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Anomalous radiation from a turbulent plasma

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It is shown that the acceleration of an electron by the electrostatic fields of the ion-wave fluctuations can lead to enhanced electromagnetic waves in a plasma. The maximum emission arises from those electrons whose velocity is close to the phase velocity of the ion-wave oscillations. Application of our investigation to laboratory and astrophysical plasmas is discussed.

I. INTRODUCTION

According to the weak-turbulence theory,[1,2] turbulence energy flows from high-frequency modes towards low-frequency modes. As a result, energy condensation occurs at the lowest frequency modes. For example, in space plasma, most of the turbulence energy is piled up in the long-period pulsations which are composed of magnetohydrodynamic (MHD) modes. On the other hand, in astrophysical[3] and laboratory plasmas,[2] the electrostatic ion-wave instability is found to play a vital role. It is well known[1] that the presence of an external current in a plasma causes a relative drift between the electrons and ions. When the relative drift speed exceeds the ion-acoustic velocity, ion-acoustic waves become unstable.[4] Linear theory shows that the wavelength of the fastest-growing mode is close to $\sqrt{8}\,\pi/\lambda_D$, where $\lambda_D = v_e/\omega_{pe}$ is the electron Debye length. Such processes as nonlinear scattering,[5] particle trapping,[6] and resonance broadening[2] can halt wave growth. Specifically, owing to the nonlinear scattering process,[5] energy cascading takes place from short-wavelength to long-wavelength ion-acoustic fluctuations. Eventually, a stationary fully developed turbulent state is attained. Since the energy density of the ion-wave fluctuations is much smaller than the thermal energy of the plasma particles (typically $|E_k|^2/4\pi n T_e \sim 10^{-2}$), the usual weak-turbulence theory can be used to describe the fully developed turbulent state.

Nonlinear interaction of a stationary turbulent plasma with electromagnetic waves can lead to such interesting phenomena as anomalous resistivity,[7] plasma laser,[8] induced scattering of waves,[9] etc. In particular, recent investigations[10,11] have demonstrated that mode coupling in a weakly turbulent plasma is significantly enhanced. This conclusion differs markedly from those derived from the well-known weak turbulence theory.[1,2] This problem is basically different than that involving the stability of a Langmuir turbulence with respect to low-frequency perturbations.[12] The latter analysis exhibits that growing density perturbations occur due to modulational instability.

In this paper, we investigate the interaction between background electrons and long-wavelength ion-acoustic stationary turbulence. Electrons whose velocity is close to the phase velocity of the ion-acoustic waves feel a strong acceleration.[13] As a result, the free energy of accelerated electrons gives rise to induced bremsstrahlung radiation. Within the framework of the linear-response theory, we obtain in Sec. II the effective dielectric constant of the electromagnetic wave in the presence of ion-wave turbulence. In Sec. III, the nonlinear dispersion relation is analyzed. It is found that unstable electromagnetic waves appear in the presence of an anisotropic ion-wave [$\omega = \pm k c_s$, where $c_s = (T_e/M)^{1/2}$ is the sound speed] turbulence in the Maxwellian plasma. This case is of physical interest since the current driven ion-wave instability is usually stabilized by nonlinear processes leading to a steady turbulent state which consists of an anisotropic turbulence spectrum. On the other hand, our calculation shows that for an isotropic turbulence spectrum, the contributions arising from the $\pm\omega_k$ modes exactly cancel each other, yielding zero growth. Finally, Sec. IV contains a brief summary and application of our investigation.

II. FORMULATION

We consider a homogeneous plasma in the presence of an enhanced ion-wave stationary turbulence. The basic equations governing the interaction of the latter with an electromagnetic test wave are the Vlasov-Maxwell equations:

$$\left[\frac{\partial}{\partial t} + \vec{v}\cdot\frac{\partial}{\partial \vec{r}} + \frac{e}{m}\left(E(\vec{r},t) + \frac{\vec{v}\times\vec{B}}{c}\right)\cdot\frac{\partial}{\partial \vec{v}}\right] f_j(\vec{r},\vec{v},t) = 0, \quad (1)$$

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}, \tag{2}$$

$$\nabla \times \vec{B} = \frac{1}{c}\frac{\partial \vec{E}}{\partial t} + \frac{4\pi}{c}\sum_j n_j e_j \int \vec{v} f_j \, d\vec{v}, \tag{3}$$

where the notation is standard.

Since the ion-wave turbulence is already present in our system, the unperturbed electron distribution function F_{0e} of the background plasma can be represented as

$$F_{0e} = f_{0e} + \epsilon f_{1e}, \tag{4}$$

where f_{0e} is the space-, time-averaged part, and f_{1e} is the low-frequency fluctuating part. Here, ϵ is a small parameter and represents, for example, the electric field strength of the turbulent fluctuations $\vec{E}_l$. Then, to order ϵ, the Vlasov equation becomes

$$\left(\frac{\partial}{\partial t} + \vec{v}\cdot\nabla\right) f_{1e} = -\frac{e}{m}\vec{E}_l \cdot \frac{\partial}{\partial \vec{v}} f_{0e}. \tag{5}$$

The Fourier component of f_{1e} follows from Eq. (5), and we have

$$f_{1e}(k,\omega) = \left(\frac{|e|}{m} E_l(k,\omega)\frac{\partial}{\partial v} f_{0e}\right) \Big/ i(\omega - kv), \tag{6}$$

where, for simplicity, the ion waves are assumed to propagate along the z direction. Eq. (6) is valid only if the ion waves have sufficiently low amplitude so that electron trapping is not important.

To investigate the problem of induced bremsstrahlung radiation from a stationary turbulent plasma, we perturb the equilibrium by introducing high-frequency electromagnetic test fields $\mu\delta\vec{E}_h$ and $\mu\delta\vec{B}_h$ in the system. Accordingly, it is legitimate to assume that $\mu \ll \epsilon$. The interaction of the test fields $\mu\delta\vec{E}_h$ and $\mu\delta\vec{B}_h$ with the finite-amplitude stationary turbulent ion-wave fields ϵE_l leads to mixed-mode (beat waves) perturbation fields $\mu\epsilon\delta\vec{E}_{lh}$, $\mu\epsilon\delta\vec{B}_{lh}$, and $\mu\epsilon\delta f_{lh}$. We note that the mixed modes are not necessarily the normal mode of the plasma. Their dynamics is, nevertheless, governed by Eqs. (1)–(3). We may therefore write the total perturbed electric and magnetic fields in the following form:

$$\delta\vec{E} = \mu\delta\vec{E}_h + \mu\epsilon\delta\vec{E}_{lh}, \tag{7}$$

$$\delta\vec{B} = \mu\vec{B}_h + \mu\epsilon\delta\vec{B}_{lh}. \tag{8}$$

The perturbed distribution function of the electron is likewise written as

$$\delta f = \mu\delta f_h + \mu\epsilon\delta f_{lh}. \tag{9}$$

Assuming $\vec{E}_l \gg \delta\vec{E}$, we linearize the Vlasov equation and obtain

$$\left(\frac{\partial}{\partial t} + \vec{v}\cdot\nabla\right)\delta f + \frac{e}{m}E_l\frac{\partial}{\partial u}\delta f + \frac{e}{m}\left(\delta\vec{E} + \frac{\vec{v}\times\delta\vec{B}}{c}\right)\cdot\frac{\partial}{\partial\vec{v}}F_{0e} = 0. \tag{10}$$

To order $\mu\epsilon^2$, Eq. (10) yields

$$\left(\frac{\partial}{\partial t} + \vec{v}\cdot\nabla\right)\delta f_h + \frac{e}{m}\left\langle E_l\frac{\partial}{\partial u}\delta f_{lh}\right\rangle + \frac{e}{m}\delta\vec{E}_h\cdot\frac{\partial}{\partial\vec{v}}f_{0e} + \frac{e}{m}\left\langle\delta\vec{E}_{lh}\cdot\frac{\partial}{\partial\vec{v}}f_{1e}\right\rangle + \frac{e}{mc}\vec{v}\times\delta\vec{B}_h\cdot\frac{\partial}{\partial\vec{v}}f_{0e} + \frac{e}{mc}\left\langle\vec{v}\times\delta\vec{B}_{lh}\cdot\frac{\partial}{\partial\vec{v}}f_{1e}\right\rangle = 0, \tag{11}$$

where $\langle\ \rangle$ denotes averaging over the low-frequency fluctuations. On the other hand, to order $\mu\epsilon$, we find an equation which determines δf_{lh}. For our purposes, we have

$$\left(\frac{\partial}{\partial t} + \vec{v}\cdot\nabla\right)\delta f_{lh} + \frac{e}{m}E_l\frac{\partial}{\partial u}\delta f_h + \frac{e}{m}\delta\vec{E}_h\cdot\frac{\partial}{\partial\vec{v}}f_{1e} + \frac{e}{m}\delta\vec{E}_{lh}\cdot\frac{\partial}{\partial\vec{v}}f_{0e} + \frac{e}{mc}\vec{v}\times\delta\vec{B}_h\cdot\frac{\partial}{\partial\vec{v}}f_{1e} + \frac{e}{mc}\vec{v}\times\delta\vec{B}_{lh}\cdot\frac{\partial}{\partial\vec{v}}f_{0e} = 0. \tag{12}$$

We now introduce the Fourier transforms in space and time for various quantities according to

$$A(\vec{k},\omega) = \int_{-\infty}^{\infty} d^3x \int_{-\infty}^{\infty} dt\, A(\vec{x},t)\exp[i\omega t - i\vec{k}\cdot\vec{x}]. \tag{13}$$

From Eq. (11) we then obtain

$$\begin{aligned} i(\Omega - Ku)\delta f_h(K,\Omega) &= \frac{|e|}{m}\sum_{k'}\left\langle E_l(k',\omega')\frac{\partial}{\partial u}\delta f_{lh}(K-k',\Omega-\omega')\right\rangle + \frac{|e|}{m}\delta\vec{E}_h(K,\Omega)\cdot\frac{\partial}{\partial\vec{v}}f_{0e} \\ &+ \frac{|e|}{m}\sum_{k'}\left\langle\delta\vec{E}_{lh}(K-k',\Omega-\omega')\cdot\frac{\partial}{\partial\vec{v}}f_{1e}(k',\omega')\right\rangle + \frac{|e|}{mc}\sum_{k'}\left\langle\vec{v}\times\delta\vec{B}_{lh}(K-k',\Omega-\omega')\cdot\frac{\partial}{\partial\vec{v}}f_{1e}(k',\omega')\right\rangle, \end{aligned} \tag{14}$$

where u is the velocity component along the z direction and we assume f_{0e} to obey the Maxwell distribution.

Using Eq. (13), we can express Eq. (12) in the form

$$(\omega - ku)\delta f_{lh}(k,\omega) = \sum_{K'}\left(\frac{e}{m}\right)^2 E_l(k-K',\omega-\Omega')\frac{\partial}{\partial u}\frac{1}{K'u-\Omega'}\delta\vec{E}_h(K',\Omega')\cdot\frac{\partial}{\partial\vec{v}}f_{0e}$$
$$+\sum_{K'}\left(\frac{e}{m}\right)^2\delta\vec{E}_h(K',\Omega')\cdot\frac{\partial}{\partial\vec{v}}\frac{1}{(k-K')u-(\omega-\Omega')}E_l(k-K',\omega-\Omega')\frac{\partial}{\partial u}f_{0e}$$
$$+\frac{1}{i}\left(\frac{|e|}{m}\right)\delta\vec{E}_{lh}(k,\omega)\cdot\frac{\partial}{\partial\vec{v}}f_{0e}+\left(\frac{e}{m}\right)^2\sum_{K'}\vec{v}\times\frac{1}{\Omega'}\vec{K}'\times\delta\vec{E}_h(K',\Omega')\cdot\frac{\partial}{\partial\vec{v}}\frac{E_l(k-K',\omega-\Omega')}{(k-K')u-(\omega-\Omega')}\frac{\partial}{\partial u}f_{0e}. \tag{15}$$

Substituting Eq. (15) into Eq. (14), we get

$$(\Omega-Ku)\delta f_h(K,\Omega)=\left(\frac{e}{m}\right)^2\sum_{k'}\left\langle E_l(k',\omega)\frac{\partial}{\partial u}\frac{1}{(K-k')u-(\Omega-\omega')}\delta\vec{E}_{lh}(K-k',\Omega-\omega')\cdot\frac{\partial}{\partial\vec{v}}f_{0e}\right\rangle$$
$$+\frac{1}{i}\frac{|e|}{m}\delta\vec{E}_h(K,\Omega)\cdot\frac{\partial}{\partial\vec{v}}f_{0e}+\left(\frac{e}{m}\right)^2\sum_{k'}\left\langle\delta\vec{E}_{lh}(K-k',\Omega-\omega')\cdot\frac{\partial}{\partial\vec{v}}\frac{E_l(k',\omega')}{k'u-\omega'}\frac{\partial}{\partial u}f_{0e}\right\rangle$$
$$+\left(\frac{e}{m}\right)^2\sum_{k'}\left\langle\vec{v}\times\frac{1}{\Omega-\omega'}[(\vec{K}-\vec{k}')\times\delta\vec{E}_{lh}(K-k',\Omega-\omega')]\cdot\frac{\partial}{\partial\vec{v}}\frac{E_l(k',\omega')}{k'u-\omega'}\frac{\partial}{\partial u}f_{0e}\right\rangle. \tag{16}$$

Since we are concerned with the weak-turbulence limit of the ion-wave fluctuations, in Eq. (16) as well as in the following, we have neglected terms of relative order $(e^2/m^2\Omega^2)|E_l|^2\partial^2/\partial u^2$. Furthermore, in Eq. (16) the electric field of the beat mode is yet unknown. Since the beat modes are driven by the test wave, we need a relation between $\delta\vec{E}_{lh}$ and $\delta\vec{E}_h$. For this purpose, we compute the x component of Eq. (3) for the mixed-mode perturbation in the Fourier space

$$K^2\delta E_{lhx}(K,\Omega)=\frac{\Omega^2}{c^2}\delta E_{lhx}(K,\Omega)-\frac{4\pi}{c^2}\sum_j n_j|e_j|\int i\Omega v_x\delta f_{lh}(K,\Omega)d^3v. \tag{17}$$

In the following, we are mainly interested in investigating the electron response to the ion-wave turbulent fields. Therefore only the contribution of the electron current in Eq. (17) need be retained.

If we substitute Eq. (15) into Eq. (17), we obtain

$$\delta E_{lhx}(K,\Omega)=D_0^{-1}(K,\Omega)\left[\left(\frac{\omega_{pe}}{\Omega}\right)^2\int\frac{i\Omega}{\Omega-Ku}\sum_{K'}\frac{|e|}{m}E_l(K-K',\Omega-\Omega')\frac{\partial}{\partial u}\frac{1}{K'u-\Omega'}f_{0e}\delta E_{hx}(K',\Omega')du\right.$$
$$+\left(\frac{\omega_{pe}}{\Omega}\right)^2\int\frac{i\Omega}{\Omega-Ku}\sum_{K'}\frac{|e|}{m}\frac{1}{(K-K')u-(\Omega-\Omega')}E_l(K-K',\Omega-\Omega')\frac{\partial}{\partial u}f_{0e}\delta E_{hx}(K',\Omega')du$$
$$\left.-\left(\frac{\omega_{pe}}{\Omega}\right)^2\int\frac{i\Omega}{\Omega-Ku}\frac{|e|}{m}\left(u+v_x^2\frac{\partial}{\partial u}\right)\sum_{K'}\frac{K'}{\Omega'}\frac{E_l(K-K',\Omega-\Omega')}{(K-K')u-(\Omega-\Omega')}\frac{\partial}{\partial u}f_{0e}\delta E_{hx}(K',\Omega')d^3v\right], \tag{18}$$

where

$$D_0(K,\Omega)=1-\left(\frac{cK}{\Omega}\right)^2-\left(\frac{\omega_{pe}}{\Omega}\right)^2\int\frac{\Omega}{\Omega-Ku}f_{0e}\,du, \tag{19}$$

and $\omega_{pe}=(4\pi ne^2/m)^{1/2}$ is the electron plasma frequency. Equation (18) shows that the electric field of the mixed-mode perturbation can be represented as the product of the turbulent field and the test-wave field.

To obtain the effective dielectric function of the electromagnetic waves in the presence of the ion-wave turbulence, we take the x component of Eq. (3) for high-frequency component, viz.,

$$K^2\delta E_{hx}(K,\Omega)=\frac{\Omega^2}{c^2}\delta E_{hx}(K,\Omega)-\frac{4\pi}{c^2}n|e|\int i\Omega v_x\delta f_h(K,\Omega)d^3v. \tag{20}$$

Inserting Eq. (18) into Eq. (16) and using the random-phase approximation, we find an expression for $\delta f_h(K,\Omega)$ in terms of $|E_l(k',\omega')|^2$. Substituting this value of δf_h into Eq. (20), and performing considerable algebra, we can put the nonlinear dispersion relation of the electromagnetic waves $[D_T(K,\Omega)]$ into the form

$$D_T(K,\Omega)=1-\left(\frac{cK}{\Omega}\right)^2-\left(\frac{\omega_{pe}}{\Omega}\right)^2\int\frac{\Omega}{\Omega-Ku}f_{0e}\,d^3v-\left(\frac{e}{m}\right)^2\left(\frac{\omega_{pe}}{\Omega}\right)^2\int\frac{d^3v\,\Omega}{\Omega-Ku}\sum_{k'}\frac{\partial}{\partial u}\frac{|E_l(k',\omega')|^2}{\Omega_s-K_s u}\frac{1}{\omega'-k'u}\frac{\partial}{\partial u}f_{0e}$$

$$+\left(\frac{e}{m}\right)^2\left(\frac{\omega_{pe}}{\Omega}\right)^2\int\frac{d^3v}{\Omega-Ku}\sum_{k'}\frac{\partial}{\partial u}\frac{|E_l(k',\omega')|^2}{\Omega_s-K_s u}K\left(u+v_x^2\frac{\partial}{\partial u}\right)\frac{1}{\omega'-k'u}\frac{\partial f_{0e}}{\partial u}$$

$$-\left(\frac{e}{m}\right)^2\frac{\omega_{pe}^4}{\Omega^2}\sum_{k'}\frac{\Omega}{\Omega-\omega'}D_0^{-1}(K-k',\Omega-\omega')|E_l(k',\omega')|^2[(R_0+R_0'+R_0'')(R_1+R_2+R_3)]=0, \tag{21}$$

where for $\omega'\ll\Omega$, $u\ll\Omega/k$, the constants, to the lowest order, are defined as

$$R_0=\int\frac{1}{\Omega-Ku}\frac{\partial}{\partial u}\frac{f_{0e}}{K_s u-\Omega_s}du=\frac{K}{\Omega^3}, \tag{22}$$

$$R_1=\int\frac{1}{\Omega_s-K_s u}\frac{\partial}{\partial u}\frac{1}{Ku-\Omega}f_{0e}\,du=\frac{K-k'}{\Omega^3}, \tag{23}$$

$$R_2=\int\frac{1}{\Omega_s-K_s u}\frac{1}{\omega'-k'u}\frac{\partial}{\partial u}f_{0e}\,du$$

$$=\frac{1}{\Omega k'}\frac{m}{T_e}\left(1+i\sqrt{\tfrac{1}{2}\pi}\frac{\omega'}{|k'|v_e}\right), \tag{24}$$

$$R_3=-\frac{K}{\Omega}\int\frac{u+v_x^2\frac{\partial}{\partial u}}{\Omega_s-K_s u}\frac{1}{\omega'-k'u}\frac{\partial}{\partial u}f_{0e}\,d^3v,$$

$$=-\frac{K}{\Omega^2}\frac{\omega'}{k'^2}\frac{m}{T_e}\left(1+i\sqrt{\tfrac{1}{2}\pi}\frac{\omega'}{|k'|v_e}\right)$$

$$+\frac{K(K-k')}{\Omega^3}\frac{1}{k'}\left(1+i\sqrt{\tfrac{1}{2}\pi}\frac{\omega'}{|k'|v_e}\right), \tag{25}$$

$$R_0'=\int\frac{1}{\Omega-Ku}\frac{1}{k'u-\omega'}\frac{\partial}{\partial u}f_{0e}\,du$$

$$=-\frac{1}{\Omega}\frac{1}{k'}\frac{m}{T_e}\left(1+i\sqrt{\tfrac{1}{2}\pi}\frac{\omega'}{|k'|v_e}\right), \tag{26}$$

$$R_0''=\int\frac{K_s}{\Omega_s}\frac{1}{\Omega-Ku}\left(u+v_x^2\frac{\partial}{\partial u}\right)\frac{1}{\omega'-k'u}\frac{\partial}{\partial u}f_{0e}\,d^3v$$

$$=\frac{(K-k')}{\Omega^2}\frac{\omega'}{k'^2}\frac{m}{T_e}\left(1+i\sqrt{\tfrac{1}{2}\pi}\frac{\omega'}{|k'|v_e}\right)$$

$$-\frac{K(K-k')}{\Omega^3}\frac{1}{k'}\left(1+i\sqrt{\tfrac{1}{2}\pi}\frac{\omega'}{|k'|v_e}\right), \tag{27}$$

and $\Omega_s=\Omega-\omega'$, $K_s=K-k'$. Here f_{0e} is assumed to be the Maxwellian distribution.

Equation (21) is the main result of this paper. Terms involving $|E_l(k',\omega')|^2$ arise owing to a nonlinear coupling of the stationary turbulence with background plasma particles.

III. ENHANCED RADIATION

Here we analyze Eq. (21) to find a complex root of Ω. It emerges from our simple calculation that an unstable electromagnetic wave can appear from a turbulent medium. In the Appendix, we see that the most dominant imaginary contribution of Eq. (21) comes from the last term, which can be written as

$$\mathrm{Im}D_T(K,\Omega)$$

$$=-\left(\frac{e}{m}\right)^2\frac{\omega_{pe}^4}{\Omega^2}\sum_{k'}D_0^{-1}(K-k',\Omega-\omega')$$

$$\times|E_l(k',\omega')|^2(\mathrm{Re}R_0'\,\mathrm{Im}R_2+\mathrm{Im}R_0'\,\mathrm{Re}R_2), \tag{28}$$

where Re and Im stand for the real and the imaginary part of the relevant terms.

For $\omega'\ll\Omega$, we can expand $D_0(K-k',\Omega-\omega')$ and find

$$D_0(K-k',\Omega-\omega')=1-\frac{c^2(K-k')^2}{(\Omega-\omega')^2}-\frac{\omega_{pe}^2}{(\Omega-\omega')^2}$$

$$=\frac{-2\omega'}{\Omega}+\frac{c^2}{\Omega^2}(2Kk'-k'^2), \tag{29}$$

where the linear dispersion relation of the test wave is used. By using Eqs. (24) and (26), we readily get

$$\mathrm{Re}R_0'=-\frac{1}{\Omega}\frac{1}{k'}\frac{m}{T_e},\quad \mathrm{Im}R_0'=-\frac{1}{\Omega}\frac{1}{k'}\frac{m}{T_e}\sqrt{\tfrac{1}{2}\pi}\frac{\omega'}{|k'|v_e}, \tag{30}$$

$$\mathrm{Re}R_2=\frac{1}{\Omega}\frac{1}{k'}\frac{m}{T_e},\quad \mathrm{Im}R_2=\frac{1}{\Omega k'}\frac{m}{T_e}\sqrt{\tfrac{1}{2}\pi}\frac{\omega'}{|k'|v_e},$$

where $|\omega'/k'v_e|\ll1$ is used.

Substituting Eqs. (29) and (30) into Eq. (28), we obtain the growth rate γ of the electromagnetic radiation. The result is

$$\frac{\gamma}{\Omega}=\sqrt{\tfrac{1}{2}\pi}\left(\frac{\omega_{pe}}{\Omega}\right)^3\sum_{k'}W(k')\left(\frac{k_e}{|k'|}\right)^3$$

$$\times\left\{1\Big/\left[1+\frac{1}{2}\left(\frac{M}{m}\right)^{1/2}\frac{c^2}{v_e}\frac{(k'-2K)}{\Omega}\right]\right\}, \tag{31}$$

where

$$\Omega^2 = \Omega_r^2 \equiv c^2K^2 + \omega_{pe}^2 , \quad W(k') = \frac{|E_l(k',\omega')|^2}{4\pi n T_e} ,$$

and the relation

$$\gamma = -\operatorname{Im} D_T(\Omega, K) \Big/ \frac{\partial}{\partial\Omega} \operatorname{Re} D_T(\Omega, K) \Big|_{\Omega=\Omega_r} \tag{32}$$

is used. Furthermore, in deriving Eq. (31) we have neglected the frequency shift due to turbulence.

The reader should note the fact that linear dispersion of ion wave $\epsilon_l(k,\omega)=0$ contains two branches, namely $\omega = \pm\omega_k = \pm k(T_e/M)^{1/2}$. The electron current drives the $+\omega_k$ mode to become unstable. Hence, in what follows, we assume that the mode $+\omega_k$ is maintained in the steady turbulent state. Subsequently, we ignore the $-\omega_k$ mode. Physically, this corresponds to the anisotropic ion-wave turbulence spectrum. Equation (31) is an odd function of ω_k. It then follows that the growth rate vanishes ($\gamma = 0$) for the isotropic ion-wave turbulence, owing to the exact cancellation between ω_k and $-\omega_k$ modes.

The order of magnitude of Eq. (31) can easily be estimated; we have

$$\frac{\gamma}{\Omega} \approx \left(\frac{\omega_{pe}}{\Omega}\right)^3 W^s \frac{k_e}{k_0'} \left(\frac{m}{M}\right)^{1/2} \frac{v_e}{c} , \tag{33}$$

where W^s and k_0' are, respectively, the total energy density and the characteristic wave number of the anisotropic ion-wave turbulence. Since in the steady state (fully developed turbulence) the ion-wave turbulence will have the fluctuation wavelength k_0^{-1} which is of the order of the size[14] of the dimension L, it follows that the growth rate given by Eq. (33) will dominate over the previous ones.[15] The amplification of the electromagnetic wave is possible only when its growth rate is larger than the collisional damping rate ν_e. Thus in order for the present mechanism to be operative, the ion-wave energy density should satisfy the inequality

$$W^s \geq \left(\frac{M}{m}\right)^{1/2} \frac{c}{v_e} \frac{k_0'}{k_e} \frac{1}{n\lambda_D^3} . \tag{34}$$

We will now comment on the physical mechanism of the amplification. Since the new contribution originates from the imaginary contribution of the velocity space integrals in Eq. (21), emission arises due to the accelerating motion of the electrons. Hence the radiation mechanism may be considered to be the induced bremsstrahlung interaction between ion-wave fields and the electrons. Consequently, the electromagnetic waves are directly emitted from accelerated electrons. Two points are worth noting. First, the contribution of the magnetic terms [Eqs. (25) and (27)] is negligible as compared to the electric field terms. Second, our mechanism works for the anisotropic ion-wave turbulence and an isotropic electron distribution function.

In the presence of an external current, the unperturbed electron distribution function can be written as

$$f_{0e} = \left(\frac{m}{2\pi T_e}\right)^{1/2} \exp\left(-\frac{m(v-v_0)^2}{2T_e}\right) , \tag{35}$$

where v_0 is the drift velocity of the electrons. The growth rate γ for this case is found to be

$$\frac{\gamma}{\Omega} = \sqrt{\tfrac{1}{2}\pi}\left(\frac{\omega_{pe}}{\Omega}\right)^3 \left(\frac{M}{m}\right)^{1/2} \left(\frac{v_0}{v_e}\right) \times \sum_{k'} W(k') \Big/ \left\{\left[-1 + \frac{1}{2}\left(\frac{M}{m}\right)^{1/2} \frac{c^2}{v_e} \frac{(-k'+2K)}{\Omega}\right]\right\} \times \left(\frac{k_e}{|k'|}\right)^3 , \tag{36}$$

where $v_0 > \omega'/k'$ is assumed. Equation (36) exhibits that the presence of the external current enhances the growth rate. The reason is that drifting electrons carry some extra free energy from the beginning.

To obtain an appreciable growth of the electromagnetic waves, we estimate the minimum scale length of the plasma. Taking typical plasma parameters, namely $n = 10^{14}\ \mathrm{cm}^{-3}$, $T_e = 1$ keV, $k_e/k_0' = 10$, $W^s = (m/M)^{1/2} = \frac{1}{43}$, we find $\gamma \approx 10^8\ \mathrm{sec}^{-1}$. Then the minimum scale length of the plasma within which the interaction can take place is $L \equiv c\gamma^{-1} = 3$m. This estimate suggests that our mechanism for generating an electromagnetic wave in a turbulent plasma is quite reasonable.

IV. DISCUSSION

Stability of a stationary high-frequency turbulent plasma against low-frequency ion-acoustic perturbations is of considerable interest with regard to the understanding of the modulational instability[12] and wave-trapping phenomena. Within the framework of a weak-turbulence approximation, one describes the evolution of randomly phased Langmuir wave packets by means of the wave kinetic equation (Liouville's equation). The form of the latter is similar to that of the Vlasov equation except that the force term here originates in the nonlinear interaction of the turbulent perturbations. When the phase velocity of the latter is equal to the group velocity of the plasmons, a strong resonance interaction sets in. This leads to the growing density perturbations (modulational instability) and was first discovered by Vedenov and Rudakov.[12]

On the other hand, generation of electromagnetic waves from a plasma is of much interest. For a plasma near thermal equilibrium, the bremsstrahlung radiation due to interparticle collisions was studied by Dupree and by Tidman and Dupree.[16] This process originates if one considers the perturbation to be of the second order in the discreteness parameter $(n\lambda_D)^{-3}$. Consequently this process does not fall within the Vlasov description. In turbulent and collisionless plasmas, numerous types of mode couplings take place. We briefly summarize a few of them in the following.

The first one is the nonlinear scattering[7] of waves on the electrons leading to the damping of the electromagnetic radiation in the Maxwellian plasma. Physically, this occurs because high-frequency electromagnetic waves interact with long-wavelength ion fluctuations—resulting in nonlinearly excited electron plasma waves. The latter resonate with the collective motion of the electrons to produce enhanced oscillations which are in turn Landau damped on particles.

The second process is the three-wave decay interaction. Here, a finite-amplitude electromagnetic wave decays into a daughter wave and a Langmuir–ion-acoustic wave. For a magnetized plasma, Hutchinson *et al.*[17] considered the decay interaction of an ordinary electromagnetic wave into a long-wavelength electron plasma wave and an ion wave in order to explain enhanced radiation in tokamak plasmas.

The third process, i.e., the induced bremsstrahlung, arises owing to the interaction of ion waves with electrons. In the presence of ion-wave turbulence, the resonant electrons can emit and absorb electromagnetic waves through induced bremsstrahlung interaction. The difference of the emission and the absorption processes causes the growth of electromagnetic radiation. The free energy of the electrons accelerated by the electric field of the ion waves is responsible for the occurrence of the unstable wave. The increment crucially depends on the slope of the electron distribution computed at the resonant velocity between electrons and the ion waves.

Moreover, since the growth rate [Eq. (33)] also depends on the real part of the dielectric constant $(\partial/\partial\Omega)\mathrm{Re}D_T$, the test wave with a particular propagation direction is amplified. The other wave with different propagation direction is damped. Therefore the induced bremsstrahlung interaction between electrons and turbulent fields gives the enhanced electromagnetic radiation for any type of electron distribution function except the plateau one.

In this paper we have emphasized the importance of the third process which, to the authors' best knowledge, was not considered earlier. For this purpose we have obtained the effective dielectric constant of the electromagnetic waves [Eq. (21)] within the framework of the linear response theory. The linear analysis shows the occurrence of the enhanced bremsstrahlung radiation from a turbulent plasma. As our process originates from the wave-particle interaction, the dominant contribution comes from the region $k' \ll k_e$ in Eq. (32). This limit is just the opposite to that of the conventional bremsstrahlung radiation[16] due to interparticle collisions, in which the main contributions arise from $k' > k_e$.

Furthermore, our investigation assumes that the steady turbulent state has a characteristic time scale which is much larger than the e-folding time ($\gamma^{-1} \approx 10^{-8}$ sec) of the instability. Hence the enhanced radiation occurs on such a short time scale that the assumption of a stationary turbulent state is always justified.

In laboratory plasmas, enhanced radiation near $\Omega \sim \omega_{pe}$ is often observed.[17] We propose that our mechanism is a possible candidate for this phenomena. According to the observations of solar radioemission, one frequently observes[3] anomalous radiation near $\Omega \sim \omega_{pe}$. Since electron bursts could be responsible for the ion-wave turbulence in the solar atmosphere, our mechanism should be a viable candidate for generating anomalous radiation from the sun.

In summary, we have pointed out the possibility of a *new mechanism* which can be responsible for the enhanced radiation from a turbulent plasma. Subsequently, the present radiation mechanism may offer an additional diagnostic technique for plasma turbulence. Moreover, consideration and understanding of enhanced radiation from a turbulent medium can be regarded as central problems in attempting to interpret numerous astrophysical radio phenomena, including pulsars.

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APPENDIX

We estimate the various terms which appear in Eq. (21). In calculating various terms, we bear in mind that the electromagnetic wave resonances are not important because $\Omega/K \gg u$. Using formula (22)–(27), it is rather easy to show that the most dominant imaginary part of the last term of

Eq. (21) comes from $\mathrm{Re}R_0'\mathrm{Im}R_2+\mathrm{Im}R_0'\mathrm{Re}R_2$. The other terms are smaller by a factor ω'/Ω.

The other two nonlinear terms of Eq. (21) are smaller as compared to Eq. (28). For example, consider the term

$$\left(\frac{e}{m}\right)^2\left(\frac{\omega_{pe}}{\Omega}\right)^2\int du\,\frac{\Omega}{\Omega-Ku}\sum_{k'}\frac{\partial}{\partial u}\frac{|E_l(k',\omega')|^2}{\Omega-(K-k')u}\,\frac{1}{\omega'-k'u}\,\frac{\partial}{\partial u}f_{0e}$$

$$=-\left(\frac{e}{m}\right)^2\left(\frac{\omega_{pe}}{\Omega}\right)^2\frac{K}{\Omega^2}\sum_{k'}|E_l(k',\omega')|^2\frac{1}{k'}\,\frac{m}{T_e}\left(1+i\sqrt{\tfrac{1}{2}\pi}\,\frac{\omega'}{|k'|v_e}\right). \tag{A1}$$

Then the ratio T [T is the imaginary part of Eq. (A1) divided by the imaginary part of Eq. (28)] can be shown to be a small quantity, i.e., $T=\omega' Kk' v_e^2/\Omega\omega_{pe}^2 \ll 1$. The other term which arises from the wave magnetic field is

$$\left(\frac{e}{m}\right)^2\left(\frac{\omega_{pe}}{\Omega}\right)^2\int\frac{d^3v}{\Omega-Ku}\sum_{k'}\frac{\partial}{\partial u}\frac{|E_l(k',\omega')|^2}{\Omega_s-K_s u}\,K\left(u+v_x^2\frac{\partial}{\partial u}\right)\frac{1}{\omega'-k'u}\,\frac{\partial}{\partial u}f_{0e}$$

$$=\left(\frac{e}{m}\right)^2\left(\frac{\omega_{pe}}{\Omega}\right)^2\frac{K^2}{\Omega^2}\sum_{k'}\frac{K-k'}{\Omega^2}|E_l(k',\omega')|^2\frac{1}{k'}\left(1+i\sqrt{\tfrac{1}{2}\pi}\,\frac{\omega'}{|k'|v_e}\right). \tag{A2}$$

The ratio Q [Q is the imaginary part of Eq. (A2) divided by the imaginary part of Eq. (28)] becomes

$$Q\equiv\frac{\omega'}{\Omega}\,\frac{Kk'v_e^2}{\omega_{pe}^2}\,\frac{K^2v_e^2}{\Omega^2}\ll 1\,. \tag{A3}$$

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