PHYSICAL REVIEW A VOLUME 20, NUMBER 5 NOVEMBER 1979

# Resonant potential scattering in a low-frequency laser field

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(Received 12 March 1979)

The problem of resonant potential scattering of a charged particle in the presence of a low-frequency laser is analyzed. A method suggested by Krüger and Jung is used and their result is obtained as part of the lowest order in an expansion in $\omega$, the laser frequency. Additional terms are obtained in lowest order and in the next order. No expansion in powers of $\omega/\Gamma$ is made where $\Gamma$ is the resonance width. A modified sum rule for the scattering is obtained.

## I. INTRODUCTION

One of the motivations for studying electron-atom scattering in the presence of a laser field is that it provides a means of measuring off-shell $T$ matrices and interference effects which are not otherwise accessible. This was recognized[1] some time ago in nuclear physics and has been the motivation for studies of nuclear bremsstrahlung. The laser, in atomic physics, makes the experiment easier by increasing the coupling to the radiation field through the stimulated process rather than reyling on the weak coupling in the spontaneous process as in bremsstrahlung.

The exotic processes mentioned above have yet to be seen. In the situation in which the laser is tuned to a transition of the free atomic target the only new quantities that have been measured[2] or calculated[3] are cross sections with initial states that are laser prepared and perhaps not otherwise accessible. The exotic effects require more-intense tunable lasers. A similar remark can be made concerning the situation in which the laser is tuned to a transition in the compound state, between resonant states,[4] or between a resonant state and a negative ion state.[5]

If the laser is low frequency and intense, (a $CO_2$ laser for example) it has been shown by Kroll and Watson[6] for potential scattering that, neglecting terms of order $\omega^2$, the only thing one obtains is the conventional cross section with no off-shell effects. The proof has been extended to atomic targets[7] and it has been shown[8] that off-shell effects will enter in order $\omega^2$. These developments neglect the possibility of scattering resonances in the absence of the laser and, as Krüger and Jung[9] have observed this effect dramatically changes the form of the $T$ matrix in the presence of the laser for potential scattering. For example, in lowest order in $\omega$, the Kroll-Watson result for the $T$ matrix for scattering with the transfer of $l$ photons is

$$J_l((\vec{p}_f-\vec{p}_i)\cdot\vec{\alpha})[\vec{p}_f\,|T(\epsilon_i)|\vec{p}_i], \tag{1.1}$$

where $T$ is the conventional $T$ matrix in the absence of the laser, $\vec{p}_i$ and $\vec{p}_f$ are the initial and final momenta, and

$$\vec{\alpha}=(e/m)\vec{E}/\omega^2 \tag{1.2}$$

is held fixed in the expansion in powers of $\omega$. The average initial electron energy $\epsilon_i=p_i^2/2m$ is related to the final energy[10] by

$$\epsilon_f=p_f^2/2m=\epsilon_i+l\omega, \tag{1.3}$$

but since we are discussing the lowest order in $\omega$ the $T$ matrix in (1.1) is evaluated on-shell. The expansion used to obtain (1.1) is in powers of $\omega/\epsilon_i$, but as Krüger and Jung have pointed out, when a resonance occurs there is also an implied expansion in powers of $\omega/\Gamma$, where $\Gamma$ is the width of a resonance, and this is usually not small. They treated the case in which the resonance energy occurs such that

$$\mathcal{E}_R\simeq\epsilon_0=\epsilon_i+m_0\omega, \tag{1.4}$$

where $m_0$ is an integer. Then when the projectile picks up $m_0$ photons it has the correct energy to resonate with the target. Their result for the $T$ matrix in lowest order can be written

$$J_{m_0-l}(\vec{p}_f\cdot\vec{\alpha})J_{m_0}(\vec{p}_i\cdot\vec{\alpha})[\vec{p}_f\,|T(\epsilon_0)|\vec{p}_i]. \tag{1.5}$$

This result has been obtained only in lowest order in $\omega$ and so the $T$ matrix is evaluated on shell.

Only one experiment[11] has been performed in this field. Values of $l$ as high as $\pm 3$ have been seen but the only quantitative result has been the sum rule

$$\sum_l \frac{d\sigma_l}{d\Omega}(\vec{p}_f,\vec{p}_i)=\frac{d\sigma}{d\Omega}(\vec{p}_f,\vec{p}_i)\Big|_{\alpha=0}. \tag{1.6}$$

That is, the total scattering for all $l$ in the presence of the laser is equal to the scattering in the absence of the laser. This is satisfied by the Kroll-Watson result (1.1) even when terms of order $\omega$ are included. It is not satisfied by (1.5), where an additional factor $J^2_{m_0}(\vec{p}_i\cdot\vec{\alpha})$ appears on the left-hand side of (1.6). The experiment is not potential scattering, but electron-argon scattering at the resonant energy of 11 eV and more important, since $m_0=0$ and[9] $(\vec{\alpha}\cdot\vec{p}_i)^2\sim 0.1$, the Bessel function is hardly different from unity.

In Sec. II the work of Krüger and Jung is extended by obtaining higher-order terms in the expansion in powers of $\omega/\epsilon_i$, but terms of order $\omega/\Gamma$ are retained to all orders. The method used closely follows a suggestion of Krüger and Jung. Their result is reproduced as part of the lowest-order result. Additional lowest-order terms are obtained and a modified sum rule is discussed.

## II. DERIVATION OF THE $T$ MATRIX

The method used by Krüger and Jung[9] to obtain (1.5) was to treat the projectile-laser interaction in the initial and final states exactly but to expand in powers of it in intermediate states. The justification was that the scattering event takes place rapidly enough so that this interaction will only perturb the projectile weakly even for intense lasers. They kept only the zero-order term but higher orders will be obtained here.

The starting point is the exact $S$ matrix

$$S=-i\int d^3r\,dt\,\chi_f^*(\vec{r},t)V(r)\psi_i^{(+)}(\vec{r},t)\,, \tag{2.1}$$

where $\psi_i^{(+)}$ is the exact wave function which evolves out of the initial state at $t=-\infty$, $V$ is the scattering potential and $\chi_f$ is the final state. It satisfies the Schrodinger equation with $V$ set equal to zero and is given by

$$\chi_f(\vec{r},t)=\exp\{i\vec{p}_f\cdot[\vec{r}-\vec{\alpha}(t)]-i\epsilon_f t\}$$
$$=\sum_\lambda J_\lambda(\vec{p}_f\cdot\vec{\alpha})\exp[i\vec{p}_f\cdot\vec{r}-i(\epsilon_f+l\omega)t]\,, \tag{2.2}$$

where the laser has been treated as a classical single-mode field in the dipole approximation and the $A^2(t)$ has been removed from the Hamiltonian by a contact transformation. The vector potential has been chosen to be

$$\vec{A}(t)=\omega(m/e)\vec{\alpha}\cos\omega t\,, \tag{2.3}$$

implying

$$\vec{\alpha}(t)=\vec{\alpha}\sin\omega t\,. \tag{2.4}$$

The expansion described above can be obtained from

$$\psi_i^{(+)}=\chi_i+\overline{G}^{(+)}V\chi_i\,, \tag{2.5}$$

where $G$ is the full Green's function, which can be expanded as

$$\overline{G}^{(+)}=G^{(+)}+G^{(+)}\frac{e}{m}\vec{p}\cdot\vec{A}G^{(+)}$$
$$+G^{(+)}\frac{e}{m}\vec{p}\cdot\vec{A}G^{(+)}\frac{e}{m}\vec{p}\cdot\vec{A}G^{(+)}+\cdots\,. \tag{2.6}$$

This generates a series for the $S$ matrix in powers of $\omega$. $G^{(+)}$ is the Green's function in the absence of the laser. It can be written

$$G^{(+)}(\vec{r}t,\vec{r}'t')=-i\theta(t-t')\sum_n\phi_n(r)\phi_n^*(r')$$
$$\times\exp[-(iW_n+\eta)(t-t')]\,, \tag{2.7}$$

where the $\phi_n$ are the stationary states (with energies $W_n$) in the presence of the potential $V$ and $\eta$ is a positive infinitesimal. Substitution of (2.5) and (2.6) into (2.1) results in a series for $S$,

$$S=\sum_{n=0}^{\infty}S^{(n)}\,. \tag{2.8}$$

The individual terms can then be simplified by the use of (2.1), since all the $t$ integrations can be performed. An energy $\delta$ function appears in all the terms, so (2.8) can be rewritten

$$S=-i\sum_{l=-\infty}^{\infty}2\pi\delta(\epsilon_f-\epsilon_i-l\omega)\sum_{n=0}^{\infty}T_l^{(n)}(\vec{p}_f,\vec{p}_i)\,, \tag{2.9}$$

thereby defining the $T$ matrix for scattering with the net absorbtion of $l$ quanta of energy from the laser. The $n=0$ term is just

$$T_l^{(0)}(\vec{p}_f,\vec{p}_i)=\sum_\lambda J_{\lambda-l}(\vec{p}_f\cdot\vec{\alpha})J_\lambda(\vec{p}_i\cdot\vec{\alpha})$$
$$\times(\vec{p}_f|V|\vec{p}_i) \tag{2.10}$$

and the $n=1$ term is

$$T_l^{(1)}(\vec{p}_f,\vec{p}_i)=\sum_\lambda J_{\lambda-l}(\vec{p}_f\cdot\vec{\alpha})J_\lambda(\vec{p}_i\cdot\vec{\alpha})$$
$$\times[\vec{p}_f|VG_0^{(+)}(\epsilon_i+\lambda\omega)V|\vec{p}_i]\,, \tag{2.11}$$

where

$$[r|G_0^{(+)}(E)|r']=\sum_n\frac{\phi_n(r)\phi_n^*(r')}{(E^+-W_n)} \tag{2.12}$$

is the time-independent Green's function with the potential $V$. These two can be combined to give

$$T_l^{(0)}(\vec{p}_f,\vec{p}_i)+T_l^{(1)}(\vec{p}_f,\vec{p}_i)$$
$$=\sum_\lambda J_{\lambda-l}(\vec{p}_f\cdot\vec{\alpha})J_\lambda(\vec{p}_i\cdot\vec{\alpha})[\vec{p}_f|T(\epsilon_i+\lambda\omega)|\vec{p}_i]\,, \tag{2.13}$$

where $T$ is the exact $T$ operator for scattering from $V$ in the absence of the laser. The $n=2$ term can be evaluated with only a little more effort, yielding

$$T_l^{(2)}(\vec{p}_f,\vec{p}_i) = \frac{\omega}{2}\sum_\lambda \sum_{x=\pm1} J_{\lambda-x-l}(\vec{p}_f\cdot\vec{\alpha})J_\lambda(\vec{p}_i\cdot\vec{\alpha}) \times [\vec{p}_f|VG_0^{(+)}(\epsilon_i+\omega(\lambda-x))\vec{\alpha}\cdot\vec{p}G_0^{(+)}(\epsilon_i+\lambda\omega)V|\vec{p}_i], \tag{2.14}$$

where the index $x$ arises from the $\cos\omega t$ in $A(t)$ by the use of

$$\cos\omega t = \tfrac{1}{2}\sum_{x=\pm1}\exp(ix\omega t).$$

With this result it is now a simple matter to obtain the general term in the series which is

$$T_l^{(n+1)}(\vec{p}_f,\vec{p}_i) = \left(\frac{\omega}{2}\right)^n \sum_{\lambda x_1\cdots x_n} J_{\lambda-l-\Sigma_{j=1}^n x_j}(\vec{p}_f\cdot\vec{\alpha})J_\lambda(\vec{p}_i\cdot\vec{\alpha}) \times\left[\vec{p}_f\left|VG_0^{(+)}\left(\epsilon_i+\omega\left(\lambda-\sum_1^n x_j\right)\right)\vec{\alpha}\cdot\vec{p}G_0^{(+)}\left(\epsilon_i+\omega\left(\lambda-\sum_1^{n-1}x_j\right)\right)\vec{\alpha}\cdot\vec{p}\cdots\vec{\alpha}\cdot\vec{p}G_0^{(+)}(\epsilon_i+\omega\lambda)V\right|\vec{p}_i\right], \tag{2.15}$$

with $(n+1)$ factors of $G_0^{(+)}$ occurring. These expressions show that this generates a series in powers powers of $\omega$, but a complication arises because of the fact that the Green's functions may be resonant. Let us hypothesize that there is an isolated pole (resonance) in $G_0^{(+)}$ at the energy $E_R=\mathcal{E}_R-\frac{1}{2}i\Gamma$ such that $\mathcal{E}_R\simeq\epsilon_0=\epsilon_i+m_0\omega$, where $m_0$ is an integer. Near that energy the Green's function can be written

$$G_0^{(+)}(E)=U_R)(U_R/(E-E_R)+G_{NR}(E), \tag{2.16}$$

where $G_{NR}$ is a slowly varying function of $E$. In the sequence of $G$'s in (2.15) it is clear that successive functions may not be resonant for a given set of the parameters $(\lambda x_1\cdots x_n)$ since the arguments of any successive pair will differ by one of the $x$'s times $\omega$, which cannot be zero. This is just an expression of the fact that the projectile-atom interaction changes the energy by $\pm\omega$. That means that at most $[\frac{1}{2}(n+2)]$ of the $G$'s can be resonant, where $[\frac{1}{2}(n+2)]$ is the largest integer $\leq\frac{1}{2}(n+2)$. This gives the number of resonant-energy denominators that can occur in (2.15). If a resonant Green's function occurs either first or last in the sequency in (2.15) then it will contribute a factor of $(\vec{p}_f|V|U_R)$ or $(U_R|V|\vec{p}_i)$. These are the matrix elements coupling the resonant state to the continuum, the mechanism of its decay, so that these can be expected to be small and of order $\Gamma^{1/2}$.

If we put these arguments together then the maximum contribution to $T^{(n+1)}$ will be

$$\frac{\omega^n}{|E-E_R|^{[(1/2)(n+2)]}}\begin{cases}\Gamma, & n+1 \text{ odd},\\ \Gamma^{1/2}, & n+1 \text{ even}\end{cases}.$$

For $n=3$ this is $\omega^3\Gamma^{1/2}/(E-E_R)^2$, and since $E-E_R\sim\Gamma$ near resonance this is of order $\omega^{3/2}(\omega/\Gamma)^{3/2}$, which we drop. Higher values of $n$ give even smaller results so only $n\leq 2$ need be considered.

Before proceeding we note that the factors $(U_R|V|\vec{p}_i)$ are taken to be small but factors such as $(\vec{p}_f|VG_{NR}\vec{\alpha}\cdot\vec{p}|U_R)$ are not necessarily small since there is no reason to assume that the $\vec{p}$ operator will only weakly couple the resonance state to the slowly varying part of the Green's function.

Now let us turn to the successive terms in the series for $T$. We first assume that neither the initial nor the final energy is resonant, that is that $m_0\neq 0$ and $l\neq m_0$. The leading term is (2.13) and we must distinguish $\lambda=m_0$ from all the others:

$$T_l^{(0)}(\vec{p}_f,\vec{p}_i)+T_l^{(1)}(\vec{p}_f,\vec{p}_i)=J_{m_0-l}(\vec{p}_f\cdot\vec{\alpha})J_{m_0}(\vec{p}_i\cdot\vec{\alpha})[\vec{p}_f|T(\epsilon_0)|\vec{p}_i]+\sum_{\lambda\neq m_0}J_{\lambda-l}(\vec{p}_f\cdot\vec{\alpha})J_\lambda(\vec{p}_i\cdot\vec{\alpha}) \times[\vec{p}_f|T(\epsilon_i+\lambda\omega)|\vec{p}_i]. \tag{2.17}$$

The second term is nonresonant and so can be expanded about $\epsilon_i$ or any other nonresonant energy. The choice is arbitrary, but for symmetry[1] we choose

$$T(\epsilon_i+\lambda\omega)=\tfrac{1}{2}[T(\epsilon_i)+T(\epsilon_f)]+\tfrac{1}{2}\omega\left(\lambda\frac{\partial}{\partial\epsilon_i}T(\epsilon_i)+(\lambda-l)\frac{\partial}{\partial\epsilon_f}T(\epsilon_f)\right)+\cdots. \tag{2.18}$$

The summations over $\lambda$ can now be carried out, with the result

$$T_l^{(0)}(\vec{p}_f,\vec{p}_i)+T_l^{(1)}(\vec{p}_f,\vec{p}_i)=J_{m_0-l}(\vec{p}_f\cdot\vec{\alpha})J_{m_0}(\vec{p}_i\cdot\vec{\alpha})[\vec{p}_f|T(\epsilon_0)|\vec{p}_i]+[J_{-l}(\vec{q}\cdot\vec{\alpha})-J_{m_0-l}(\vec{p}_f\cdot\vec{\alpha})J_{m_0}(\vec{p}_i\cdot\vec{\alpha})]$$
$$\times\tfrac{1}{2}[\vec{p}_f|T(\epsilon_i)+T(\epsilon_f)|\vec{p}_i]-\frac{\omega}{2}\frac{l}{\vec{q}\cdot\vec{\alpha}}J_{-l}(\vec{q}\cdot\vec{\alpha})\left[\vec{p}_f\left|\vec{p}_i\cdot\vec{\alpha}\frac{\partial}{\partial\epsilon_i}T(\epsilon_i)+\vec{p}_f\cdot\vec{\alpha}\frac{\partial}{\partial\epsilon_f}T(\epsilon_f)\right|\vec{p}_i\right]$$
$$-\tfrac{1}{2}\omega m_0 J_{m_0-l}(\vec{p}_f\cdot\vec{\alpha})J_{m_0}(\vec{p}_i\cdot\vec{\alpha})\left[\vec{p}_f\left|\frac{\partial}{\partial\epsilon_i}T(\epsilon_i)+\frac{\partial}{\partial\epsilon_f}T(\epsilon_f)\right|\vec{p}_i\right],\quad \vec{q}=\vec{p}_f-\vec{p}_i. \tag{2.19}$$

The next order, $T^{(2)}$, can have either of the two Green's function resonant. This can be written

$$T_l^{(2)}(\vec{p}_f,\vec{p}_i)=\frac{\omega}{2}\sum_{x=\pm1}J_{m_0-l-x}(\vec{p}_f\cdot\vec{\alpha})J_{m_0}(\vec{p}_i\cdot\vec{\alpha})[\vec{p}_f|VG_0^{(+)}(\epsilon_f+\omega(m_0-l-x))\vec{\alpha}\cdot\vec{p}G_0^{(+)}(\epsilon_0)V|\vec{p}_i]$$
$$+\frac{\omega}{2}\sum_{x=\pm1}J_{m_0-l}(\vec{p}_f\cdot\vec{\alpha})J_{m_0+x}(\vec{p}_i\cdot\vec{\alpha})[\vec{p}_f|VG_0^{(+)}(\epsilon_0)\vec{\alpha}\cdot\vec{p}G_0^{(+)}(\epsilon_i+\omega(m_0+x))V|\vec{p}_i]$$
$$+\frac{\omega}{2}\sum_{\lambda,x}{}'J_{\lambda-l-x}(\vec{p}_f\cdot\vec{\alpha})J_\lambda(\vec{p}_i\cdot\vec{\alpha})[\vec{p}_f|VG_0^{(+)}(\epsilon_f+\omega(\lambda-l-x))\vec{\alpha}\cdot\vec{p}G_0^{(+)}(\epsilon_i+\omega\lambda)V|\vec{p}_i], \tag{2.20}$$

where the prime on the last summation deletes the terms $\lambda=m_0$ and $\lambda=m_0+x$, which appear explicitly in the first two terms. The Green's function $G(\epsilon_0)$ is resonant and given by (2.16), where $G_{NR}$ is a slowly varying function of $\epsilon_0$ and $U_R$ is interpreted as the resonance wave function.[12] The remaining Green's functions in (2.20) are all nonresonant and can be expanded in powers of $\omega$. The contribution from the terms linear in $\omega$ is of order $\omega^2\Gamma^{-1/2}$, which is dropped. So each of these Green's functions can be evaluated at some nonresonant energy $\epsilon_i$ or $\epsilon_f$ and the difference (of order $\omega$) is not significant. Moreover, the nonresonant part of the resonant wave function $G_{NR}$ is expected (to order $\omega$) to be the same as $G(\epsilon_f)$ and $G(\epsilon_i)$, so the distinction among the three will now be dropped. The summations over $x$ and $\lambda$ in (2.20) can all be carried out, with the result

$$T_l^{(2)}(\vec{p}_f,\vec{p}_i)$$
$$=\frac{\omega}{\epsilon_0-E_R}J_{m_0-l}(\vec{p}_f\cdot\vec{\alpha})J_{m_0}(\vec{p}_i\cdot\vec{\alpha})\left(\frac{m_0-l}{\vec{p}_f\cdot\vec{\alpha}}(\vec{p}_f|VG\,\vec{\alpha}\cdot\vec{p}|U_R)(U_R|V|\vec{p}_i)+\frac{m_0}{\vec{p}_i\cdot\vec{\alpha}}(\vec{p}_f|V|U_R)(U_R|\vec{\alpha}\cdot\vec{p}GV|\vec{p}_i)\right)$$
$$-\frac{\omega l}{\vec{q}\cdot\vec{\alpha}}J_{-l}(\vec{q}\cdot\vec{\alpha})(\vec{p}_f|VG\vec{\alpha}\cdot\vec{p}GV|\vec{p}_i). \tag{2.21}$$

The last term in (2.21) can be rewritten with the aid of the identity[8]

$$\alpha\cdot(\vec{\nabla}_{p_i}+\vec{\nabla}_{p_f})[\vec{p}_f|T(E)|\vec{p}_i]$$
$$=\frac{1}{m}\left[p_f\left|T(E)\frac{\vec{\alpha}\cdot\vec{p}}{(E^+-p^2/2m)^2}T(E)\right|\vec{p}_i\right] \tag{2.22}$$
$$=\frac{1}{m}[\vec{p}_f|VG_0^{(+)}(E)\vec{\alpha}\cdot\vec{p}G_0^{(+)}(E)V|\vec{p}_i].$$

Then it can be combined with the two terms of (2.19) which contain the factor $J_{-l}(\vec{q}\cdot\vec{\alpha})$ to give

$$J_{-l}(\vec{q}\cdot\vec{\alpha})\tfrac{1}{2}\{[\vec{p}_f-\vec{\lambda}|T(\epsilon_i-\vec{\lambda}\cdot\vec{p}_i/m)|\vec{p}_i-\vec{\lambda}]$$
$$+[\vec{p}_f-\vec{\lambda}|T(\epsilon_f-\vec{\lambda}\cdot\vec{p}_f/m)|\vec{p}_i-\vec{\lambda}]\}, \tag{2.23}$$

where $\vec{\lambda}=\omega l m\vec{\alpha}/\vec{q}\cdot\vec{\alpha}$. In obtaining (2.23) from the two terms of (2.19) and the last term of (2.21) explicit use has been made of the fact that the $T$ matrices occurring in (2.23) are nonresonant and can therefore be expanded in powers $\vec{\lambda}$. These $T$ matrices are on shell (with error of order $\omega^2$) and are the simple generalization of the Kroll-Watson result arising from the averaging procedure adopted in (2.18).

The next term, $T^{(3)}$, has contributions from the situation where two of the Green's functions (the first and last) are simultaneously resonant and another in which only the middle $G$ is resonant. In each case the remaining $G$'s can be expanded in powers of $\omega$ and all but the lowest-order terms can be dropped. This results in

$$T_l^{(3)}(\vec{p}_f,\vec{p}_i)=\frac{\omega^2}{4}\sum_x J_{m_0-l}(\vec{p}_f\cdot\vec{\alpha})J_{m_0}(\vec{p}_i\cdot\vec{\alpha})[\vec{p}_f|VG_0^{(+)}(\epsilon_0)\vec{\alpha}\cdot\vec{p}G_0^{(+)}(\epsilon_i)\vec{\alpha}\cdot\vec{p}G_0^{(+)}(\epsilon_0)V|\vec{p}_i]$$
$$+\frac{\omega^2}{4}\sum_{x_1x_2}J_{m_0-l-x_2}(\vec{p}_f\cdot\vec{\alpha})J_{m_0+x_1}(\vec{p}\cdot\vec{\alpha})[\vec{p}_f|VG_0^{(+)}(\epsilon_f)\vec{\alpha}\cdot\vec{p}G_0^{(+)}(\epsilon_0)\vec{\alpha}\cdot\vec{p}G_0^{(+)}(\epsilon_i)V|\vec{p}_i]. \tag{2.24}$$

The resonant Green's function is now replaced by the first term of (2.16) and the summations over $x$ are then done. The result is

$$T_l^{(3)}(\vec{p}_f,\vec{p}_i)=\frac{1}{2}\,\frac{\omega^2}{\epsilon_0-E_R}\,J_{m_0-l}(\vec{p}_f\cdot\vec{\alpha})J_{m_0}(\vec{p}_i\cdot\vec{\alpha})\times\left((\vec{p}_f|V|U_R)\,\frac{(U_R|\vec{\alpha}\cdot\vec{p}G\vec{\alpha}\cdot\vec{p}|U_R)}{\epsilon_0-E_R}\,(U_R|V|\vec{p}_i)+\frac{2m_0(m_0-l)}{(\vec{p}_i\cdot\vec{\alpha})(\vec{p}_f\cdot\vec{\alpha})}\,(\vec{p}_f|VG\vec{\alpha}\cdot\vec{p}|U_R)(U_R|\vec{\alpha}\cdot\vec{p}GV|\vec{p}_i)\right), \tag{2.25}$$

where the distinction between the nonresonant Green's functions which appear here has been neglected since that would introduce terms of order $\omega^3$. As pointed out above, higher-order terms such as $T^{(4)}$ can be dropped.

Before we proceed it should be noted that, although the expansion has been in powers of $\omega/\epsilon_i$ with a retention of only the zeroth and first orders, $(\omega/\Gamma)$ has been treated as a quantity of order one. In fact for a $CO_2$ laser $\omega\sim 0.1$ eV, and a typical Feshach resonance width is of order $5\times10^{-3}$ eV, so that $\omega/\Gamma\sim 20$. It then could be that the higher-order terms in the expansion in $\omega/\epsilon_i$ can be compensated by powers of $\omega/\Gamma$. That this is not the case can be demonstrated in the following way. Let us investigate the leading terms in the resonant part of the expansion for $T^{(n)}$. For example, the odd terms $T^{(2j+1)}$ can be obtained in essentially the same way as was $T^{(3)}$, with the result

$$T_l^{(2j+1)}(\vec{p}_f,\vec{p}_i)=\frac{1}{2^{j-1}}\,\frac{\omega^{2j}}{\epsilon_0-E_R}\,J_{m_0-l}(\vec{p}_f\cdot\vec{\alpha})J_{m_0}(\vec{p}_i\cdot\vec{\alpha})\left((\vec{p}_f|V|U_R)\left(\frac{(U_R|\vec{\alpha}\cdot\vec{p}G\vec{\alpha}\cdot\vec{p}|U_R)}{\epsilon_0-E_R}\right)^j(U_R|V|\vec{p}_i)+\frac{2(m_0-l)m_0}{\vec{p}_i\cdot\vec{\alpha}\vec{p}_f\cdot\vec{\alpha}}\,(\vec{p}_f|VG\vec{\alpha}\cdot\vec{p}|U_R)\left(\frac{(U_R|\vec{\alpha}\cdot\vec{p}G\vec{\alpha}\cdot\vec{p}|U_R)}{\epsilon_0-E_R}\right)^{j-1}(U_R|\vec{\alpha}\cdot\vec{p}GV|\vec{p}_i)\right), \tag{2.26}$$

which sums from $j=1-\infty$ with the result

$$\sum_{j=1}^{\infty}T_l^{(2j+1)}(\vec{p}_+,\vec{p}_i)=\frac{1}{2}\,\frac{\omega^2J_{m_0-l}(\vec{p}_f\cdot\vec{\alpha})J_{m_0}(\vec{p}_i\cdot\vec{\alpha})}{\epsilon_0-E_R-\frac{1}{2}\omega^2(U_R|\vec{\alpha}\cdot\vec{p}G\vec{\alpha}\cdot\vec{p}U_R)}\times\left((\vec{p}_f|V|U_R)\,\frac{(U_R|\vec{\alpha}\cdot\vec{p}G\vec{\alpha}\cdot\vec{p}|U_R)}{\epsilon_0-E_R}\,(U_R|V|\vec{p}_i)+\frac{2(m_0-l)m_0}{\vec{p}\cdot\vec{\alpha}\,\vec{p}_i\cdot\vec{\alpha}}\,(\vec{p}_f|VG\vec{\alpha}\cdot\vec{p}|U_R)(U_R|\vec{\alpha}\cdot\vec{p}GV|\vec{p}_i\right). \tag{2.27}$$

From this it is clear that this expansion parameter is

$$(\omega^2/\Gamma)(U_R|\vec{\alpha}\cdot\vec{p}G\vec{\alpha}\cdot\vec{p}|U_R)\sim(\omega^2/\Gamma\epsilon_i)|\vec{\alpha}\cdot\vec{p}|^2,$$

which is small for the experiment in question,[11] but it is also clear that a more intense laser will make this effect significant. The effect is just the "dressing" of the resonance state by the laser, and the term

$$\tfrac{1}{2}\omega^2(U_R|\vec{\alpha}\cdot\vec{p}G\vec{\alpha}\cdot\vec{p}|U_R)$$

can be interpreted as the dynamic Stark shift of the complex energy of the resonance.

If we now assemble the result $T^{(0)}\cdots T^{(3)}$, we obtain

$$\begin{aligned}T_l(\vec{p}_f,\vec{p}_i)=(-1)^l\Bigg(&\tfrac{1}{2}J_l(\vec{q}\cdot\vec{\alpha})\left\{\left[\vec{p}_f-\vec{\lambda}\,\middle|\,T\left(\epsilon_i-\frac{\vec{\lambda}\cdot\vec{p}_i}{m}\right)\middle|\,\vec{p}_i-\vec{\lambda}\right]+\left[\vec{p}_i-\vec{\lambda}\,\middle|\,T\left(\epsilon_f-\frac{\vec{\lambda}\cdot\vec{p}_f}{m}\right)\middle|\,\vec{p}_i-\vec{\lambda}\right]\right\}\\&+J_{l-m_0}(\vec{p}_f\cdot\vec{\alpha})J_{m_0}(-\vec{p}_i\cdot\vec{\alpha})\Bigg\{[\vec{p}_f|T(\epsilon_0)|\vec{p}_i]-\tfrac{1}{2}[\vec{p}_f|T(\epsilon_i)+T(\epsilon_f)|\vec{p}_i]\\&+\frac{\omega(m_0-l)}{\vec{p}_f\cdot\vec{\alpha}}\,\frac{(\vec{p}_f|VG\vec{\alpha}\cdot\vec{p}|U_R)(U_R|V|\vec{p}_i)}{\epsilon_0-E_R}+\frac{\omega m_0}{\vec{p}_i\cdot\vec{\alpha}}\,\frac{(\vec{p}_f|V|U_R)(U_R|\vec{\alpha}\cdot\vec{p}GV|\vec{p}_i)}{\epsilon_0-E_R}\\&+\tfrac{1}{2}\omega m_0\left[\vec{p}_f\,\middle|\,\frac{\partial}{\partial\epsilon_i}T(\epsilon_i)+\frac{\partial}{\partial\epsilon_f}T(\epsilon_f)\middle|\,\vec{p}_i\right]+\tfrac{1}{2}\omega^2(\vec{p}_f|V|U_R)\,\frac{(U_R|\vec{\alpha}\cdot\vec{p}G\vec{\alpha}\cdot\vec{p}|U_R)}{(\epsilon_0-E_R)^2}\,(U_R|V|\vec{p}_i)\\&+\omega^2\,\frac{m_0(m_0-l)}{(\vec{p}_i\cdot\vec{\alpha})(\vec{p}_f\cdot\vec{\alpha})}\,\frac{(\vec{p}_f|VG\vec{\alpha}\cdot\vec{p}|U_R)(U_R|\vec{\alpha}\cdot\vec{p}GV|\vec{p}_i)}{\epsilon_0-E_R}\Bigg\}\Bigg).\end{aligned} \tag{2.28}$$

The first term $[\sim J_l(\vec{q}\cdot\vec{\alpha})]$ is essentially the Kroll-Watson result discussed above. The first term in the second expression in curly brackets is the resonant contribution calculated by Druger and Jung; the next is a nonresonant contribution which tends to cancel the first as we move away from resonance. The next two are of order $\omega\Gamma^{1/2}/\epsilon_0 - E_R$ and the remaining are of order $\omega$, $\omega^2\Gamma/(\epsilon_0 - E_R)^2$, and $\omega^2/(\epsilon_0 - E_R)$. [It should be pointed out here that in the absence of resonance, i.e., as $\Gamma\to\infty$, the result (2.28) returns to its usual form.[6]]

The cross section is proportional to the absolute square of the $T$ matrix (2.28), which evidently contains interference between the resonant and nonresonant terms. In the absence of the laser this interference is also present:

$$\frac{d\sigma(\vec{p}_f,\vec{p}_i)}{d\Omega}\bigg|_{\alpha=0} = \left(\frac{m}{2\pi}\right)|[\vec{p}_f\,|\,T_{NR}+T_R(\epsilon_0)|\,\vec{p}_i\,]|^2, \quad (2.29)$$

where we have used

$$T(\epsilon_0) = T_{NR} + T_R(\epsilon_0)\ . \quad (2.30)$$

If we retain only the terms of zero order in $\omega$ in (2.28) but continue to distinguish between resonant and nonresonant terms, then the sum rule (1.6) can be written

$$\sum_l \frac{d\sigma_l(\vec{p}_f,\vec{p}_i)}{d\Omega} = \left(\frac{m}{2\pi}\right)^2 \{|(\vec{p}_f|\,T_{NR}|\,\vec{p}_i)|^2 + J^2_{m_0}(\vec{p}_i\cdot\vec{\alpha})|[\vec{p}_f\,|\,T_R(\epsilon_0)|\,\vec{p}_i\,]|^2 + 2J^2_{m_0}(\vec{p}_i\cdot\alpha)\,\mathrm{Re}[\vec{p}_f\,|\,T_{NR}|\,\vec{p}_i][\vec{p}_f\,|\,T_R(\epsilon_0)|\,\vec{p}_i\,]\}, \quad (2.31)$$

which, with the aid of (2.29), is

$$\sum_l \frac{d\sigma_l(\vec{p}_f,\vec{p}_i)}{d\Omega} = [1 - J^2_{m_0}(\vec{p}_i\cdot\vec{\alpha})]\frac{d\sigma(\vec{p}_f,\vec{p}_i)}{d\Omega}\bigg|_{NR} + J^2_{m_0}(\vec{p}_i\cdot\vec{\alpha})\frac{d\sigma(\vec{p}_f,\vec{p}_i)}{d\Omega}\bigg|_{\alpha=0}, \quad (2.32)$$

where

$$\frac{d\sigma(\vec{p}_f,\vec{p}_i)}{d\Omega}\bigg|_{NR} = \left(\frac{m}{2\pi}\right)^2|(\vec{p}_f|T_{NR}|\,\vec{p}_i)|^2\ . \quad (2.33)$$

This result already provides a new tool since in the absence of the laser only (2.29) is measurable, whereas the laser, through the parameter $\vec{p}_i\cdot\alpha$, allows for an additional measurement of the resonant part of the cross section.

The summation over $l$ in (2.31) extends over all integers and in particular includes $l=m_0$, which was excluded before the start of calculation (2.17). However in lowest order in $\omega$, (2.28) applies just so long as the distinction between resonant and nonresonant energies is preserved. So (2.32) is correct. A procedure similar to the one used above yields, for $l=m_0\neq 0$,

$$\begin{aligned} T_{m_0}(\vec{p}_f,\vec{p}_i) = (-1)^{m_0}\Bigg(& J_{m_0}(\vec{q}\cdot\vec{\alpha})\left[\vec{p}_f-\vec{\lambda}\,\middle|\,T\left(\epsilon_i-\frac{\vec{\lambda}\cdot\vec{p}_i}{m}\right)\middle|\,\vec{p}_i-\vec{\lambda}\right] + J_0(\vec{p}_f\cdot\alpha)J_{m_0}(-\vec{p}_i\cdot\vec{\alpha}) \\ &\times\Bigg\{[\vec{p}_f\,|\,T(\epsilon_0)|\vec{p}_i\,] - [\vec{p}_f\,|\,T(\epsilon_i)|\,\vec{p}_i\,] + \frac{w\,m_0}{\vec{p}_i\cdot\vec{\alpha}}\,\frac{(\vec{p}_f\,|\,V|\,U_R)(U_R|\,\vec{\alpha}\cdot\vec{p}GV|\,\vec{p}_i)}{\epsilon_0-E_R} \\ &- m_0\omega\left(\vec{p}_f\,\middle|\,\frac{\partial T(\epsilon_i)}{\partial\epsilon_i}\middle|\,\vec{p}_i\right) + \tfrac{1}{2}\omega^2(\vec{p}_f|V|\,U_R)\frac{(U_R|\,\vec{\alpha}\cdot\vec{p}G\vec{\alpha}\cdot\vec{p}|U_R)}{(\epsilon_0-E_R)^2}(U_R|V|\vec{p}_i)\Bigg\}\Bigg). \end{aligned} \quad (2.34)$$

The other values of $l$ and $m_0$ can be obtained similarly but will not be discussed here.

If the next order, $\omega\Gamma^{1/2}/\epsilon_0 - E_R$, is retained in the $T$ matrix then an additional term is added to the sum rule, the right-hand side of (2.32). It is

$$\begin{aligned} 2J^2_{m_0}(\vec{p}\cdot\vec{\alpha})\frac{\omega m_0}{\vec{p}_i\cdot\vec{\alpha}}\,\mathrm{Re}\Bigg(& \frac{(\vec{p}_f\,|\,T_{NR}|\vec{p}_i)^*}{\epsilon_0-E_R}\,[(\vec{p}_f|V|\,U_R)(U_R|\,\vec{\alpha}\cdot\vec{p}GV|\,\vec{p}_i) + (\vec{p}_f\,|VG\vec{\alpha}\cdot\vec{p}|\,U_R)(U_R|V|\,\vec{p}_i)] \\ &+ \frac{[\vec{p}_f\,|\,T_R(\epsilon_0)|\vec{p}_i\,]^*}{\epsilon_0-E_R}\,(\vec{p}_f|V|\,U_R)(U_R|\,\vec{\alpha}\cdot\vec{p}GV|\vec{p}_i)\Bigg), \end{aligned} \quad (2.35)$$

which is the lowest-order off-shell information obtainable via the sum rule. More detailed information can be obtained by investigating the individual $l$ values.

Before attempting a comparison with experiment it is necessary that one extend the above calculations to include the fact that the target is an atom not a potential and to average over the energy distribution of the incident beam. In the usual case in which the beam energy width is much larger than the resonance width, the resonance structure will then be averaged away and the additional information contained in (2.35) will be much more difficult to obtain. This will be discussed in a subsequent publication.


## ACKNOWLEDGMENT

This research was supported by the U. S. Office of Naval Research through Contract No. 0014-76C-0014.



[1]See for example H. Feshbach and D. Yennie, Nucl. Phys. 37, 150 (1962).

[2]P. Langendam, M. Gavrila, H. Kaandorp, and M. Van der Wiel, J. Phys. B 9, L453 (1976).

[3]See, for example, M. H. Mittleman, Phys. Rev. A 18, 685 (1978).

[4]I. V. Hertel and W. Stoll, J. Phys. B 7, 570 (1974); 7, 583 (1974); N. D. Bhaskar, B. Jaduszliwer, and B. Bederson, Phys. Rev. Lett. 38, 14 (1977).

[5]M. H. Mittleman, J. Phys. B 12, 1781 (1979).

[6]N. M. Kroll and K. M. Watson, Phys. Rev. A 8, 804 (1973).

[7]M. H. Mittleman, (unpublished).

[8]M. H. Mittleman, Phys. Rev. A 19, 134 (1979).

[9]H. Krüger and C. Jung, Phys. Rev. A 17, 1706 (1978).

[10]Units of $\hbar = 1$ are used.

[11]A. Weingartshofer, J. K. Holmes, G. Caudle, E. M. Clarke, and H. Krüger, Phys. Rev. Lett. 39, 269 (1977).

[12]See, for example, R. G. Newton, *Scattering of Waves and Particles* (McGraw-Hill, New York, 1966), and R. M. More and E. Gerjuoy, Phys. Rev. A 7, 1288 (1973).