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Can beats of electron waves at optical frequencies drive a free-electron laser?

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A recent proposal by H. Schwarz is analyzed. General principles of quantum mechanics are used to limit the possibilities for radiation from beats in an electron beam.

Schwarz[1] has recently proposed that a new kind of free-electron laser might be based on radiation from what he describes as quantum beats in a free-electron beam. Each electron is to be prepared with the modulated single-particle wave function

$$\psi_{sp}(x,t)=(\phi_1+\phi_2)/\sqrt{2}\,, \tag{1}$$

where

$$\phi_j=\exp[i(k_jx-\omega_jt)]\,.$$

Schwarz observes that the quantum-mechanical expectation value of the charge density and current density are given by

$$j_{sp}=v\rho_{sp}=ev[1+\cos(k_bx-\omega_bt)]\,, \tag{2}$$

where the beat frequency and the beat wave number are defined by $\omega_b=\omega_2-\omega_1$ and $k_b=k_2-k_1$, and $v=\hbar(k_1+k_2)/2m$. He then treats j_{sp} as a classical current, as in an antenna. In some approximate classical calculation, he finds the single-particle power P_{sp} which is radiated at frequency ω_b by such a current to be

$$P_{sp}(\omega_b;\text{mod.})=C\omega_1/\omega_b^2\,, \tag{3}$$

where C is a certain positive constant. For n coherently radiating electrons he uses $j_{np}=nj_{sp}$ so that $P_{np}=n^2P_{sp}$, and that value of P_{np} forms the basis for the proposed free-electron laser.

Actually, the power radiated into a free electromagnetic field by the source in Eq. (2) is exactly zero in classical electromagnetic theory.[2] The value of P_{sp} given in Eq. (3) is an artifact of the approximate classical calculation. Similarly in quantum mechanics, a free electron cannot radiate a free photon because of the conservation of energy and momentum. Using a modulated initial state does not help because the superposition of two zeros can only give zero.

One could generalize the question raised by Schwarz. Suppose radiation is made possible by having the electrons or the photons interact with some other object. Such a process could, for instance, be bremsstrahlung or synchrotron radiation when the electron is not free, or it could be transition radiation or Čerenkov radiation when the photon is not free. Would the modulated electron state (1) then radiate significantly more light at the beat frequency than a plane-wave state of the same energy? For a single electron, the answer is no. For n coherently radiating electrons, radiation at frequency ω_b is enhanced by a factor proportional to n^2 over the amount emitted by one electron, but that radiation then depends upon the single-electron mechanism, whatever it is, and the power is not proportional to P_{sp} of Eq. (3).

The single-electron case. If energy is conserved between the electron and the photon alone, then the two components of the modulated electron state feed final states containing electrons of different energy, and there is no interference contribution to the radiation of a photon of any frequency.[3] Therefore, we have

$$P_{sp}(\omega_b;\text{mod.})=P_{sp}(\omega_b;\text{plane wave}). \tag{4}$$

Even if energy is not conserved between the electron and photon, for instance if radiation is stimulated by an external field at the beat frequency, the rate of radiation can at most be doubled by interference because the square of the sum of two amplitudes is at most twice the sum of the squares.[4]

The many-electron case. First, consider two electrons which can radiate coherently. The initial-state wave function is

$$\begin{aligned}\psi_{2p}(x_1,x_2,t)&=\psi_{sp}(x_1,t)\psi_{sp}(x_2,t)\\&=\tfrac{1}{2}[\phi_1(1)\phi_1(2)+\phi_2(1)\phi_2(2)\\&\quad+\phi_1(1)\phi_2(2)+\phi_2(1)\phi_1(2)]\,.\end{aligned} \tag{5}$$

The last two terms in Eq. (5) have the same energy. They can both emit a photon at the frequency ω_b to leave two electrons in the final state $\phi_1(1)\phi_1(2)$. For that photon frequency only, two contributions to the amplitude for producing the final state add coherently. In the best case for interference, where the amplitudes for emitting the photon from the two plane-wave states are equal, the rate of emission P_{2p} obtained from Eq. (5) is $P_{2p}=2.5P_{sp}$ instead of $P_{2p}=2P_{sp}$ without interference. For n coherently radiating electrons, there

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are many interfering terms in the product wave function and counting out the terms gives

$$P_{np}(\omega_b;\,\text{mod.}) = \tfrac{1}{4}(3n+n^2)P_{\text{sp}}(\omega_b;\,\text{plane wave})\,. \qquad (6)$$

The important point is that the right-hand side of Eq. (6) represents the rate of emission of a photon through interaction of either the electron or the photon with another object, and it has no relation to Eq. (3). In more physical language, the source of the enhancement is the superradiance[5] of the n-electron initial state, rather than the beat phenomenon in the one-electron current j_{sp}. In fact, the n^2 term in the radiation comes from the interference of components in the n-particle wave function, all of which have the same momentum and the same energy. That interference can therefore contribute no spatial or temporal beats[4] to the n-particle expected current j_{np}(mod.).

Conventional "free-electron" lasers avoid the objections presented here by having the electrons interact with a magnetic field.[6] A recent proposal of Fradkin[7] avoids those objections by having the electrons interact with an optical field. In his field, the right-hand side of Eq. (4) can be doubled because energy is not conserved in a time-dependent external field.

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[1]H. Schwarz, Phys. Rev. Lett. 42, 1141 (1979).

[2]The velocity v defines a moving frame in which j_{sp} vanishes everywhere and ρ_{sp} is stationary.

[3]This point was made in the same context by L. D. Favro, D. M. Fradkin, P. K. Kuo, and W. B. Rolnick, Nuovo Cimento Lett. 4, 1147 (1970).

[4]General limitations on the enhancement of radiation by interference in modulated wave functions are discussed by H. J. Lipkin and M. Peshkin, Appl. Phys. Lett. 19, 313 (1971); J. Appl. Phys. 43, 3037 (1972).

[5]R. H. Dicke, Phys. Rev. 93, 99 (1954).

[6]W. B. Colson, Phys. Lett. A 59, 187 (1976); W. H. Louisell, J. F. Lam, D. A. Copeland, and W. B. Colson, Phys. Rev. 19, 288 (1979).

[7]D. M. Fradkin, Phys. Rev. Lett. 42, 1209 (1979).