# Ratio of diffusion coefficient to mobility for electrons in liquid argon

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The ratio of the diffusion coefficient $D$ to the mobility $\mu$ for electrons has been measured in liquid argon by using a modified Townsend apparatus. The measured ratio $eD/\mu$ monotonically increases from $\sim 0.1$ to $\sim 0.4$ eV with an increase in the electric field from $\sim 2$ to $\sim 10$ kV/cm. The ratio $eD/\mu$ is smaller than that in gaseous argon at the same value of $E/N$, where $N$ is the atomic density. The mean momentum-transfer cross section $\langle\sigma\rangle$ is estimated as a function of $\langle\epsilon\rangle$ on the assumption $\langle\epsilon\rangle = 3eD/2\mu$, where $\langle\epsilon\rangle$ is the mean electron agitation energy. Thus the $\langle\sigma\rangle$ obtained is nearly constant ($3\sim 4\times 10^{-17}$ cm$^2$) for $\langle\epsilon\rangle$ from 0.15 to 0.6 eV. This value of $\langle\sigma\rangle$ is almost the same as that estimated from the low-field mobility of electrons in liquid argon. This result is consistent with both Derenzo's experiment and Lekner's theory. It is concluded that there is no Ramsauer minimum at about 0.4 eV of $\langle\epsilon\rangle$, where the minimum exists in gaseous argon.

## I. INTRODUCTION

Electron drift velocities in liquified rare gases have been measured for over twenty years. Ten years ago Miller *et al.*[1] extensively measured the drift velocity of electrons in condensed states of rare gases, carefully eliminating the effect of impurities. Recently our group[2] and Yoshino *et al.*[3] found a considerable increase in the electron drift velocity in rare-gas liquids, when molecular impurities were doped. Such an effect is well known in the gas phase[4,5] and is explained by the reduction of the mean electron energy due to the inelastic collision of electrons with molecular impurities, and by the resulting decrease of momentum-transfer cross section $\sigma$ due to the Ramsauer-Townsend effect. For liquid argon, however, the increase in the electron drift velocity was found to be smaller than that for gaseous argon. It was thought that this fact might suggest the absence of the Ramsauer-Townsend effect in liquid argon.[2]

In 1967 Cohen and Lekner[6] gave a solution of the Boltzmann equation for electrons in gases, liquids, and solids. Lekner[7] applied the solution to the electron motion in liquid argon and derived basic transport parameters for electrons, such as drift velocity $w$, mean agitation energy $\langle\epsilon\rangle$, momentum-transfer mean free path $\lambda$ of electrons, etc., as functions of the electric field $E$. As a result, it is shown that a Ramsauer minimum does not occur in liquid argon.

Information experimentally obtained so far, however is almost limited to the drift velocities of electrons in liquid argon. The value of $\langle\epsilon\rangle$ is known for "cool" electrons in thermal equilibrium with liquid argon in low $E$, where the mobility $\mu$ is independent of $E$. Christophorou and McCorkle[8] estimated the mean scattering cross section $\langle\sigma\rangle_L$ from $\mu$ for cool electrons in liquid argon at various temperatures and in the high-pressure gas and claimed a shift of the Ramsauer minimum to the lower side of $\langle\epsilon\rangle$, with an increase in atomic density. Except for Derenzo's data,[9] no information on $\epsilon$ has been available for "hot" electrons, which gain energy from the electric field. The value of $\langle\epsilon\rangle$ for hot electrons can be estimated by measuring the ratio of the diffusion coefficient $D$ to $\mu$ from Einstein's relation[10]

$$eD/\mu = kT, \tag{1}$$

where $e$ is the electron charge, $k$ is Boltzmann's constant, and $T$ is the electron temperature.

In this paper we present experimental data on the ratio $eD/\mu$ as a function of $E$ for electrons in liquid argon, obtained by a modified Townsend apparatus,[4] using a parallel-plate pulse ionization chamber. Results were compared with those obtained by Lekner and Derenzo. Furthermore, the momentum-transfer cross section is discussed in comparison with that in gaseous argon.[11-13]

 

## II. EXPERIMENTAL

### A. General

A parallel-plate pulse ionization chamber designed as a modified Townsend apparatus was used. We will consider an isolated group of $\mathfrak{N}$ electrons traveling from the cathode to the collector under the influence of a uniform and constant electric field $E$ in the chamber. Suppose that these electrons are concentrated at a point $(x, y, z) = (0, 0, 0)$ at time $t = 0$; then their distribution at $t$ is described by[10]

$$n = \frac{\mathfrak{N}}{4\pi D t (4\pi D_L t)^{1/2}} \exp\left(-\frac{\rho^2}{4Dt}\right) \exp\left(-\frac{(z+wt)^2}{4D_L t}\right), \tag{2}$$

where the $z$ axis is chosen in the direction of $E$, $w$ is the drift velocity of the group, $D_L$ is the longitudinal coefficient of diffusion, and $\rho^2 = x^2 + y^2$. The average of $\rho^2$ is easily obtained to be $4Dt$. Thus $(4Dt)^{1/2}$ gives a measure of the lateral spread of the group.

In the configuration shown in Fig. 1 the number of electrons $(\ldots, n_{k-1}, n_k, n_{k+1}, \ldots)$ collected by each separated collector $(\ldots, c_{k-1}, c_k, c_{k+1}, \ldots)$ does not depend on $D_L$, when the distance $d$ between the cathode and the collector is sufficiently large compared to the longitudinal spread of the group $(4D_L\tau)^{1/2}$, where $\tau = d/w$. We may then use $D$ for $D_L$ for convenience in the calculations. Thus the distribution in the group at $t = \tau$ is written as

$$n(r) = \mathfrak{N} \exp(-r^2/R^2)/(\pi^{1/2}R)^3, \tag{3}$$

where $r^2 = x^2 + y^2 + (z + w\tau)^2$ and

$$R^2 = 4D\tau. \tag{4}$$

Now we can calculate the number of electrons $n_k$ that arrive at the collector $c_k$ at $z = -d$, occupying the region from $X - \frac{1}{2}s$ to $X + \frac{1}{2}s$ along the $x$ axis and having sufficient length along the $y$ axis. For example, when $X - \frac{1}{2}s > 0$, this number is given by

$$n_k = 2\pi\left(\int_{X-s/2}^{X+s/2} r(r - X + \tfrac{1}{2}s)n(r)dr + \int_{X+s/2}^{\infty} rsn(r)dr\right), \tag{5}$$

which is derived in Appendix A.

Curves giving $n_k/\mathfrak{N}$ obtained from Eq. (5) are shown in Fig. 2 for some typical values of $R$, as functions of the ratio $X/s$ for $s = 60$ μm. This value of $s$ corresponds to the condition of the present experiment as will be described in Sec. II B. The calculated curves are fitted to the experimental data by adjusting $R$ and $X$. Thus a value of $R$ is determined.

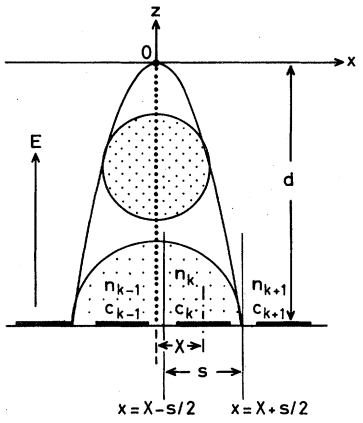


FIG. 1. Schematic drawing showing the diffusion process of drift electrons in the space between parallel plate electrodes. The split collectors are denoted by $\ldots, c_{k-1}, c_k, c_{k+1}, \ldots$, and $\ldots, n_{k-1}, n_k, n_{k+1}, \ldots$ express the number of electrons collected by $\ldots, c_{k-1}, c_k, c_{k+1}, \ldots$. For example, $n_k$ is equal to the number of electrons contained in the volume between two planes at $x = X - \frac{1}{2}s$ and $x = X + \frac{1}{2}s$.

Finally, we obtain the ratio $eD/\mu$ from the relation

$$eD/\mu = e(R^2/4\tau)(E/w) = eR^2V/4d^2, \tag{6}$$

where $V$ is the voltage applied between the cathode and the collector.

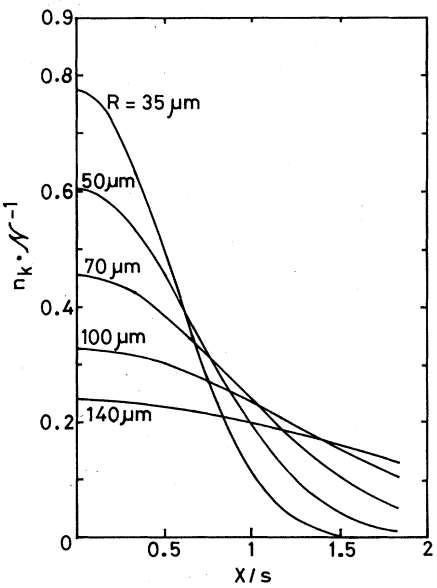


FIG. 2. Calculated distributions of the relative number of electrons collected by each collector of 60 μm in width against the normalized distance $X/s$.

### B. Apparatus and procedure

The pulse ionization chamber is filled with liquid argon and has split collectors and a cathode, on which a narrow linelike source of $\alpha$ particles was deposited. The parameter $R$ was determined by measuring the charge collected by each collector.

#### *1. Ionization chamber*

The cathode is a flat-surfaced stainless-steel plate, with an effective area of $25\times25$ mm$^2$. A source of $^{210}$Po is deposited at the center of the cathode. The length of the source is about 1 mm and the width less than 20 $\mu$m.[14]

The collector, which has an area of $24\times24$ mm$^2$, consists of eight split collectors,[15] denoted by $\ldots, c_{-1}, c_0, c_{+1}, \ldots$, as shown in Fig. 3. These collectors have an effective width of 60 $\mu$m, except for two outer collectors.[15]

The distance between the cathode and the collector was set to be 1.7, 2.7, 3.7, and 4.8 mm. The electric field in the electron drift space was sufficiently uniform without a guard ring.

Commercial argon of 99.999% stated purity was further purified by barium-titanium getters, described in our previous papers.[2,16] The ionization chamber was inserted into a vacuum-tight vessel made of stainless steel and copper, was baked out at 100 °C, and was evacuated to a pressure of less than $1.0\times10^{-7}$ Torr, together with the gas-filling system. Purified argon condensed into the vessel when the vessel was cooled down with liquid oxygen.

#### *2. Electric circuit*

Signals induced on the strip collectors were amplified by a charge-sensitive preamplifier and fed to a main amplifier having differential and integral time constants of 2 $\mu$sec, and analyzed by a multichannel pulse-height analyzer. To reduce the background noise, the pulses from the collectors, other than the collector $c_0$, which is the nearest to the center line of $\alpha$ source, were gated by using the pulses from the collector $c_0$, as shown in Fig. 4. A pulse corresponding to all electrons $\mathfrak{N}$ due to one $\alpha$ particle was measured by connecting all the collectors. Thus the ratios $n_{-1}/\mathfrak{N}$, $n_0/\mathfrak{N}$, and $n_{+1}/\mathfrak{N}$ were obtained.

## III. RESULTS AND DISCUSSION

The insert in Fig. 5 shows a typical pulse-height spectrum due to $^{210}$Po $\alpha$ particles obtained when all collectors were connected. The pulse height at the peak of the spectrum corresponds to $e\mathfrak{N}$. The pulse height versus electric field strength curve is shown in Fig. 5 for different distances $d$ between the cathode and the collector. As seen

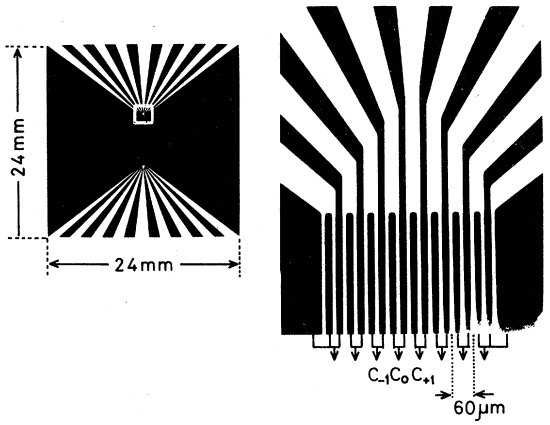


FIG. 3. Photographs of gold collector electrode deposited on ceramic substrate. (a) Top view of collector electrode. (b) Magnified view of the central part of the white square in (a). The lower part shows the scheme of electrical connections.

from the figure, the pulse height does not depend on $d$. This shows that electronegative impurities, which should reduce the pulse height, are sufficiently removed by our purification method.

Figure 6 shows the relation between $d$ and $R^2$, experimentally obtained. In Sec. II we assumed that the group of electrons were concentrated at the starting point of the drift. Here let us consider that at $t=0$, the group of electrons have a Gaussian radial distribution, expressed by Eq. (3), in which $R$ is replaced by an initial mean-square radius $R_0$. Then Eq. (4) is modified to

$$R^2=4D\tau+R_0^2=4Dd/w(E)+R_0^2, \tag{7}$$

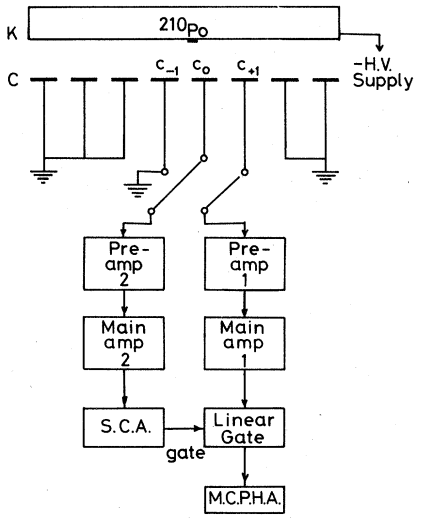


FIG. 4. Schematic diagram of the apparatus. The number of electrons collected by the collector $c_{+1}$ is measured by gating with the pulse from $c_0$.

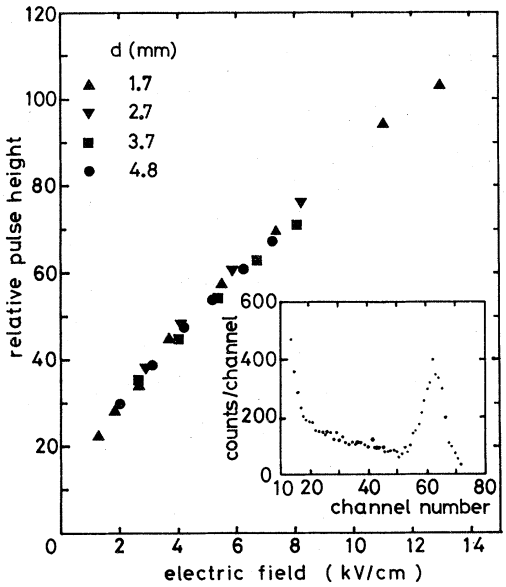


FIG. 5. The variation of total collected charge due to $^{210}$Po $\alpha$ particles vs electric field $E$ for different distances $d$ between the cathode and the collector. The inserted figure shows a typical pulse-height spectrum, when $d = 1.7$ mm and $E = 5.9$ kV/cm.

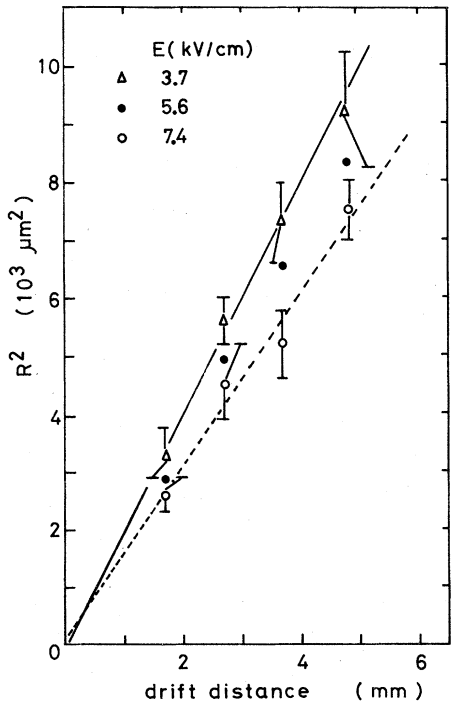


FIG. 6. Square of the measured mean width of the electron distribution $R^2$ against the drift distance $d$ for several values of electric field strength $E$. The solid line ($R_0^2 = -60 \pm 710$ $\mu$m$^2$) and dashed line ($R_0^2 = 90 \pm 530$ $\mu$m$^2$) are drawn for the results at $E = 3.7$ and $E = 7.4$ kV/cm, respectively, by the least-squares fitting.

and Eq. (6) is modified to

$$eD/\mu = e(R^2 - R_0^2)V/4d^2 . \quad (8)$$

From Eq. (7) and Fig. 6 we can estimate the value of $R_0$ to be less than ~25 $\mu$m by extrapolating $\tau$ to 0. Actually, $R_0$ is determined by the range (~50 $\mu$m) of $\alpha$ particles isotropically emitted from $^{210}$Po and by the spread[14] ($\lesssim$20 $\mu$m) of $^{210}$Po on the cathode.

Effects of positive ions on the drifting electrons can be neglected because $d$ is sufficiently large compared with the range of $\alpha$ track in liquid argon. The space-charge effects of electrons in the diffusion measurement are more complicated. If the space-charge effect of electrons is taken into account, the relation between $R^2$ and $d$ is expected to be nonlinear, as shown in Appendix B. In the experiment, however, a linear relation was obtained, as shown in Fig. 6. This fact suggests that the effect of the electron space charge is negligible, although the reason is not known at the present.

In Fig. 7, $eD/\mu$ obtained from Eq. (8) are plotted against $E$ for the different distances in liquid argon. The value of $eD/\mu$ increases from ~0.1 to ~0.4 eV with the increase in $E$ from ~2 to ~10 kV/cm. The experimental error is estimated to be less than 15%. The difference in $eD/\mu$ for the

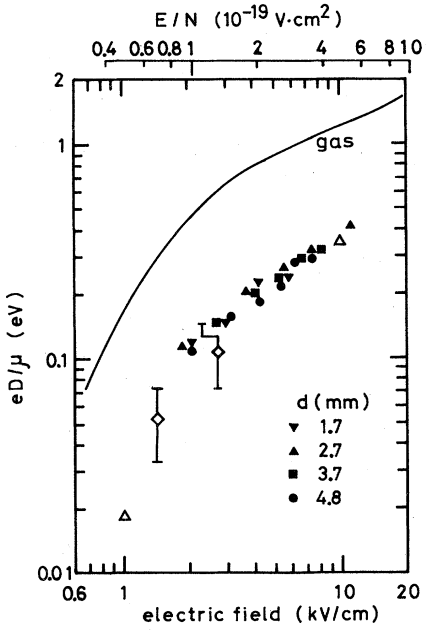


FIG. 7. Field dependence of the ratio $eD/\mu$ in liquid argon. The symbols ▲, ▼, ■, and ● represent the present results. Points ◇ and △ are the results according to Derenzo (Ref. 9) and Lekner (Ref. 7), respectively. Solid curve shows the results for gaseous argon. In the upper horizontal scale, $N$ is the atomic density in liquid argon ($2.1 \times 10^{22}$ cm$^{-3}$).

different distances is negligible. This means that Eq. (7), neglecting the space-charge effect, applies well. The space-charge effect in our experiment might have slightly increased the observed value of $eD/\mu$. Figure 7 also shows the results obtained by Derenzo,[9] with a method similar to ours. In Derenzo's experiment, in contrast to our experiment, the space-charge effect might have decreased the value of $eD/\mu$ because he used, as ionizing radiation, high-energy charged particles penetrating the detector along the electric field, and so the electrons and ions produced were attracting each other during the electron drift. Nevertheless, the agreement between our results and Derenzo's is fairly good. This fact also suggests that the effect of space charge is small compared with the experimental error.

In Fig. 7 the results of Warren and Parker[11] for gaseous argon at 77 K are also shown. The figure shows that the mean agitation energy $\langle\epsilon\rangle$ of electrons in liquid argon is smaller than that in gaseous argon over the region of $E/N$ in this experiment, where $N$ is the atomic density. Values of $\langle\epsilon\rangle$ theoretically obtained by Lekner[7] are also given in the same figure by using the approximate relation, $eD/\mu = \frac{2}{3}\langle\epsilon\rangle$. Rigorously, this relation can apply only to electrons having a Maxwellian distribution, but will be roughly valid for other plausible distributions.[17] The figure shows that the present result is in good agreement with that of Lekner.

We will estimate the mean momentum-transfer cross section $\langle\sigma\rangle$ of electrons from the results shown in Fig. 7, using the data on the electron drift velocity $w$ in liquid argon.[1] For the estimate we adapt the relation

$$\langle\sigma\rangle = eE/Nw(2m\langle\epsilon\rangle)^{1/2}, \tag{9}$$

which was derived from the relation $w = eE\lambda/m\langle v\rangle$ used by McDaniel,[4] where $m$ is the mass of electron, $\langle v\rangle$ is the average velocity of electrons, $\lambda$ is the mean free path, and $\frac{1}{2}m\langle v\rangle^2 = \langle\epsilon\rangle = 3eD/2\mu$. Thus obtained, $\langle\sigma\rangle$ is shown in Fig. 8. We can also estimate $\langle\sigma\rangle$ from the data on $w$ at low electric field, where mobility is constant and $\langle\epsilon\rangle$ is determined not by the electric field but by the temperature $T$ of the liquid. The plotted values of $\langle\sigma\rangle$ ($3\sim4\times10^{-17}$ cm$^2$) for small $\langle\epsilon\rangle$ (~0.01 eV) are thus obtained from Eq. (9), on the assumption $\langle\epsilon\rangle = \frac{3}{2}kT$, by using the values of mobility at low field obtained by Miller *et al.*,[1] Shibamura *et al.*,[2] and Yoshino *et al.*[3] Let us compare $\langle\sigma\rangle$ with the momentum-transfer cross section[12] $\sigma(\epsilon)$ for monoenergetic electrons. The solid curve shows $\langle\sigma\rangle$ from the experimental data of $eD/\mu$ (Ref. 11) and $w$ (Ref. 18) for gaseous argon. The dot-dash curve shows $\sigma(\epsilon)$.[12] Comparison of these two curves shows that $\langle\sigma\rangle(\langle\epsilon\rangle)$ reflects $\sigma(\epsilon)$ well for gaseous argon. Such a relation should be expected for liquid argon. The dashed curve in Fig. 8 also shows $\sigma$ translated from the mean free path that Lekner[7] theoretically obtained for liquid argon.

Figure 8 clearly shows the difference in the tendency of $\langle\sigma\rangle$ against $\langle\epsilon\rangle$ in both phases of argon. For gaseous argon $\langle\sigma\rangle$ shows a minimum corresponding to a Ramsauer-Townsend effect at about 0.4 eV of $\langle\epsilon\rangle$. For liquid argon, on the other hand, $\langle\sigma\rangle$ is nearly constant around this energy region.

The momentum-transfer cross sections estimated in the present experiment and those derived from

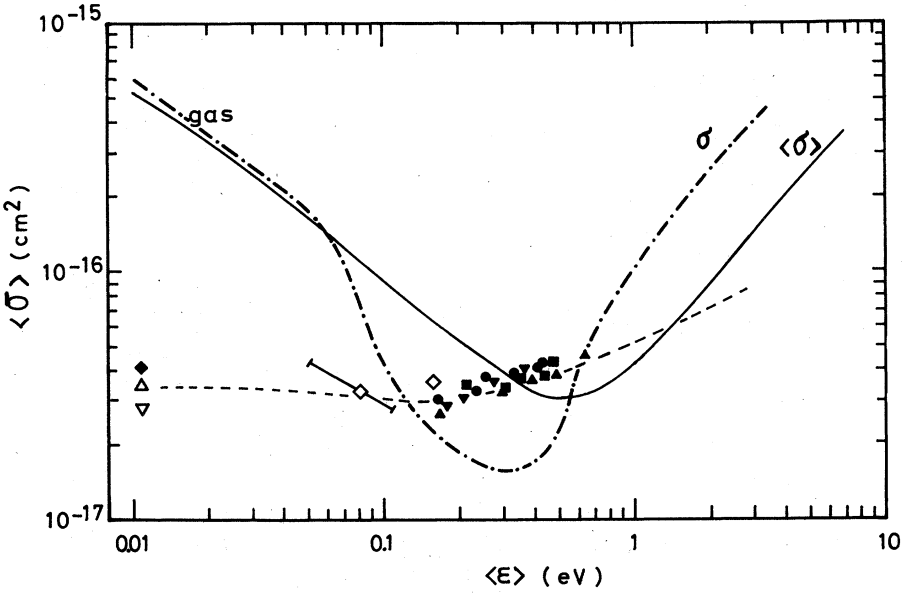


FIG. 8. Variation of the momentum-transfer cross section $\langle\sigma\rangle$ as a function of mean electron energy $\langle\epsilon\rangle$. The points represented by ▲, ▼, ■, ●, and ◇ correspond to the results shown by the same marks in Fig. 7. The symbols ◆, ▽, and △ represent $\langle\sigma\rangle$ translated from the results obtained by Shibamura *et al.* (Ref. 2), Miller *et al.* (Ref. 1), and Yoshino *et al.* (Ref. 3), respectively. The dashed curve shows $\sigma$ obtained from Lekner's theory. The solid curve shows $\langle\sigma\rangle$ and the dot-dash curve shows $\sigma(\epsilon)$ for gaseous argon.

low-field mobility[1,2,3] are in good agreement with Lekner's theory in which absence of the Ramsauer minimum is claimed. The result of Derenzo's experiment[9] is also consistent with Lekner's calculation. On the other hand, Christophorou and McCorkle[8] obtained a mean scattering cross section $\langle\sigma\rangle_L$, for $\langle\epsilon\rangle$ from ~0.01 to ~0.02 eV, from the data of low-field mobility for electrons in liquid argon at various temperatures, taking the variation of the structure factor into account. They have claimed that there is a sharp Ramsauer minimum[19] at $\langle\epsilon\rangle$ ~0.02 eV. Unfortunately, the present experiment cannot cover the region below ~0.15 eV of $\langle\epsilon\rangle$. It is concluded by taking Derenzo's data into account that there is no Ramsauer minimum of $\langle\epsilon\rangle$ at least in the region from ~0.08 to ~0.6 eV, although the minimum might shift to the lower energy side of $\langle\epsilon\rangle$, as claimed by Christophorou and McCorkle.[8]

Extending our considerations to liquid krypton and xenon, we estimated mean cross sections for electrons of low energy (corresponding to the temperatures of the liquids) from Eq. (9), using the data of the low-field mobility[3] as done for liquid argon. The cross section is $1.2\times10^{-17}$ cm$^2$ for liquid krypton and $0.8\times10^{-17}$ cm$^2$ for liquid xenon. According to Frost and Phelps[12] the momentum-transfer cross section is $6\times10^{-16}$ cm$^2$ in argon, $2\times10^{-15}$ cm$^2$ in krypton, and $7\times10^{-15}$ cm$^2$ in xenon, all in gases at the electron energy corresponding to the temperature of each liquid. In the gases the cross section is larger for atoms with larger atomic number. In the liquids the situation is different. The cross section in argon ($3\sim4\times10^{-17}$ cm$^2$) is larger than that in krypton or xenon (~$10^{-17}$ cm$^2$). Such a difference between the liquid and the gas might be explained by the band structure in the liquids. Furthermore, if it is assumed that the variation of the cross section with electron energy in liquid krypton or xenon is similar to that in liquid argon, we can estimate the cross section in liquid krypton or xenon to be nearly constant and take a value of about $10^{-17}$ cm$^2$ for electron energies from 0.1 to 0.6 eV. To test this expectation we are planning an experiment on $eD/\mu$ in liquid krypton and xenon.

## IV. CONCLUSION

The following conclusions are deduced from the present experiment.

In liquid argon the value of $eD/\mu$ for electrons, which is a good measure of the mean electron energy, monotonically increases from ~0.1 to ~0.4 eV, with the increase in the electric field from ~2 to ~10 kV/cm. For these electric fields $eD/\mu$ is smaller than that in gaseous argon; in other words, the electron temperature in the liquid phase is lower than that in the gas phase at the same $E/N$.

The mean momentum-transfer cross section $\langle\sigma\rangle$ estimated from $eD/\mu$ is nearly constant ($3\sim4\times10^{-17}$ cm$^2$) at electron energies $\langle\epsilon\rangle$ from 0.15 to 0.6 eV in liquid argon. The value of $\langle\sigma\rangle$ is almost the same as that estimated from low-field mobility of electrons in liquid argon. There is no Ramsauer minimum at about 0.4 eV of $\langle\epsilon\rangle$, where the minimum exists in gaseous argon.

## ACKNOWLEDGMENTS

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## APPENDIX A

Consider a spherically symmetric distribution $n(r)$ of electrons. For $X-\frac{1}{2}s\geq0$ we can calculate the number of electrons between two planes, $x=X-\frac{1}{2}s$ and $x=X+\frac{1}{2}s$, as the sum of two terms, $n_k^{(1)}$ and $n_k^{(2)}$. The first term is the number of electrons inside the sphere of radius $r=X+\frac{1}{2}s$, and the second term is the number of electrons outside the sphere of radius $r=X+\frac{1}{2}s$. The origin of the coordinate is chosen to be at the center of the distribution. As shown in Fig. 9, the number of electrons $dn$ contained in the volume element of the ring between $r$ and $r+dr$, and $\theta$ and $\theta+d\theta$, is written as

$$dn=n(r)\,2\pi r^2\sin\theta\,d\theta\,dr\,,$$

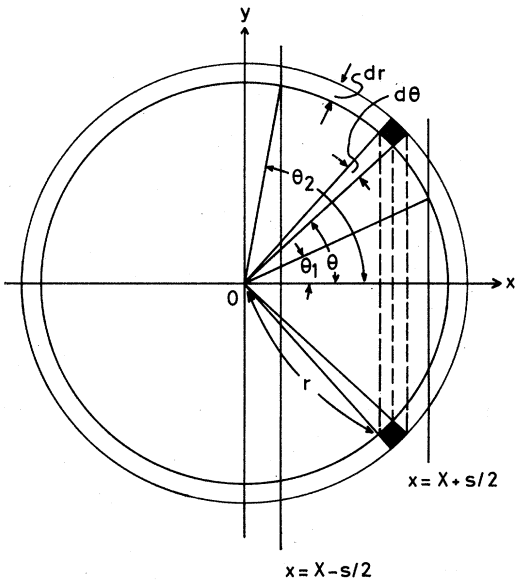


FIG. 9. Cross sectional view of the volume element $dV=2\pi r^2\sin\theta\,d\theta\,dr$ for $r>X+\frac{1}{2}s$.

where $\theta$ is the angle measured from $x$ axis. Then we obtain

$$n_k^{(1)} = \int_{X-s/2}^{X+s/2} n(r) \int_0^{\theta_2} 2\pi r \sin\theta \, d\theta \, r \, dr$$

and

$$n_k^{(2)} = \int_{X+s/2}^{\infty} n(r) \int_{\theta_1}^{\theta_2} 2\pi r \sin\theta \, d\theta \, r \, dr ,$$

where $\cos\theta_1 = (X + \frac{1}{2}s)/r$ and $\cos\theta_2 = (X - \frac{1}{2}s)/r$. From these relations Eq. (5) is derived. Also, for $X - \frac{1}{2}s < 0$, $n_k$ is easily obtained in a similar manner.

## APPENDIX B

Let us estimate the spread of the electron group due to the mutual repulsion of electrons (the space-charge effect) in the absence of electron diffusion. We will again treat a spherical symmetric group of electrons having a Gaussian radial distribution of density, starting from an initial mean-square radius $R_0$. The group will be divided into ten subgroups, each of which contains $\frac{1}{10}$ of the total of the electrons in the group, by nine concentric spheres with radii $r_i$ $(i = 1, 2, \ldots, 9)$, determined by the relation

$$\int_0^{r_i} 4\pi r^2 n(r) dr = P_i \mathfrak{N} , \qquad \text{(B1)}$$

where $P_i = \frac{1}{10} i$. The value of $r_i$ is easily computed numerically for a given $R_0$.

Next let us calculate the spreading velocity of the sphere. The velocity $dr_i/dt$ is determined not only by the electric field $\vec{E}_s$ due to the internal space charge of electrons, but also by the externally applied electric field $\vec{E}$. By Gauss's theorem, $\vec{E}_s$ at point $\vec{r}_i$ is given by

$$\vec{E}_s = -(\vec{r}_i / 4\pi\epsilon_A r_i^3) P_i \mathfrak{N} ,$$

where $\vec{r}_i$ is measured from the center of the group and $\epsilon_A$ is the dielectric constant. Thus the drift velocity of electrons at $\vec{r}$ is

$$\vec{w} = \vec{w}[\vec{E}_s(\vec{r}_i) + \vec{E}] .$$

The velocity $\vec{v}_r$ relative to the center of the group is written as

$$\vec{v}_r = d\vec{r}_i/dt = \vec{w}(\vec{E}_s + \vec{E}) - \vec{w}(\vec{E}) .$$

Here we are interested in $v_\perp$, the component of the velocity perpendicular to $\vec{E}$, which may be estimated approximately as

$$v_\perp = |\vec{w}(\vec{E})| \, |\vec{E}_s| / |\vec{E}| ,$$

so long as $\vec{E}_s$ is sufficiently small compared with $\vec{E}$.

Finally, we can calculate the spread of radii starting at $t = 0$ from $r_i(0)$ determined by Eq. (B1), again on the assumption of the spherical symmetry. The radius at $\Delta t$ is calculated by

$$r_i(\Delta t) = r_i(0) + v\Delta t .$$

In this manner, the time variation of the radius $r_i(t)$ is followed up to $t = \tau$. Here $\tau$ is the drifting time of electrons from the cathode to the collector. Thus we can obtain the radius $R_s$ corresponding to the root mean square of the electron distribution in the group. The value of $R_s^2$ is shown in Fig. 10.

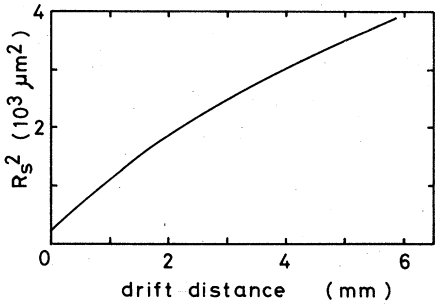


FIG. 10. Variation of $R_s^2$ as a function of distance $d$ when $R_0 = 25$ μm, $E = 5.6$ kV/cm, and $N = 2.0 \times 10^4$, where $N$ is the number of electrons in the group. Variation of $R_s^2$ is almost the same when $R_0 < 25$ μm and $2 < E < 10$ kV/cm for $d > 1$ mm.

The value of $R^2$ shown in Fig. 6 might be supposed to consist of $4D\tau$ $(= 4Dd/w)$, $R_0^2$, and $R_s^2$, namely, $R^2 = 4D\tau + R_0^2 + R_s^2$. The term $4D\tau$ is linear against $d$ at a given $E$ and $R_0^2$ is constant. Figure 10 shows that $R_s^2$ is nonlinear against $d$. Therefore it is expected that the relation between $R^2$ and $d$ is nonlinear, if $R_s^2$ is taken into account.

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[14]A source of $\alpha$ particles was fabricated in the following way: first, $^{210}$Po was deposited on the cathode, coated with a plastic mask, through a photoetched rectangular hole (15 $\mu$m × 1 mm) at the center of the mask. Then the mask was dissolved with acetone. Autoradiography using x-ray films showed that the width of the $^{210}$Po source was narrower than 20 $\mu$m. The photoetching of both the plastic mask and the collector was carried out by Iwatsu Electric Co., Ltd.
[15]The collector consists of 18 actual electrodes, 16 of which are strip electrodes having a width of 20 $\mu$m, 2 of which are outer electrodes of 12 mm in width, as shown in Fig. 3. The 16 strip electrodes are arranged at 30 $\mu$m center-to-center spacings. These 18 electrodes are photoetched gold films deposited on the ceramic substrate and are insulated from each other. The 18 electrodes were divided into eight groups denoted by ..., $c_{-1}$, $c_0$, $c_{+1}$, ..., as shown in Fig. 3. The 2 (or 3) electrodes composing one group were electrically connected outside. Thus, these groups are regarded to be strip collectors having an effective width of 60 $\mu$m, except for two outer groups.
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[19]The sharpness of the minimum in Ref. 8 is not understood, as the authors of Ref. 8 themselves stated.

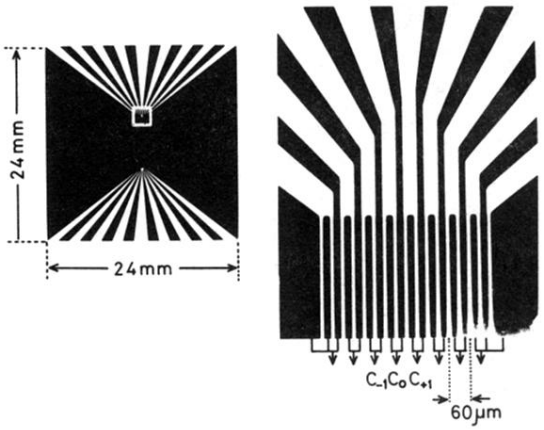


FIG. 3. Photographs of gold collector electrode deposited on ceramic substrate. (a) Top view of collector electrode. (b) Magnified view of the central part of the white square in (a). The lower part shows the scheme of electrical connections.