^{-10}$ sec.

The deflection pattern calculated is plotted in Fig. 3. With $\delta = 10^{-5}$ rad for the detecting system, we obtain the angular dependency of the integrated deflection probability as shown in Fig. 4 which is

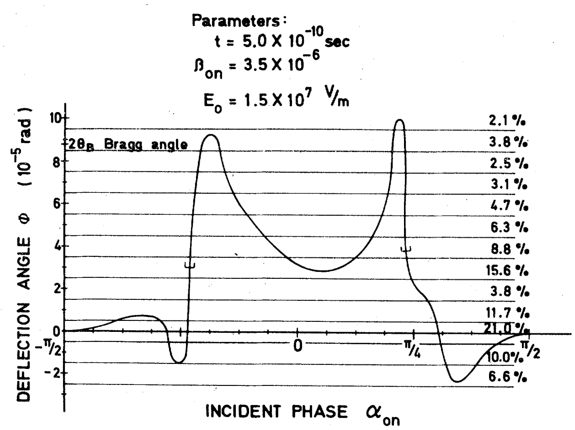

FIG. 3. Classical prediction of the electron deflections of Bartell's experiment for $-\frac{1}{2}\pi \leq \alpha_{0n} \leq \frac{1}{2}\pi$. (The motion of the electrons with α_{0n} between [ ] are described by the closed curves in Fig. 2.)

in good agreement with the experimental result reported in the Fig. 4 of Ref. 6. Most of the scattered electrons do not satisfy the Bragg condition but rather appear at smaller angles. Since in recent theoretical treatments with quantum-mechanical approach, the maximum deflection probability must occur at the Bragg condition, our classical theory then gives a better explanation. Furthermore, Bartell also reported that "Spicules were not observed with normal burst mode with peak power of 0.3 MW. With 80 MW spicules up to 20% of the incident beam height were often observed. With 15 to 40 MW spicules were observed less frequently and were lower in height than those observed at higher laser powers." This can be well explained with Eq. (15). For $\beta_{nc} \geq \beta_{nB} = 3.5 \times 10^{-6}$, the power required should be at least of 39 MW. It is unequivocal that 0.3 MW is much too low to produce any noticeable deflection. For a laser beam with peak power between 15–40 MW,

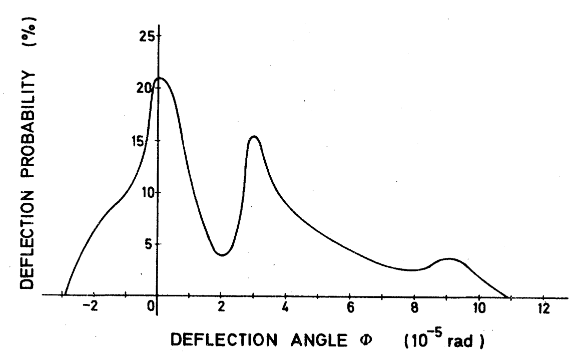

FIG. 4. Angular distribution of the integrated deflection probability for Bartell's experiment with resolving power 10^{-5} rad.

β_{0n} of some of the misaligned electrons may well be within the critical values β_{nc} corresponding to these laser powers. Thus deflection can be observed. As the laser power is increased, β_{nc} is also increased and the requirement $\beta_{0n} < \beta_{nc}$ is easily obtained. This is the very reason why Bartell could observe that "The reflection probability of electrons encountering a hyper-intense portion of a giant pulse would be high even if alignment were imperfect."

B. Takeda's experiment

The following is an account of Takeda's experiment[7]:

Laser beam: Peak power $P = 20$ MW, $D = 0.5$ cm, $E_0 = 1.96 \times 10^7$ V/m, and $\omega = 2.72 \times 10^{15}$ rad/sec.

Electron beam: Incident energy = 300 eV, $\beta_0 = 3.43 \times 10^{-2}$, $\beta_{0n} = 3.5 \times 10^{-6}$, $\beta_{nc} = 6.0 \times 10^{-6}$, and $t = 4.86 \times 10^{-10}$ sec.

The Φ-α_{0n} curve is plotted in Fig. 5. In this experiment, the detector only detected those electrons which were not stopped by the movable metal plate. When the movable stopper was set to the position of the Bragg angle, $\cong 2.0 \times 10^{-4}$ rad, the fraction of electrons detected was 0.6%. According to Fig. 5, our theoretical result is 0.9% for electrons deflected with an angle greater than the Bragg angle.

C. Pfeiffer's experiment

The following describes Pfeiffer's experiment[8]:

Laser beam: Intensity $I = 5 \times 10^7$ W/cm^2, $D = 1.1$ cm, $E_0 = 9.7 \times 10^6$ V/m, and $\omega = 2.72 \times 10^{15}$ rad/sec.

Electron beam: Incident energy = 36 eV, $\beta_0 = 1.19 \times 10^{-2}$, $\beta_{0n} = 3.5 \times 10^{-6}$, $\beta_{nc} = 2.9 \times 10^{-6}$, and $t = 3.1 \times 10^{-9}$ sec.

Since $\beta_{0n} > \beta_{nc}$, there should be no Bragg reflection. In this experiment, no deflected electron was observed at the Bragg angle.

D. Schwarz's experiment

The following explains Schwarz's experiment[4,5,9,25]:

Laser beam: Peak power $P = 32.5$ kW (normal nonstanding wave operation), $D = 0.3$ cm, $\omega = 1.78 \times 10^{15}$ rad/sec.

Electron beam: Incident energy = 10 eV, $\beta_0 = 6.26 \times 10^{-3}$, $\beta_{nB} = 2.29 \times 10^{-6}$, $t = 1.6 \times 10^{-9}$ sec.

In this experiment, Schwarz reported that the laser intensity in the resonator was at least a factor of 6 higher if a mirror was used to form a standing wave because the Q value of the system was increased although the actual power used was uncertain. However, according to our theory, we can find that for high deflection probability at Bragg condition, E_0 of the laser beam should be much greater than 5×10^6 V/m which corresponds to a peak power of 0.45 MW, a factor of 14 higher than that without a mirror. Such a high power could not be obtained from the laser used by Schwarz. Therefore, he found minimum deflection probabilities when the center lines of the electron beam and the laser beam fulfilled the Bragg condition ($\beta_{0n} = \beta_{nB}$). In order to obtain a maximum number of electrons deflected to one side, Schwarz had to rearrange the orientation of the two beams. Thus the Bragg condition was violated.

By observing the deflection distributions with different sets of E_0 and β_{0n}, it can be found that

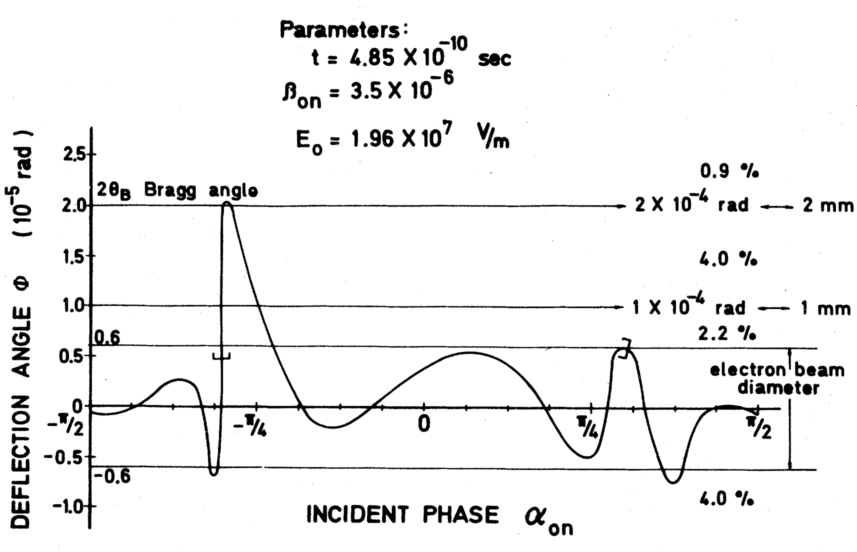

FIG. 5. Classical prediction of the electron deflections of Takeda's experiment for $-\frac{1}{2}\pi \leq \alpha_{0n} \leq \frac{1}{2}\pi$.

TABLE I. Comparison of the classical deflection probabilities with the experimental results for different laser powers in Schwarz's experiment.

Peak power (MW)	Effective filter factor	E_0 (10^6 V/m)	Deflection probability obtained from Figs. 6(a) and 6(b)	Probability measured [a]
0.243	$(1.33)^2$	3.6	0.78	0.80
0.137	$(1.00)^2$	2.7	0.61	0.60
0.104	$(0.87)^2$	2.4	0.54	0.55
0.045	$(0.57)^2$	1.5	0.25	0.22
0.019	$(0.37)^2$	1.0	0.14	0.08
0.006	$(0.21)^2$	0.6	0	0.06

[a] Reference 9.

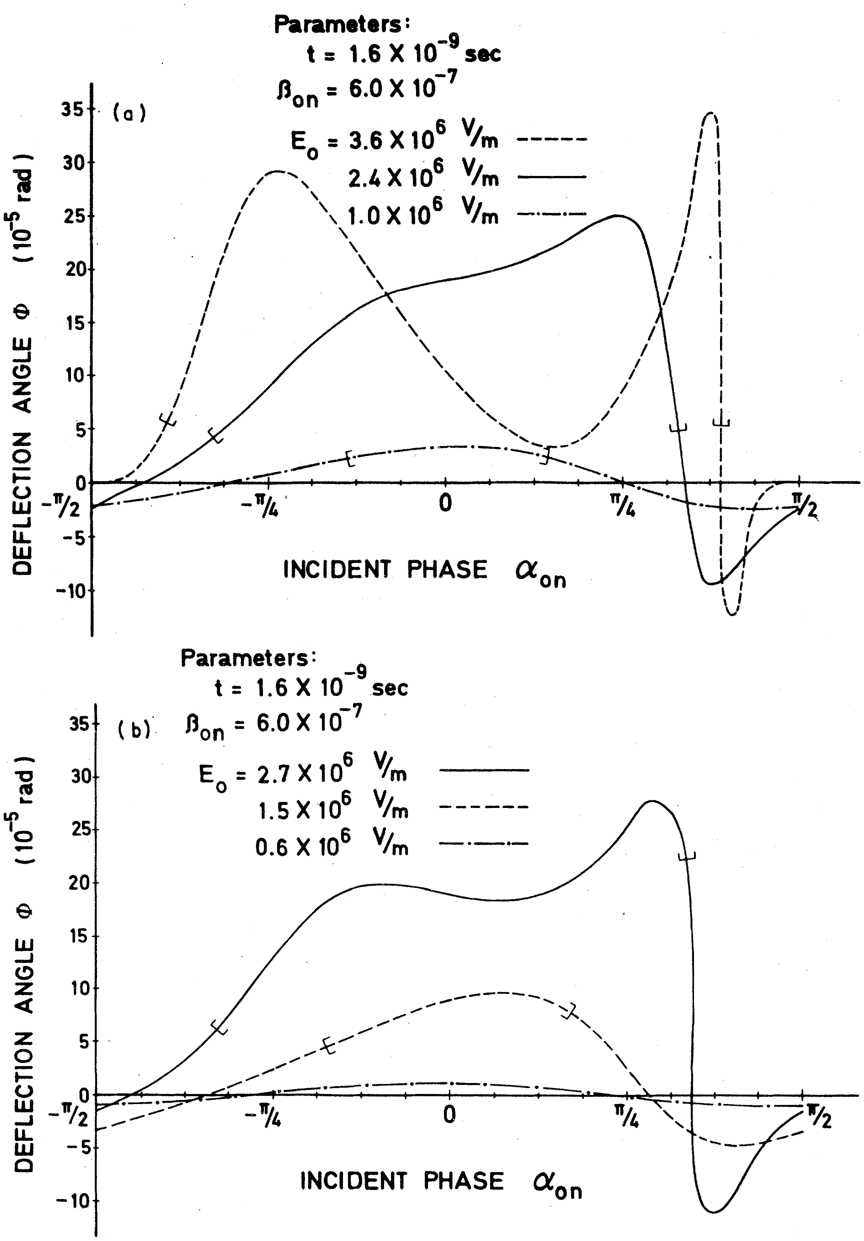

FIG. 6. (a) Classical prediction of the electron deflections of Schwarz's experiment for $-\frac{1}{2}\pi \leq \alpha_{0n} \leq \frac{1}{2}\pi$ with $E_0 = 3.6\times10^6$, 2.4×10^6, 1.0×10^6 V/m. (b) Classical prediction of the electron deflections of Schwarz's experiment for $-\frac{1}{2}\pi \leq \alpha_{0n} \leq \frac{1}{2}\pi$ with $E_0 = 2.7\times10^6$, 1.5×10^6, 0.6×10^6 V/m.

the probability deflected to one side has a maximum of 78% when $E_0 = 3.6\times10^6$ V/m ($\beta_{nc} = 1.7\times10^{-6}$) and $\beta_{0n} = 6\times10^{-7}$. This corresponds to a peak power 0.24 MW of the laser beam, a factor of 7.4 higher than that without the mirror reflector. Since Schwarz stated[25] that 80% of the deflection probability to one side could be obtained if the laser was used in its maximum, it is reasonable to assume that the maximum peak power is 0.24 MW.

In order to reduce the laser intensity, Schwarz inserted filters between the laser rod and the reflecting mirror. However, this would also decrease the Q value of the system. Thus the effective filter factors are assumed to be squares of those reported in Ref. 9. In Table I, the experimental results and the theoretical predictions corresponding to different peak powers and E_0 are listed. The maximum power is 0.243 MW. All other laser powers in column 1 are calculated with the factors in column 2. The deflection distribution curves corresponding to different E_0 in column 3 are plotted in Figs. 6(a) and 6(b) from which the theoretical results in column 4 are obtained by subtracting the lower-half deflection probabilities from that of the upper-half for the region outside $\pm1.5\times10^{-5}$ rad from the undeflected electron beam. The value 1.5×10^{-5} rad is determined by the resolving power of the detection system. The experimental results are listed in column 5. Although the experimental measurements of the actual peak powers are not known, the close agreement of the experimental and theoretical results indicates that the experiment, the theory, and the assumptions are self-consistent. It may suggest that this method can be used to measure the laser intensity inside a resonator.

Finally, all the experimental results and the theoretical predictions of the scattering effect are summarized in Table II. Very good agreement between the experimental and theoretical results is noted.

IV. DISCUSSION AND CONCLUSION

(a) According to our classical theory, the orientation of the polarization of the laser light affects the deflection of the electron beam. As shown in Fig. 1, if the polarization is perpendicular to the incident electron path, then $\xi = 90°$ and the first term on the right-hand side of Eq. (7) will vanish. The motion of the electron along the $\vec{n}$ direction will obey Eq. (8). However, the electron also moves in the $\vec{k}$ direction. From Eq. (5a), we obtain

$$\beta_k = -(2eE_0/mc\omega)[\sin(\omega t+\varphi)\sin\omega\rho_n - \sin\varphi\,\sin\omega\rho_{0n}]. \quad (17)$$

The phase factor φ varies arbitrarily as the electrons enter the laser beam at arbitrary time, hence the deflected electrons will spread over a certain region in the $\vec{k}$ direction. However, the speed of the electrons along this direction must be within $\pm(4eE_0/mc\omega)$. As far as the Bragg scattering condition is concerned, we find that $(4eE_0/mc\omega) > \beta_{nc} > \beta_{nB}$. Thus the deflection along the $\vec{k}$ direction is easily detected. However, in most of the recent experiments except that of Takeda, the motion of the electrons in this direction has been overlooked. Although the effect in the $\vec{k}$ direction in Takeda's experiment is not apparent because he has adjusted the polarization nearly parallel to the electron path, the small deflection observed in the $\vec{k}$ direction does give evidence that the polarization effect does exist. The classical theory gives a clear description of this effect.

TABLE II. Comparison of the experimental results and the classical theoretical predictions.

	Experimental results		Classical predictions	
	Deflection angle Φ (rad)	Deflection probability P	Deflection angle Φ (rad)	Deflection probability P
Bartell	8.8×10^{-5} (Bragg angle)	0.05	8.8×10^{-5}	0.036
	5.0×10^{-5}	0.10 (max)	3.0×10^{-5}	0.155 (max)
Takeda	2×10^{-4} (Bragg angle)	0.006	2×10^{-4}	0.009
Pfieffer	Bragg angle	0	Bragg angle	0
Schwarz	Bragg condition	minimum (no quantitative result)	Bragg condition	0
	Bragg condition is violated	0.80 (max)	Incident angle θ_0 $=9.6\times10^{-5}$ rad	0.78 (max)

On the other hand, if the polarization is parallel to $\vec{l}$, $\xi = 0$. The motion of the electrons in $\vec{k}$ direction will vanish, but the first term on the right-hand side of Eq. (7) affects the motion of the electrons in the $\vec{n}$ direction. The relative contribution of this term, in comparison with that of the second term is given by the ratio $R = (2mc\beta_{0l}/eE_0t)$. Using the data of the four experiments, we obtain that R(Schwarz) = 0.4%, R(Bartell) = 4%, R(Takeda) = 1%, and R(Pfeiffer) = 0.2%. Thus the effect of the first term in Eq. (7) is to broaden the deflected electron beam in the $\vec{n}$ direction only by a factor of a few percent. As the deflection probability being measured has been taken by integrating over the unresolved region of the system, this broadening effect can in general be neglected.

(b) Finally, we conclude that the classical theory actually gives a better explanation to the experiments than those recent quantum-mechanical and semi-quantum-mechanical theories. The scattering process seems to be in classical nature. However, in order to have a deeper understanding of this process, more work should be done in the following directions. First, since the measurements in recent experiments have been concentrated at the Bragg angle, information of the scattering process has not yet been sufficient. Therefore we suggest a reexamination of the problem experimentally in the light of the classical point of view. The angular distribution of the deflection probability should be obtained for different sets of parameters (β_{0n}, t, E_0) and the dependence of the critical value β_{nc} on the electric field strength should be investigated. Second, the polarization effect on the scattering along the $\vec{k}$ direction should be measured carefully as discussed in (a). Third, more work should be done in the quantum mechanical approach to see whether a more satisfactory theory can be found to explain the experiments. This is of particular interest if the polarization effect is considered. However, it must be noted that the Ehrenfest theorem may not be applied to our classical theory. The replacement of the position and momentum coordinates in classical canonical equation of Hamiltonian with the Hamiltonian of Eq. (13) by the average values of the position and momentum operators in Ehrenfest equations is not justified[26] because the interaction potential is in the form of $\cos^2 k_z z$ which is a polynomial exceeding the second degree in z. In addition, since a monochromatic and spatially coherent electron beam is used in our classical theory, the dimensions of the corresponding electron wave packet is not sufficiently small at all times to satisfy the Ehrenfest's theorem. Therefore, our classical theory may not be explained as just the classical approximation in quantum mechanics.

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APPENDIX

Under the experimental conditions given by the recent experiments, the first-order deflection probabilities of the "Kapitza-Dirac effect" predicted by those quantum-mechanical and semi-quantum-mechanical theories which are not limited by the experimental conditions are given in Table III for comparison.

TABLE III. First-order deflection probabilities of the "Kapitza-Dirac effect."

Theories / Experiments	Schoenebeck [a] P_S	Ezawa-Namaizawa [b] P_{EN}	Gush-Gush [c] P_{GG}	Ehlotzky-Leubner [d] P_{EL}	Measurement $\underline{P}$
Schwarz	0.04	0.58	0.74	0.32	0.80
Bartell	0.08	0.74	0.75	0.02	0.05
Takeda	0.012	0.007	0.025	0.016	0.006
Pfeiffer	0.03	0.71	0.69	0.01	0

[a] Reference 14.
[b] Reference 15.
[c] Reference 16.
[d] Reference 18.

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