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Excitation of the hydrogen atom in a modified Glauber theory

T. T. Gien

Department of Physics, Memorial University of Newfoundland, St. John's, Newfoundland A1C 5S7, Canada

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A modified Glauber method proposed recently for electron-atom scattering is extended to the analysis of the 1-2 excitation of atomic hydrogen by electron impact. Exchange effects are not neglected, but included in the calculation through the exact eikonal exchange formulas. The results are compared with experimental data acquired by absolute measurements. Good agreement with experimental data is found. The differential cross sections calculated with the conventional Glauber amplitude, but without neglecting the exact eikonal exchange effects, are also obtained and shown for comparison.

I. INTRODUCTION

In the recent papers,[1,2] a modified Glauber amplitude has been proposed for the analysis of electron-atom scatterings and considered for various elastic scattering processes with some degree of success. By a careful review of the derivation of the Glauber amplitude,[1] I found that the so-called straight-line approximation affects the second-order eikonal term of the Glauber amplitude most seriously. It is the consideration of this approximation, coupled with the assumption of a zero value for all the excitation energies of the target intermediate states, which makes the second-order term of the elastic electron-atom scattering amplitude become divergent in the forward direction on the one hand and its real part (which is of a significant magnitude) disappear altogether from the Glauber amplitude on the other hand. As was well known, the Glauber method is essentially a small-angle approximation.[3,4] The approximations considered in the method should, thereby, produce a scattering amplitude which is more successful at small than at large scattering angles. It is not, therefore, at all surprising to find that the differential cross sections calculated with the conventional Glauber amplitude for both elastic and inelastic processes[2,5] are much smaller than the experimental ones[6,7] at larger scattering angles. At intermediate and large scattering angles, not all the eikonal terms may adequately represent the corresponding Born terms anymore. Thus the avoidance of an automatic consideration of eikonal approximation for all Born terms may be a correct answer to these problems. The divergence in the forward direction of the second-order scattering term due to eikonalization is a very serious deficiency for the Glauber amplitude, as this would mean a violation of the conservation of probability which is a main principle of quantum theory. Thus the eikonal approximation should not be considered for this term at all. A reasonably good correction for the deficiency of the Glauber amplitude could, therefore, be achieved by considering an eikonal approximation restricted to scattering orders from the third up only (instead of the second as in the conventional Glauber method). Note that the arguments usually used to justify the eikonal approximation in the conventional Glauber theory can again be used here for higher orders of scattering. Within this spirit, a modified Glauber amplitude has been proposed as follows for the analysis of electron-atom scattering at intermediate energies:

$$F_{\mathrm{GM}} = f_{B1} + f_{B2} + F'_G\,, \tag{1a}$$

where F'_G is the eikonal approximation of higher-order scattering amplitude, and f_{B1} and f_{B2} are the first and second Born amplitudes, respectively. From a physical point of view, one may argue that another reason for the limited consideration of eikonal approximation is that the value of momentum k at these intermediate energies has not been high enough to justify an eikonal approximation for all orders of scattering. Since f_{B1} is identical to the first-order eikonal term of the conventional Glauber amplitude, in practice, the following equivalent expression of F_{GM} is used for evaluation instead:

$$F_{\mathrm{GM}} = F_G - F_{G2} + f_{B2} \tag{1b}$$

Thus, within the modified Glauber method, the second-order eikonal term in F_G has been corrected with its counterpart prior to eikonalization, i.e., the second Born term. The consideration of the second form of F_{GM} in an analysis of scattering is to be able to make use of the various methods of calculation of F_G and F_{G2} already available in the literature.

With this simple modification of the Glauber amplitude, the theoretical cross sections were found to agree quite well with experimental data when the amplitude is considered for the analysis of elastic scatterings of electrons by hydrogen and helium atoms.[1,2] These results indicate that the sole rectification of the second-order eikonal term is indeed sufficient to remove the discrepancy between the theoretical results calculated with the conventional Glauber amplitude and experimental data at intermediate and large scattering angles. Thus the conventional Glauber method first introduced by Franco[8] and his coworkers[9] to the domain of atomic and molecular collision may be retained as a good approximation for electron-atom scatterings at intermediate and high energies provided that the eikonalization of the second-order scattering term is not considered in order to avoid some very serious deficiencies arising from it.

At first sight, the modified Glauber method seems to be closely related to a method proposed by Byron and Joachain[10] for electron-atom scatterings [eikonal-Born series (EBS) method]. However, a closer look reveals immediately that this is actually not so.[11] While the two methods may only formally resemble each other in the similar treatment of the second Born term as usually considered in the second Born scattering theory, they differ from each other in all aspects, including their numerical values, the procedure of evaluation, and especially the basis for the consideration of the approximation. In fact, the theoretical basis for the consideration of the modified Glauber approximation is completely different from that of the EBS. While, as was stressed by authors of the EBS method, the purpose of their consideration of the EBS amplitude in electron-atom scattering is to provide a consistent leading order correction ($1/k^2$) to the second Born method, the modified Glauber approximation was designed (as was discussed in some details above and elsewhere[1,2]) to correct a serious defect of the conventional Glauber amplitude which I still regard as an acceptable approximation for electron-atom scatterings at intermediate energies, provided that its serious deficiencies are singled out and adequately remedied. Furthermore, this serious defect may also be related to the consideration of the so-called straight-line approximation which is required in the derivation of the conventional Glauber amplitude. Thus the exclusion of f_{B2} from eikonalization in the modified Glauber method is, to some extent, equivalent to the relaxation of a strict consideration of the straight-line approximation in the Glauber method. In this connection, one may now understand why there seems to exist a concordance in the results of the Glauber-angle method[12] and those of other methods (such as modified Glauber, EBS) in which the second Born term rather than f_{G2} is chosen to be evaluated. Even on the numerical value side, the modified Glauber direct amplitude differs very much from the eikonal Born series amplitude by the presence of an *infinite* number of higher-order eikonal terms implicitly contained in the Glauber amplitude. These higher-order terms are usually found to be of a significant magnitude. For instance, in the case of elastic e-H scattering, the numerical value of f_{G4} is found to be of the order of $\frac{1}{3}$ of f_{G1} at small scattering angles and double of $\mathrm{Im} f_{B2}$ at large scattering angles. The abrupt cutoff of these terms from the Glauber amplitude at fourth order would seem quite arbitrary and unjustifiable. Note that even if each of these higher-order eikonal terms is of small magnitude, their effect, as a whole through F'_G, on the scattering amplitude might not be negligible either, since the number of these terms is infinite. In fact, a comparable situation worth mentioning here is the case of the Born-Oppenheimer and Ochkur exchange amplitudes: while these amplitudes also differ from each other by an infinite number of terms presumably of small magnitude, the numerical values of the two amplitudes are found actually drastically different from each other. Furthermore, since amplitudes such as $\mathrm{Re} f_{B2}$, $\mathrm{Im} f_{B2}$, and f_{G3} also contain, beside their leading order term, terms of order higher than $1/k^2$, the sole neglect of the higher-order eikonal terms on that basis (while retaining those of the similar nature in $\mathrm{Re} f_{B2}$, $\mathrm{Im} f_{B2}$, and f_{G3}) would seriously jeopardize the consistency in the procedure of making an approximation. Finally, the two methods of approximation (modified Glauber and EBS) also differ in the calculation of exchange effects. While in the modified Glauber method, the exchange effect is calculated with the exact eikonal exchange amplitude,[13,14] in the eikonal-Born series method, because of the philosophy of the model, the exchange effect is calculated with the approximate Ochkur form[15] instead. Regarding this, it should be stressed that the values of these two exchange amplitudes are also significantly different from each other. It is also worth mentioning that these higher-order eikonal terms imply a much drastic difference in cross section between the two methods of approximation in the case of positron scattering[2] (not discussed here, also see my paper presented at XI ICPEAC, Kyoto, Japan). Encouraged by the good agreement with experimental data in the case of elastic scattering,[2] in this paper, I extend the consideration of the modified Glauber amplitude to the study of the 1-2 excitation of a hydrogen atom by electron impact to find out whether the modified Glauber amplitude still remains a good approximation for the inelastic case.

Recently,[7] experimental data for the 1-2 excitation of hydrogen by electron impact have been made available in the literature by absolute measurement. The comparison of these sets of data with results calculated with the conventional Glauber amplitude[9] shows that the theoretical values are much smaller at large scattering angles. In all these calculations, the Glauber exchange effect was, however, neglected. The main reason for the neglect of eikonal exchange effect is that then one did not know exactly how to reduce the order of the integrals of the Glauber exchange amplitudes to a lesser, while the so-called Glauber-Ochkur exchange amplitude available then was shown to be unacceptable to represent correctly the exchange effect in an eikonal theory.[16] Through an ingenious application of a method similar to the one initiated by Gau and Macek[17] for the direct eikonal scattering amplitude, Madan[13] and then Foster and Williamson[14] have succeeded in reducing the eikonal exchange amplitude to a two-dimensional integral which can be computed with much less difficulty. Although in principle, the Glauber exchange amplitude for e^{-}-H scatterings can now be evaluated, no work on the excitation of hydrogen atom by electron impact done within the conventional Glauber method and with the inclusion of exchange effect has been reported in the literature. The lack of the Glauber calculations with exchange is perhaps due to the fact that although the order of the integral involved in the eikonal exchange amplitudes has been reduced to two, the derivations of those amplitudes which can be ready for numerical evaluation in the inelastic e^{-}-H scatterings are still not quite simple. For these processes, in general, one needs to take the derivative of the integrand three times and carry out an integration by parts once. As a result, a number of terms which constitute the amplitude increases drastically after each derivative or integration by parts is taken and the final form of the amplitude to be evaluated becomes quite complex. Obviously, one is tempted to find out whether the inclusion of the exact eikonal exchange effect improves the Glauber results. In this paper, I shall, therefore, also report the results of an analysis of the 1-2 excitation of hydrogen atom by electron impact with the conventional Glauber method in which the Glauber exchange effects are no longer neglected. To my knowledge, no such work has been carried out in the literature before. It will be seen that although at lower scattering energies, the new theoretical results with exchange improve somewhat in so far as to compare with experimental data at small and intermediate scattering angles, these results are still far from being in good agreement with experimental data. Thus, to improve the agreement, the consideration of the Glauber approximation modified as described above must definitely be explored.

The 1-2 excitation of a hydrogen atom is composed of four processes which are $1s$-$2s$, $1s$-$2p\pm$, and $1s$-$2p,0$, where $\pm$ and 0 denote the bound states of a hydrogen atom which have as eigenvalues of l_z, ± 1, and 0, respectively. In Sec. II, I shall first analyze the four processes $1s$-$2s$, $1s$-$2p\pm$ and $1s$-$2p,0$ separately. The results will then be used to derive the differential cross sections of the 1-2 excitation. I shall make a brief review on the explicit forms of various scattering amplitudes which are needed in my subsequent calculations. I shall also describe briefly how these forms are reduced to the simple final expressions which can be ready for numerical computations. The procedure of calculating approximately the second Born term will also be discussed in some details. A brief outline of the basic equations for the Glauber exchange amplitude is given as well. The results of the analysis will be presented in Sec. III together with discussions. Comparisons with the results obtained in some other theoretical models (recalculated by me) and with experimental data will also be made. Some main conclusions drawn from this work are summarized in Sec. IV.

II. FORMALISM

The 1-2 excitation of a hydrogen atom by electron impact is composed of four different processes, namely, $1s$-$2s$, $1s$-$2p\pm$, and $1s$-$2p,0$. In the following, I shall successively analyze these processes with the modified Glauber amplitude.

A. $1s$-$2s$ excitation

The simplified form of the Glauber amplitude for the process $1s$-$2s$ is well known in the literature.[9,18] One may use the integral form[9] or the closed form by Gerjoy and Thomas[18] to evaluate the amplitude. The z direction of this amplitude has been chosen, as usual, perpendicular to the momentum transfer direction. The second-order eikonal amplitude F_{G2} for this process can also be evaluated with ease by using its closed form.[19] The second Born scattering amplitude is given by[20]

$$\bar{f}_{B2}=\frac{2}{\pi^2}\int d^3\vec{k}\,\frac{1}{K_\mu^2 K_\nu^2}\,\frac{1}{k^2-p_\mu^2-i\epsilon}\times\langle\nu|e^{i\vec{q}\cdot\vec{r}}-e^{i\vec{K}_\mu\cdot\vec{r}}-e^{-i\vec{K}_\nu\cdot\vec{r}}+1|\mu\rangle\,, \qquad (2)$$

where $p_\mu = k_\mu - 2\bar{\omega}$. The procedure of calculating the second Born amplitude has been described in details in the literature.[20] By integrating over $\vec{r}$

and then using the Feynman's technique,[21] one can transform the second Born amplitude into an one-dimensional integral which can be evaluated with ease. Pertinent expressions which are useful in the computation of $\bar{f}_{B2}$ may also be found, for instance, in the paper by Byron and Latour.[19] In order to evaluate $\bar{f}_{B2}$ for the $1s$-$2s$ process, one also needs to calculate the first- and second-order derivatives of $I_\mu(a^2)$ and $I_\nu(a^2)$ where $I_\mu(a^2)$ and $I_\nu(a^2)$ are given by

$$I_\mu(a^2)=\int_0^1 \frac{dx}{[a^2x+x(1-x)q^2]^{1/2}\{(a^2+2\omega_{\mu\nu})x+2(\bar{\omega}-\omega_{\mu\nu})-2ip_\mu[a^2x+x(1-x)q^2]\}}\,. \tag{3a}$$

$$I_\nu(a^2)=\int_0^1 \frac{dx}{[a^2x+x(1-x)q^2]^{1/2}\{(a^2-2\omega_{\mu\nu})x+2\bar{\omega}-2ip_\mu[a^2x+x(1-x)q^2]^{1/2}\}}\,. \tag{3b}$$

This can either be done numerically or more conveniently by using directly the following explicit forms of the derivatives,

$$\frac{dI_\mu}{da^2}=\int_0^1 \frac{1}{\Lambda v}\left(\frac{ip_\mu}{\Lambda v}-\frac{1}{2\Lambda^2\sqrt{x}}-\frac{\sqrt{x}}{v}\right)dx \tag{4a}$$

and

$$\begin{aligned}\frac{d^2I_\mu}{d(a^2)^2}=\int_0^1 \frac{1}{\Lambda v}\bigg[&\left(\frac{ip_\mu\sqrt{x}}{\Lambda v}-\frac{1}{2\Lambda^2}-\frac{x}{v}\right)\\ &\times\left(\frac{ip_\mu}{\Lambda v}-\frac{\sqrt{x}}{v}-\frac{1}{2\Lambda^2\sqrt{x}}\right)\\ &+\frac{ip_\mu}{\Lambda v}\left(\frac{ip_\mu\sqrt{x}}{\Lambda v}-\frac{1}{2\Lambda^2}-\frac{x}{v}\right)\\ &+\frac{1}{2\Lambda^4\sqrt{x}}+\frac{x}{v^2}\left(\sqrt{x}-\frac{ip_\mu}{\Lambda}\right)\bigg]dx\,,\end{aligned} \tag{4b}$$

where

$$\Lambda=[a^2+(1-x)q^2]^{1/2} \tag{5a}$$

and

$$\begin{aligned}v=&(a^2+2\omega_{\mu\nu})x+2(\bar{\omega}-\omega_{\mu\nu})\\ &-2ip_\mu[a^2x+x(1-x)q^2]^{1/2}\,.\end{aligned} \tag{5b}$$

$\omega_{\mu\nu}$ is the energy difference between the ground state and the second level excited state of a hydrogen atom, i.e., $\omega_{\mu\nu}=\frac{3}{8}$ a.u. The expressions for dI_ν/da^2 and $d^2I_\nu/d(a^2)^2$ can be obtained from those of dI_μ/da^2 and $d^2I_\mu/d(a^2)^2$ by simply substituting $\omega_{\mu\nu}$ with $-\omega_{\mu\nu}$ and $\bar{\omega}-\omega_{\mu\nu}$ with $\bar{\omega}$.

B. 1s-2p excitation

The $1s$-$2p$ excitation of hydrogen atom is composed of three processes: $1s$-$2p\pm$ and $1s$-$2p,0$, where $\pm$ and 0 indicate the eigenvalues ±1 and 0 of the z component of the orbital angular momentum of the bound electron. I shall deal with the $1s$-$2p\pm$ processes first.

As is well known, the Glauber amplitude for the $1s$-$2p\pm$ process can be calculated easily by using either its closed form or its one-dimensional integral expression.[9,18] Again, the z direction of the system of axes is consistently chosen to be perpendicular to the momentum transfer. The second-order eikonal term for this process can also be calculated with its closed form.[19] It should be noted that if the phase factor $e^{\mp i\phi_q}$ is ignored for all the scattering amplitudes involved in the calculation, F_{G2} for the two processes $1s$-$2p\pm$ will be a real quantity and equal to each other. As for the second Born term of the $1s$-$2p\pm$ processes, similar arguments will lead to the same formula as given in Eq. (2) above in the average closure summation approximation, with the exception that $|\nu\rangle$ is now the $2p\pm$ bound states of the hydrogen atom instead of the $2s$. After the explicit expressions of the $1s$ and $2p\pm$ states have been substituted into Eq. (2), the integration over the coordinate variables in the matrix element part can be done analytically and one obtains

$$\begin{aligned}\bar{f}_{B2}(1s\text{-}2p\pm)=\frac{12i}{\pi^2}\int d^3\vec{k}\,\frac{1}{k^2-p_\mu^2-i\epsilon}\,\frac{1}{K_\mu^2K_\nu^2}\\ \times\left[\frac{q_x\mp iq_y}{(q^2+a^2)^3}-\frac{K_{\mu x}\mp iK_{\mu y}}{(K_\mu^2+a^2)^3}\right.\\ \left.+\frac{K_{\nu x}\mp iK_{\nu y}}{(K_\nu^2+a^2)^3}\right].\end{aligned} \tag{6}$$

Furthermore, one may put $\bar{f}_{B2}$ in a nicer form as follows

$$\begin{aligned}\bar{f}_{B2}(1s\text{-}2p\pm)=6i\,\frac{d^2}{d(a^2)^2}\bigg[&\frac{1}{a^2}(J_{\mu x}\mp iJ_{\mu y})-\frac{1}{a^2}(J_{\nu x}\mp iJ_{\nu y})\\ &-\frac{q^2}{a^2(q^2+a^2)}(q_x\mp iq_y)\bigg],\end{aligned} \tag{7}$$

where $J_{\mu x}$, $J_{\mu y}$, $J_{\nu x}$, and $J_{\nu y}$ are components of the vectors $\vec{J}_\mu$ and $\vec{J}_\nu$ on the x and y axes, respectively. $\vec{J}_\mu$ and $\vec{J}_\nu$ are given by

$$\vec{J}_\mu=\frac{1}{\pi^2}\int d^3\vec{k}\,\frac{1}{k^2-p_\mu^2-i\epsilon}\,\frac{1}{K_\nu^2}\,\frac{\vec{K}_\mu}{K_\mu^2+a^2} \tag{8}$$

and

$$\vec{J}_\nu = \frac{1}{\pi^2}\int d^3\vec{k}\,\frac{1}{k^2-p_\mu^2-i\epsilon}\,\frac{1}{K_\mu^2}\,\frac{\vec{K}_\nu}{K_\nu^2+a^2}\,. \tag{9}$$

Finally, the well-known Feynman's technique can again be used to transform the vectors $\vec{J}_\mu$ and $\vec{J}_\nu$ into an expression of one-dimensional integral

$$\vec{J}_\mu = A_1\vec{q}+B_1\vec{k}_\nu \tag{10}$$

and

$$\vec{J}_\nu = C_1\vec{q}+D_1\vec{k}_\mu\,, \tag{11}$$

where A_1 and B_1 are, respectively, given by

$$\begin{aligned} A_1 = I_\mu(a^2) &+ \int_0^1 \frac{x\,dx}{\Lambda(p_\mu-P+i\Lambda)(p_\mu+P+i\Lambda)} \\ &- \frac{i}{2}\int_0^1 \frac{x\,dx}{P^3}\ln\left(\frac{p_\mu-P+i\Lambda}{p_\mu+P+i\Lambda}\right) \\ &- \frac{i}{2}\int_0^1 \frac{x\,dx}{P^2}\left(\frac{1}{p_\mu-P+i\Lambda}+\frac{1}{p_\mu+P+i\Lambda}\right). \end{aligned} \tag{12}$$

and

$$\begin{aligned} B_1 = &-\frac{i}{2}\int_0^1 dx\,\frac{1}{P^3}\ln\left(\frac{p_\mu-P+i\Lambda}{p_\mu+P+i\Lambda}\right) \\ &-\frac{i}{2}\int_0^1 \frac{dx}{P^2}\left(\frac{1}{p_\mu-P+i\Lambda}+\frac{1}{p_\mu+P+i\Lambda}\right) \end{aligned} \tag{13}$$

in which

$$P = (k_\nu^2 + x^2q^2 + 2x\vec{k}_\nu\cdot\vec{q})^{1/2} \tag{14}$$

and

$$\Lambda = \{[a^2+q^2(1-x)]x\}^{1/2}\,. \tag{15}$$

To obtain the expressions for C_1 and D_1, one needs only to interchange $\vec{k}_\mu$ with $\vec{k}_\nu$ everywhere, and $I_\mu(a^2)$ with $I_\nu(a^2)$. In order to calculate $\bar{f}_{B2}$, one may also need to know the explicit expressions of $d\vec{J}_\mu/da^2$, $d\vec{J}_\nu/da^2$, $d^2\vec{J}_\mu/d(a^2)^2$, and $d^2\vec{J}_\nu/d(a^2)^2$. They are given by

$$\frac{d\vec{J}_\mu(a^2)}{da^2} = A_2\vec{q}+B_2\vec{k}_\nu\,, \tag{16}$$

$$\frac{d\vec{J}_\nu(a^2)}{da^2} = -C_2\vec{q}+D_2\vec{k}_\mu\,, \tag{17}$$

$$\frac{d^2\vec{J}_\mu(a^2)}{d(a^2)^2} = A_3\vec{q}+B_3\vec{k}_\nu\,, \tag{18}$$

and

$$\frac{d^2\vec{J}_\nu(a^2)}{d(a^2)^2} = -C_3\vec{q}+D_3\vec{k}_\mu\,, \tag{19}$$

where

$$\begin{aligned} A_2 = \frac{dI_\mu(a^2)}{da^2} &- \int_0^1 dx\,\frac{x^2}{2\Lambda^2}\,\frac{1}{(p_\mu-P+i\Lambda)(p_\mu+P+i\Lambda)}\left(\frac{1}{\Lambda}+\frac{i}{p_\mu-P+i\Lambda}+\frac{i}{p_\mu+P+i\Lambda}\right) \\ &+\frac{1}{4}\int_0^1 dx\,\frac{x^2}{P^3\Lambda}\left(\frac{1}{p_\mu-P+i\Lambda}-\frac{1}{p_\mu+P+i\Lambda}\right) \\ &-\frac{1}{4}\int_0^1 dx\,\frac{x^2}{P^2\Lambda}\left(\frac{1}{(p_\mu-P+i\Lambda)^2}+\frac{1}{(p_\mu+P+i\Lambda)^2}\right). \end{aligned} \tag{20}$$

$$B_2 = -\frac{1}{4}\int_0^1 dx\,\frac{x}{P^2\Lambda}\left(\frac{1}{(p_\mu-P+i\Lambda)^2}+\frac{1}{(p_\mu+P+i\Lambda)^2}\right)+\frac{1}{4}\int_0^1 dx\,\frac{x}{P^3\Lambda}\left(\frac{1}{p_\mu-P+i\Lambda}-\frac{1}{p_\mu+P+i\Lambda}\right) \tag{21}$$

$$\begin{aligned} A_3 = \frac{d^2I_\mu(a^2)}{d(a^2)^2} &+\frac{1}{4}\int_0^1 dx\,\frac{x^3}{\Lambda^3}\,\frac{1}{(p_\mu-P+i\Lambda)(p_\mu+P+i\Lambda)}\left[\frac{1}{\Lambda}+i\left(\frac{1}{p_\mu-P+i\Lambda}+\frac{1}{p_\mu+P+i\Lambda}\right)\right]^2 \\ &+\frac{1}{4}\int_0^1 dx\,\frac{x}{\Lambda(p_\mu-P+i\Lambda)(p_\mu+P+i\Lambda)}\left[\frac{2x^2}{\Lambda^4}+i\frac{x}{\Lambda(p_\mu-P+i\Lambda)}\left(\frac{x}{\Lambda^2}+i\frac{x}{\Lambda(p_\mu-P+i\Lambda)}\right)\right. \\ &\left.+i\frac{x}{\Lambda(p_\mu+P+i\Lambda)}\left(\frac{x}{\Lambda^2}+i\frac{x}{\Lambda(p_\mu+P+i\Lambda)}\right)\right] \\ &+\frac{1}{8}\int_0^1 dx\,\frac{x^3}{P^3\Lambda^2}\left[-\frac{1}{\Lambda}\left(\frac{1}{p_\mu-P+i\Lambda}-\frac{1}{p_\mu+P+i\Lambda}\right)+i\left(-\frac{1}{(p_\mu-P+i\Lambda)^2}+\frac{1}{(p_\mu+P+i\Lambda)^2}\right)\right] \\ &+\frac{1}{8}\int_0^1 dx\,\frac{x^3}{P^2\Lambda^2}\left[\frac{1}{\Lambda}\left(\frac{1}{(p_\mu-P+i\Lambda)^2}+\frac{1}{(p_\mu+P+i\Lambda)^2}\right)+2i\left(\frac{1}{(p_\mu-P+i\Lambda)^3}+\frac{1}{(p_\mu+P+i\Lambda)^3}\right)\right] \end{aligned} \tag{22}$$

and

$$\begin{aligned} B_3 = \frac{1}{8}\int_0^1 dx\,\frac{x^2}{P^2\Lambda^2}&\left[\frac{1}{\Lambda}\left(\frac{1}{(p_\mu-P+i\Lambda)^2}+\frac{1}{(p_\mu+P+i\Lambda)^2}\right)+2i\left(\frac{1}{(p_\mu-P+i\Lambda)^3}+\frac{1}{(p_\mu+P+i\Lambda)^3}\right)\right] \\ +\frac{1}{8}\int_0^1 dx\,\frac{x^2}{P^3\Lambda^2}&\left[\frac{1}{\Lambda}\left(\frac{1}{p_\mu+P+i\Lambda}-\frac{1}{p_\mu-P+i\Lambda}\right)+i\left(\frac{1}{(p_\mu+P+i\Lambda)^2}-\frac{1}{(p_\mu-P+i\Lambda)^2}\right)\right] \end{aligned} \tag{23}$$

P and Λ are given by Eqs. (14) and (15) above. To obtain the expressions for C_2, D_2, C_3, and D_3 again, one needs only to interchange $\vec{k}_\mu$ with $\vec{k}_\nu$ and $I_\mu(a^2)$ with $I_\nu(a^2)$ in the expressions for A_2, B_2, A_3, and B_3. The combined use of the appropriate expressions above in $\bar{f}_{B2}$ [Eq. (7)] gives the expression for f_{B2} which can be evaluated easily,

$$\bar{f}_{B2}(1s-2p\pm)=6ie^{\mp i\phi_q}(m_1q_\perp+m_2k_{\nu\perp}+m_3k_{\mu\perp}), \qquad (24)$$

where m_1, m_2, and m_3 are given by

$$m_1=2(A_1+C_1)/a^6-2(A_2+C_2)/a^4+(A_3+C_3)/a^2 - [2q^2/a^6(q^2+a^2)^3](q^2+3a^2q^2+3a^2)I, \qquad (25)$$

$$m_2=2B_1/a^6-2B_2/a^4+B_3/a^2, \qquad (26)$$

and

$$m_3=-2D_1/a^6+2D_2/a^4-D_3/a^2. \qquad (27)$$

$q_\perp$, $k_{\nu\perp}$, and $k_{\mu\perp}$ are algebraic components of $\vec{q}$, $\vec{k}_\nu$, and $\vec{k}_\mu$ on the plane perpendicular to the z axis. In this plane, $q_\perp$, $k_{\nu\perp}$, and $k_{\mu\perp}$ are all referred to $\vec{q}$ as the common positive direction of axis. Thus,

$$q_\perp=q, \quad k_{\nu\perp}=(k_\mu\cos\theta-k_\nu)k_\nu/q, \quad k_{\mu\perp}=(k_\mu-k_\nu\cos\theta)k_\mu/q, \qquad (28)$$

where θ is the scattering angle. All the vectors $\vec{q}$, $\vec{k}_\nu$, and $\vec{k}_\mu$ have the same azimuthal angle ϕ_q. Again, when the amplitude $\bar{f}_{B2}$ is computed, one ignores the phase factor $e^{\mp i\phi_q}$ altogether.

It is worth noting that with the direction of the z axis taken to be perpendicular to $\vec{q}$, as was usually considered in the conventional Glauber theory,[3,8,9] the Glauber amplitude for the process $1s$-$2p$, 0 is vanishing. The second-order eikonal term of the amplitude is also vanishing. However, the corresponding second Born term is not. The expression for $\bar{f}_{B2}$ of the process $1s$-$2p$, 0 can be derived with ease. One finds that

$$\bar{f}_{B2}(1s\text{-}2p,0)=6\sqrt{2}\,i(m_1q_z+m_2k_{\nu z}+m_3k_{\mu z}) \qquad (29)$$

where, with the usual choice of z-axis direction,

$$q_z=0, \quad k_{\nu z}=k_{\mu z}=k_\mu k_\nu\sin\theta/q. \qquad (30)$$

C. Eikonal exchange amplitude

As for the eikonal exchange amplitudes, the following general formulas for "post" and "prior" e-H scatterings have been given by Foster and Williamson,[14]

$$F_{if}^{\text{post exch.}}=-\pi 2^{4-i\eta_i}\frac{\Gamma(1-i\eta_i)}{\Gamma(-i\eta_i)}C_f^*C_iD_i(\mu,\vec{\gamma})D_f(M,\vec{\Gamma})\int_0^{+\infty}d\lambda\lambda^{-i\eta_i-1}\int_0^1 dx x^{-1} \times\left[\mu\left(\frac{d}{d\mu^2}\right)^2\mathfrak{F}_{(+)}(1,0,0,0,0)-x^{-1}\left(\frac{d}{d\mu^2}\right)^2\mathfrak{F}_{(+)}(1,0,0,1,0)\right]_{\vec{\gamma}=\vec{\Gamma}=0}, \qquad (31)$$

and

$$F_{if}^{\text{prior exch.}}=-\pi 2^{4-i\eta_f}\frac{\Gamma(1-i\eta_f)}{\Gamma(-i\eta_f)}C_f^*C_iD_i(\mu,\vec{\gamma})D_f(M,\vec{\Gamma})\int_0^{+\infty}d\lambda\lambda^{-i\eta_f-1}\int_0^1 dx\, x^{-1} \times\left[M\left(\frac{d}{dM^2}\right)^2\mathfrak{F}_{(-)}(1,0,0,0,0)-x^{-1}\left(\frac{d}{dM^2}\right)^2\mathfrak{F}_{(-)}(1,0,0,1,0)\right]_{\vec{\gamma}=\vec{\Gamma}=0}, \qquad (32)$$

where $\eta_i=1/k_i$, $\eta_f=1/k_f$ and the initial and final bound states of hydrogen atom are written as

$$u_i(\vec{r})=D_i(\mu,\vec{\gamma})C_i\exp(-\mu r+i\vec{\gamma}\cdot\vec{r})|_{\vec{\gamma}=0} \qquad (33a)$$

$$u_f(\vec{R})=D_f(M,\vec{\Gamma})C_f^*\exp(-MR-i\vec{\Gamma}\cdot\vec{R})|_{\vec{\Gamma}=0}. \qquad (33b)$$

C_i and C_f are normalization constants and $D_i(\mu,\vec{\gamma})$ and $D_f(M,\vec{\Gamma})$ are appropriate differential operators which generate the required wave functions. Other notations have their usual meanings.[14] To be consistent to the case of direct scattering, here the z axis is also chosen to be perpendicular to the momentum transfer $\vec{q}$. It is worth mentioning that with this choice of z axis, the discrepancy between the values of "post" and "prior" exchange amplitudes will be minimized, especially at higher scattering energies.[13] For the $1s$-$2s$ process, one has

$$C_i=1/\sqrt{\pi}, \quad D_i(\mu,\vec{\gamma})=1, \quad \mu=1, \quad C_f^*=(1/4\sqrt{2})(1/\sqrt{\pi}), \quad D_f(M,\vec{\Gamma})=2+d/dM, \quad M=\tfrac{1}{2}. \qquad (34)$$

For the $1s$-$2p\pm$ process,

$$C_f^*=\frac{1}{8\sqrt{\pi}}, \quad D_f(M,\vec{R})=i\left(\frac{\partial}{\partial\Gamma_x}\mp i\frac{\partial}{\partial\Gamma_y}\right), \quad M=\tfrac{1}{2}. \qquad (35)$$

Finally for the $1s$-$2p,0$ process,

$$C_f^* = \frac{1}{4\sqrt{2}}\,\frac{1}{\sqrt{\pi}}\,, \quad D_f(M,\vec{\mathrm{R}}) = i\frac{\partial}{\partial\Gamma_z}\,, \quad M = \tfrac{1}{2}\,. \tag{36}$$

In order to reduce the eikonal exchange amplitudes to the forms which can be ready for numerical integrations, one needs to use the following expressions of the derivatives of $\mathfrak{F}_{(+)}(m,p,r,s,t)$,

$$\frac{d}{dM}\mathfrak{F}_{(+)}(m,p,r,s,t) = 2(i\eta_i - m)\mathfrak{F}_{(+)}(m+1,p,r,s,t+1) - (i\eta_i + r)\mathfrak{F}_{(+)}(m,p,r+1,s,t) + t\mathfrak{F}_{(+)}(m,p,r,s,t-1)\,, \tag{37}$$

$$\frac{d}{d\mu^2}\mathfrak{F}_{(+)}(m,p,r,s,t) = -\tfrac{1}{2}xp\mathfrak{F}_{(+)}(m,p+2,r,s,t) + (i\eta_i - m)x\mathfrak{F}_{(+)}(m+1,p+1,r,s,t+1) - \tfrac{1}{2}x(i\eta_i + r)\mathfrak{F}_{(+)}(m,p+1,r+1,s,t) + \tfrac{1}{2}xt\mathfrak{F}_{(+)}(m,p+1,r,s,t-1)\,, \tag{38}$$

$$\left(\frac{\partial}{\partial\Gamma_x} \mp i\frac{\partial}{\partial\Gamma_y}\right)\mathfrak{F}_{(+)}(m,p,r,s,t) = -2(i\eta_i - m)(k_{i\perp} - xk_{f\perp})e^{\mp i\phi_q}\mathfrak{F}_{(+)}(m+1,p,r,s,t)\,, \tag{39}$$

and

$$i\frac{\partial}{\partial\Gamma_z}\mathfrak{F}_{(+)}(m,p,r,s,t) = (i\eta_i + r)\mathfrak{F}_{(+)}(m,p,r+1,s,t) - 2iQ_{+z}(i\eta_i - m)\mathfrak{F}_{(+)}(m+1,p,r,s,t) \tag{40}$$

as well as the similar expressions of the derivatives of $\mathfrak{F}_{(-)}(m,p,r,s,t)$. Besides the above derivatives, in order to guarantee the numerical convergence of the integration over λ at $\lambda = 0$, it is preferable to take an integration by parts in terms of the variable λ first. This integration implies the need for the knowledge of the explicit expressions of $d\mathfrak{F}_{(+)}/d\lambda$ and $d\mathfrak{F}_{(-)}/d\lambda \cdot d\mathfrak{F}_{(+)}/d\lambda$ is given by

$$\begin{aligned}\frac{\partial}{\partial\lambda}&\mathfrak{F}_{(+)}(m,p,r,s,t)\\ &= (1-x)\{[p\mathfrak{F}_{(+)}(m,p+2,r,s,t) + 2M(i\eta_i - m)\mathfrak{F}_{(+)}(m+1,p+1,r,s,t) - (i\eta_i + r)\mathfrak{F}_{(+)}(m,p+1,r+1,s,t)\\ &\qquad + t\mathfrak{F}_{(+)}(m,p+1,r,s,t-1)][\lambda(1-x) - ixk_{fz}] + s\mathfrak{F}_{(+)}(m,p,r,s-1,t)\\ &\qquad - 2i(i\eta_i - m)k_{iz}\mathfrak{F}_{(+)}(m+1,p,r,s,t) + (i\eta_i + r)\mathfrak{F}_{(+)}(m,p,r+1,s,t)\}\end{aligned} \tag{41}$$

$\partial\mathfrak{F}_{(-)}(m,p,r,s,t)/\partial\lambda$ can be obtained by simply interchanging $\vec{k}_i$ with $\vec{k}_f$, $\vec{\gamma}$ with $\vec{\Gamma}$ and μ with M in Eq. (41). The combined use of these derivatives of $\mathfrak{F}_{(+)}$ and $\mathfrak{F}_{(-)}$ in Eqs. (31) and (32) for each of the processes $1s$-$2s$, $1s$-$2p\pm$, and $1s$-$2p,0$ yields after some very tedious algebra the appropriate final forms for the Glauber exchange amplitudes. It should be noted that an alternative method of obtaining the values of exchange amplitudes may be available by using directly the numerical derivatives of the integrals involved. The numerical evaluation of the exchange amplitudes by this method is, however, also rather cumbersome.

III. RESULTS AND DISCUSSIONS

The Glauber exchange effects have been evaluated with the post eikonal exchange formulas for the four processes $1s$-$2s$, $1s$-$2p,0$, and $1s$-$2p,\pm$ and included into the direct conventional Glauber amplitudes to obtain the differential cross sections with exchange at 100, 200, and 300 eV. In Tables I and II, I present the results of the 1-2 excitation of hydrogen atom at scattering energies of 200 and 300 eV. The exchange effect is mostly enhanced at intermediate scattering angles from around 30° up. Within the conventional Glauber method, while, as was well known, the direct scattering amplitude of the process $1s$-$2p,0$ is vanishing; the eikonal exchange amplitude is not. The eikonal exchange effect $1s$-$2p,0$ is also tentatively included into the differential cross sections of the 1-2 excitation. However, the results are not modified much by such an inclusion.

The second-order eikonal term F_{G2} of the processes $1s$-$2s$ and $1s$-$2p\pm$ have been calculated[22] for scattering angles from 0° to 180° and at scattering energies of 100, 200, and 300 eV by using the appropriate expressions already well known in the literature. The results show that F_{G2} of the $1s$-$2s$ process can be a good substitute for the imaginary part of the second Born term calculated with the average closure summation method and that the real part of $\bar{f}_{B2}$ contributes significantly to the differential cross sections. The differential cross sections of $1s$-$2s$ excitation of hydrogen

TABLE I. Differential cross sections of 1-2 excitation of hydrogen atom by electron impact at 200 eV in $a_0^2 sr^{-1}$ unit, calculated with the conventional Glauber method with exchange. The numbers in parentheses indicate the possible error in the least significant digits of the cross section experimental value.

Angle in degree	First Born	Glauber	Glauber with exchange (without including $1s$-$2p$, 0)	Glauber with exchange	Experimental data[a]
20	7.125×10^{-2}	5.868×10^{-2}	5.282×10^{-2}	5.309×10^{-2}	$4.97(57) \times 10^{-2}$
25	1.569×10^{-2}	1.529×10^{-2}	1.476×10^{-2}	1.488×10^{-2}	...
30	3.848×10^{-3}	5.633×10^{-3}	6.111×10^{-3}	6.164×10^{-3}	$6.72(61) \times 10^{-3}$
40	3.218×10^{-4}	1.646×10^{-3}	1.981×10^{-3}	1.994×10^{-3}	$2.54(38) \times 10^{-3}$
50	4.006×10^{-5}	7.302×10^{-4}	8.593×10^{-4}	8.632×10^{-4}	$1.32(24) \times 10^{-3}$
60	6.947×10^{-6}	3.809×10^{-4}	4.283×10^{-4}	4.298×10^{-4}	$7.27(72) \times 10^{-4}$
70	1.580×10^{-6}	2.212×10^{-4}	2.382×10^{-4}	2.390×10^{-4}	$4.48(66) \times 10^{-4}$
80	...	1.399×10^{-4}	1.454×10^{-4}	1.459×10^{-4}	$2.71(33) \times 10^{-4}$
90	...	9.501×10^{-5}	9.609×10^{-5}	9.634×10^{-5}	$1.99(29) \times 10^{-4}$
100	...	6.855×10^{-5}	6.796×10^{-5}	6.811×10^{-5}	$1.66(36) \times 10^{-4}$
110	...	5.213×10^{-5}	5.102×10^{-5}	5.112×10^{-5}	$1.51(28) \times 10^{-4}$
120	...	4.152×10^{-5}	4.040×10^{-5}	4.050×10^{-5}	$1.20(29) \times 10^{-4}$
130	...	3.448×10^{-5}	3.360×10^{-5}	3.380×10^{-5}	$1.04(31) \times 10^{-4}$
140	...	2.973×10^{-5}	2.931×10^{-5}	2.987×10^{-5}	$1.17(39) \times 10^{-4}$

[a]Experimental data by Williams and Willis (Ref. 7).

atom by electron impact are then computed first without and then with the inclusion of the Glauber exchange effect. In Fig. 1, I present these results along with those calculated with the conventional Glauber amplitude as well as those calculated with the first Born approximation at 100 eV. It is found that the values calculated with the modified Glauber amplitude differ considerably from those evaluated with the conventional Glauber amplitude at all angles. The inclusion of the eikonal exchange effect does not modify significantly the differential cross sections at small scattering angles, but alters considerably these values at larger scattering angles. For the second

TABLE II. Same as in Table I at 300 eV.

Angle in degree	First Born	Glauber	Glauber with exchange (without including $1s$-$2p$, 0)	Glauber with exchange	Experimental data[a]
10	8.989×10^{-1}	8.034×10^{-1}	7.661×10^{-1}	7.664×10^{-1}	...
15	1.169×10^{-1}	1.016×10^{-1}	9.300×10^{-2}	9.311×10^{-2}	...
20	1.752×10^{-2}	1.696×10^{-2}	1.556×10^{-2}	1.560×10^{-2}	$1.16(25) \times 10^{-2}$
25	3.074×10^{-3}	4.386×10^{-3}	4.335×10^{-3}	4.350×10^{-3}	...
30	6.349×10^{-4}	1.754×10^{-3}	1.872×10^{-3}	1.878×10^{-3}	$1.57(27) \times 10^{-3}$
40	4.235×10^{-5}	5.452×10^{-4}	6.041×10^{-4}	6.054×10^{-4}	$6.11(82) \times 10^{-4}$
50	4.624×10^{-6}	2.358×10^{-4}	2.555×10^{-4}	2.560×10^{-4}	...
60	...	1.200×10^{-4}	1.265×10^{-4}	1.267×10^{-4}	$1.85(30) \times 10^{-4}$
70	...	6.862×10^{-5}	7.065×10^{-5}	7.074×10^{-5}	...
80	...	4.305×10^{-5}	4.350×10^{-5}	4.355×10^{-5}	$1.01(18) \times 10^{-4}$
90	...	2.911×10^{-5}	2.900×10^{-5}	2.903×10^{-5}	...
100	...	2.096×10^{-5}	2.067×10^{-5}	2.069×10^{-5}	$8.33(97) \times 10^{-5}$
110	...	1.592×10^{-5}	1.561×10^{-5}	1.563×10^{-5}	...
120	...	1.268×10^{-5}	1.241×10^{-5}	1.242×10^{-5}	$6.42(142) \times 10^{-5}$
130	...	1.052×10^{-5}	1.033×10^{-5}	1.035×10^{-5}	...
140	...	9.077×10^{-6}	8.970×10^{-6}	9.000×10^{-6}	$4.10(122) \times 10^{-5}$

[a]Experimental data by Williams and Willis (Ref. 7).

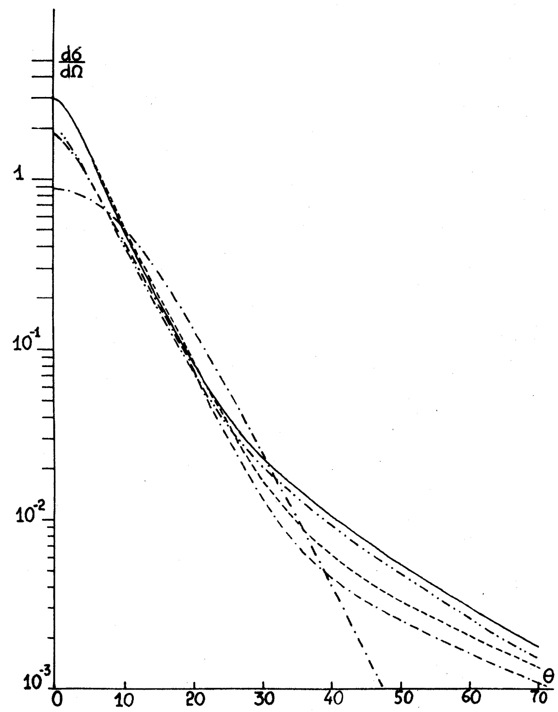

FIG. 1. Differential cross sections of $e-H1s-2s$ inelastic scattering in $a_0^2\,Sr^{-1}$ unit at 100 eV. —•—•—•—•— first Born approximation; ——•——•——• conventional Glauber; —••—••—•• conventional Glauber with eikonal exchange; ------- modified Glauber, $\omega=0.50$; ———— modified Glauber with eikonal exchange.

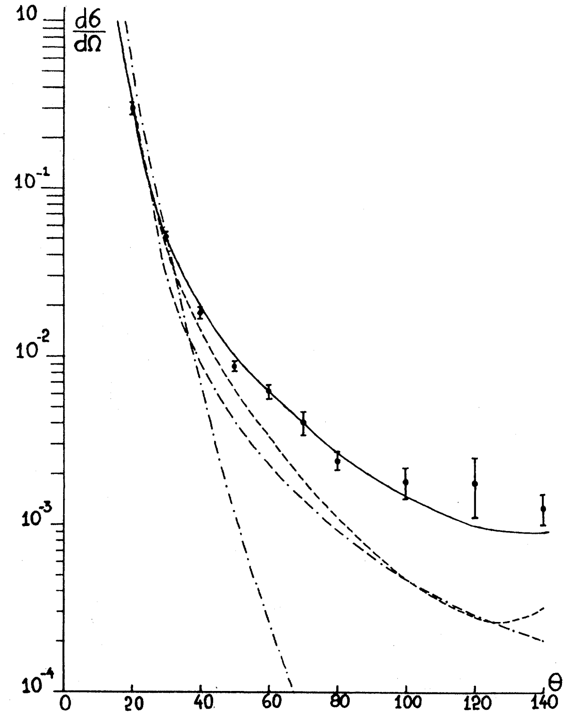

FIG. 3. Differential cross sections of $e-H1-2$ excitation at 100 eV in $a_0^2\,Sr^{-1}$ unit. —•—•—•— first Born approximation; ——•——•——• conventional Glauber; --------- conventional Glauber with eikonal exchange; modified Glauber with eikonal exchange.

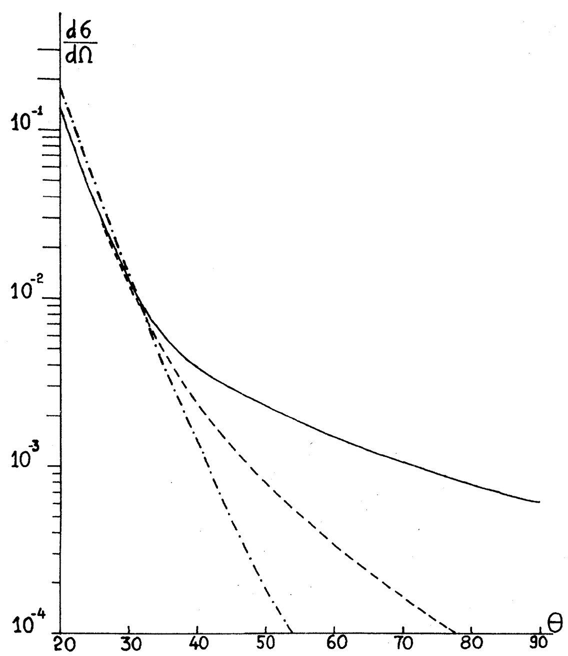

FIG. 2. Differential cross sections of $e-H1s-2p\pm$ inelastic scattering in $a_0^2 Sr^{-1}$ unit at 100 eV. —•—•—•—•— first Born approximation; --------- conventional Glauber; ———— modified Glauber, $\omega=0.50$.

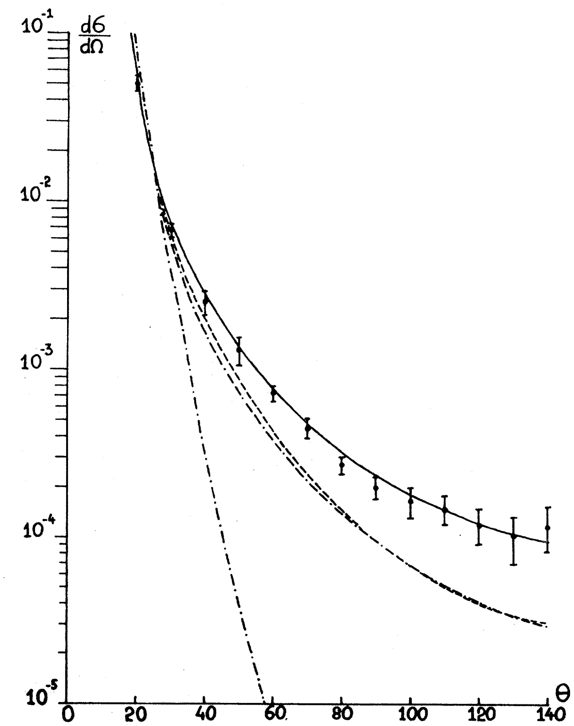

FIG. 4. Same as in Fig. 3 at 200 eV.

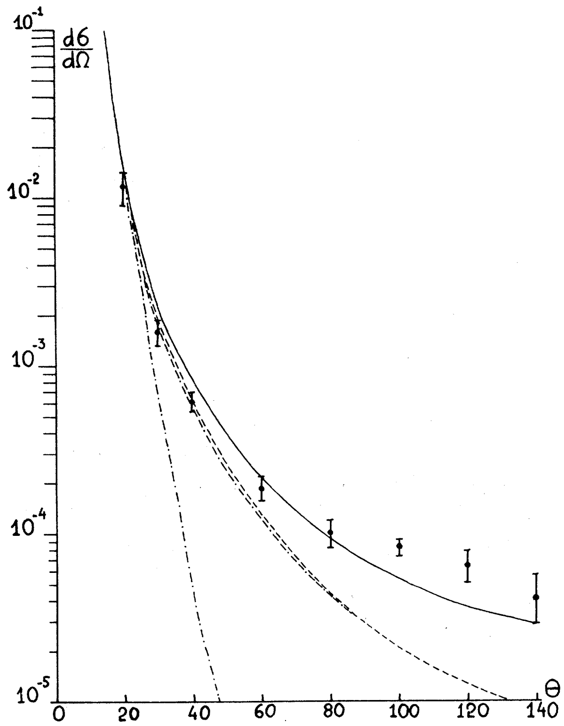

FIG. 5. Same as in Fig. 3 at 300 eV.

Born term f_{B2} of the $1s$-$2p\pm$ processes, one calculates it approximately with Eq. (24) and with the help of the expressions of A_1, A_2, A_3, B_1, B_2, B_3, C_1, C_2, and C_3 given in Sec. III. In all the amplitudes of interest, the factor $e^{\mp i\phi_q}$ has been ignored. It should be noted that with the disregard of the factor $e^{\mp i\phi_q}$, F_{G2} is a real quantity. I find that the characteristic behaviors of F_{G2} as well as its magnitude can be reproduced adequately by the real part of the approximate second Born term, although the magnitude of F_{G2} is somewhat deficient in comparison with $\mathrm{Re}\bar{f}_{B2}$, especially at larger scattering angles. The results of differential cross sections of the $1s$-$2p\pm$ scattering at 100 eV are shown in Fig. 2 together with those calculated with the conventional Glauber and first Born amplitudes. Within the modified Glauber method, F_{GM} for the process $1s$-$2p, 0$ is identical to the second-Born amplitude. f_{B2} of the process $1s$-$2p, 0$ is calculated with Eq. (29) shown in Sec. II. The values of differential cross section of this process are not so large, compared to those of the other processes which constitute the 1-2 excitation of a hydrogen atom. The combination of the differential cross sections of the above four processes give the differential cross sections of the 1-2 excitation of hydrogen atom by electron impact which are to be compared with experimental data acquired by absolute measurement.[7] In Figs. 3–5, I present the differential cross sections of the 1-2 excitation of hydrogen atom calculated with the modified Glauber amplitude with the inclusion of the Glauber exchange effects at 100, 200, and 300 eV. Also shown along are the differential cross sections calculated within the conventional Glauber method with and without the inclusion of exchange, as well as those given by the first Born approximation. It is found that the results of the modified Glauber method agree quite well with experimental data. The deficiency of the differential cross sections at intermediate and large scattering angles suffered by the conventional Glauber amplitude has been completely removed by the replacement of F_{G2} by $\bar{f}_{B2}$. Thus,

TABLE III. Differential cross sections in $a_0^2 sr^{-1}$ unit of the 1–2 excitation of hydrogen atom by electron impact, calculated with the modified Glauber method at 100 eV.

Angle in degree	Modified Glauber	Modified Glauber (with $1s$-$2p,0$)	Modified Glauber with exchange	Modified Glauber with exchange (incl. $1s$-$2p,0$)	Experimental data[a]
20	3.542×10^{-1}	3.599×10^{-1}	3.188×10^{-1}	3.247×10^{-1}	$2.97(25)\times10^{-1}$
25	1.122×10^{-1}	1.160×10^{-1}	1.077×10^{-1}	1.112×10^{-1}	···
30	4.295×10^{-2}	4.542×10^{-2}	4.856×10^{-2}	5.071×10^{-2}	$5.19(23)\times10^{-2}$
40	1.370×10^{-2}	1.477×10^{-2}	1.960×10^{-2}	2.044×10^{-2}	$1.82(16)\times10^{-2}$
50	7.794×10^{-3}	8.291×10^{-3}	1.069×10^{-2}	1.106×10^{-2}	$8.73(62)\times10^{-3}$
60	5.074×10^{-3}	5.323×10^{-3}	6.287×10^{-3}	6.477×10^{-3}	$6.19(57)\times10^{-3}$
70	3.483×10^{-3}	3.616×10^{-3}	3.945×10^{-3}	4.050×10^{-3}	$4.07(41)\times10^{-3}$
80	2.505×10^{-3}	2.580×10^{-3}	2.648×10^{-3}	2.708×10^{-3}	$2.38(29)\times10^{-3}$
90	1.885×10^{-3}	1.930×10^{-3}	1.896×10^{-3}	1.931×10^{-3}	···
100	1.481×10^{-3}	1.508×10^{-3}	1.440×10^{-3}	1.460×10^{-3}	$1.82(37)\times10^{-3}$
110	1.209×10^{-3}	1.225×10^{-3}	1.153×10^{-3}	1.167×10^{-3}	···
120	1.022×10^{-3}	1.032×10^{-3}	9.715×10^{-4}	9.868×10^{-4}	$1.79(68)\times10^{-3}$

[a] Experimental data by Williams and Willis (Ref. 7).

the sole correction of the second-order term of the conventional Glauber amplitude with its counterpart prior to eikonization, i.e., $\bar{f}_{B2}$ is sufficient to make up for the deficiency of the amplitude at larger scattering angles in both elastic and inelastic scatterings. The inclusion of the $1s$-$2p$, 0 process into the total 1-2 excitation of hydrogen atom only modifies slightly the theoretical results of cross sections. In Table III, I show the modified Glauber results with and without including the $1s$-$2p$, 0 process at 100 eV for comparison. Also presented in this table are the theoretical results of the first Born approximation, those of the conventional Glauber method with and without exchange and experimental data. The consideration of the eikonal exchange effects improves the agreement between the theoretical values and experimental data. As expected, the exchange becomes less significant as the scattering energy increases.

IV. CONCLUSIONS

In this paper, the modified Glauber method proposed earlier for electron-atom scattering has been extended to the analysis of the 1-2 excitation of hydrogen atom by electron impact. The exchange effects have not been neglected. They are included into the calculations by using the exact eikonal exchange formulas. The differential cross sections of the conventional Glauber method with the inclusion of eikonal exchange effect have also been obtained. I found that the deficiency of the theoretical cross sections calculated with the conventional Glauber amplitude in so far as to compare with experimental data at intermediate and large scattering angles can be corrected by a restricted consideration of eikonal approximation for the scattering amplitude. The agreement with experimental data of the theoretical results given by the modified Glauber method is rather good. Since the conventional Glauber method is essentially a small-angle approximation, its ability of providing a good agreement with experimental data, as expected, decreases gradually as the scattering angles become greater. This is because the eikonalization of the exact scattering has transformed its higher-order Born terms beyond the first one into the corresponding eikonal terms and some of them at this intermediate range of energies are no longer good approximations for the Born terms at larger scattering angles. The nonconsideration of the eikonal approximation for the second-order scattering term appears to be sufficient to make up for the deficiency of the conventional Glauber amplitude in both elastic and inelastic scatterings.

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