$$a_{\vec{K}\vec{K}'} = \int_{-\infty}^{\infty} e^{i(K_X + k - K'_X)X}\, dX\, \delta(K_Y - K'_Y)\, \delta(K_Z - K'_Z)$$
$$= \delta(K_X + k - K'_X)\, \delta(K_Y - K'_Y)\, \delta(K_Z - K'_Z)$$
$$\approx \delta(\vec{K} - \vec{K}') \, ,$$

since k is much smaller than K_X. This result merely states there will be essentially no change in the momentum of the center of mass, and whatever change there is will be in the X component of the momentum. The second operator gives

$$a_{\vec{K}\vec{K}'} = \frac{\hbar}{iM} \int_{-\infty}^{\infty} e^{-iK'_Z Z} \frac{\partial}{\partial Z} e^{iK_Z Z} dZ\, \delta(K_X + k - K'_X) \delta(K_Y - K$$
$$= (\hbar K_Z/M)\, \delta(K_Z - K'_Z)\, \delta(K_X + k - K'_X)\, \delta(K_Y - K'_Y)$$
$$\approx v_Z\, \delta(\vec{K} - \vec{K}')$$

since $\hbar K_Z$ is the momentum in the Z direction.

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Resonant Scattering of Radiation from Collision-Damped Two-Level Systems

B. R. Mollow
Department of Physics, The University of Massachusetts, 100 *Arlington Street, Boston, Massachusetts* 02116
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The power spectrum of the radiation emitted by a driven collision-damped two-level system is evaluated. The driving field, which is treated classically, is assumed to oscillate harmonically near the atomic resonance frequency, and its intensity is allowed to assume arbitrary values. The collisions are assumed to be strong, i.e., to instantaneously thermalize the state of the atom. The limiting forms of the power spectrum of the radiated field are discussed for the cases of low and high excitation of the atomic system.

I. INTRODUCTION

The effect of collisions on the response of a collection of atoms to a monochromatic incident electromagnetic field has been extensively studied, in both classical and quantum-mechanical contexts.[1-8] For the most part, previous analyses have been devoted to the evaluation of quantities which depend only upon the mean values of atomic operators, such as the electromagnetic susceptibilities or the absorption line-shape function, i.e., the rate of attenuation of the incident field as a function of its frequency. It has been found, in particular, that the widths[9] of the peaks (centered at the atomic resonance frequencies) in the line-shape function are proportional to the collision rate for relatively weak incident fields, but that they are proportional to the intensity of the incident field when it is great enough to lead to an appreciable degree of saturation.

In the present paper our interest lies in describing the spectral properties of the field radiated by the driven atoms, and hence in evaluating the correlation function which represents the product of atomic dipole moments at two different times. We assume that the incident field oscillates at a fixed frequency ω which lies near an atomic resonance frequency ω_0, and allow the field intensity to be arbitrarily great. Our analysis is carried out within the context of a simple model consisting of a single two-level atom driven by a classical electric field and subject to strong random collisions which abruptly thermalize its state. We assume that the collision rate κ is much greater than all other relaxation rates, in particular, that it is much greater than the radiative decay rate, the effect of which has been analyzed in a previous paper.[10]

The results we find for the case of collisional relaxation differ markedly in the limit of weak driving fields from those for the case of radiative relaxation. A principal difference is that in the collisional case the radiated field contains, in addition to a coherent monochromatic spectral component oscillating at the driving frequency ω, incoherent components oscillating within an interval κ of the resonance frequency ω_0. The incoherent components are appreciable even at low temperatures, and in fact they are equal in intensity to that of the coherent component in the zero-temperature limit. In the limit of strong driving fields,[11] on the other hand, the solutions for the radiative and collisional cases resemble each other quite strongly, each case being characterized by three spectral components in the radiated field, one centered at the driving frequency ω, and one at each of the displaced frequencies $\omega \pm \Omega$, where Ω is the frequency of field-induced atomic transitions.

The basic strong-collision model is introduced in Sec. II, and the equilibrium expectation values of the atomic dipole moment and excitation probability are evaluated. The equations of motion for the atomic density operator are expressed in a form suitable for use in the analysis of Sec. III, where the two-time dipole moment correlation function, which specifies the power spectrum of the radiated field, is evaluated by means of a Markov-type approximation.[12]

II. STRONG-COLLISION MODEL OF ATOMIC RELAXATION

Let us consider a two-level atom with energy eigenstates $|0\rangle_a$ and $|1\rangle_a$ and corresponding eigenvalues 0 and $\hbar\omega_0$, respectively. In the presence of a prescribed classical field

$$E_c(t) = (1/\sqrt{2})\,[\mathcal{E}(t) + \mathcal{E}^*(t)]\hat{e}_0, \tag{2.1}$$

with spectral components near the resonance frequency ω_0, the Hamiltonian for the atom may be approximated by the expression

$$H(t) = \hbar\,\omega_0\, a^\dagger a - \hbar\lambda\mathcal{E}(t)\, a^\dagger - \hbar\,\lambda^*\mathcal{E}^*(t)\, a\,, \tag{2.2}$$

where $a^\dagger$ and a are the atomic raising and lowering operators

$$\text{(a)}\ a^\dagger \equiv |1\rangle_a\, {}_a\langle 0|\,, \quad \text{(b)}\ a \equiv |0\rangle_a\, {}_a\langle 1|\,, \tag{2.3}$$

and the coupling parameter λ is related to the dipole matrix element μ by means of the expression

$$\lambda = (\vec{\mu}\cdot\hat{e}_0)/\hbar\sqrt{2}\,. \tag{2.4}$$

Let us now assume that the atom suffers random strong collisions[13] which abruptly thermalize its state. It is thus described immediately after a collision by the density operator[14]

$$\rho_T = \bar{n}_T\, a^\dagger a + (1 - \bar{n}_T)\, a a^\dagger\,, \tag{2.5}$$

where $\bar{n}_T$ is the thermal-equilibrium probability of finding the atom in its excited state

$$\bar{n}_T = 1/(1 + e^{\hbar\omega_0/kT})\,. \tag{2.6}$$

If we assume that the probability that a collision takes place between the times t and $t+dt$ is $\kappa\, dt$, then we may represent the effect of the collision process on the atomic density operator $\rho_a(t)$ by adding to the equation for its time derivative a term equal to κ times its change $\{\rho_T - \rho_a(t)\}$ during a collision. We have then[5]

$$\frac{\partial}{\partial t}\,\rho_a(t) = \frac{1}{i\hbar}\,[H(t),\ \rho_a(t)] + \kappa\{\rho_T - \rho_a(t)\}\,, \tag{2.7}$$

where $H(t)$ is the Hamiltonian (2.2) for the driven atom in the absence of collisions.

In order to carry out our evaluation of the two-time dipole moment correlation function in Sec. III, it will be necessary to express the time evolution of the atomic density operator in a form which does not depend upon its being a Hermitian operator with unit trace, but which remains consistent when it is allowed, in a formal sense, to be a general operator in the state space of the atom. To accomplish this, it is sufficient to note that the collisional process must be represented by a transformation on a general atomic operator $\rho_a(t)$ which preserves its trace, is linear, and transforms it into ρ_T if it is Hermitian and has unit trace. It is not difficult to show that the only transformation which satisfies these requirements is

$$\rho_a(t) \to [\mathrm{Tr}\rho_a(t)]\rho_T\,. \tag{2.8}$$

We must therefore change Eq. (2.7) to the more general relation

$$\frac{\partial}{\partial t}\,\rho_a(t) = \frac{1}{i\hbar}\,[H(t),\ \rho_a(t)] + \kappa\{(\mathrm{Tr}\rho_a(t))\rho_T - \rho_a(t)\}\,, \tag{2.9}$$

in order to carry out our evaluation of the two-time atomic correlation function.

When the relation (2.2) for the Hamiltonian $H(t)$ is substituted into Eq. (2.9), it is found that the atomic expectation values

$$\bar{n}(t) \equiv \mathrm{Tr}\{\rho_a(t)a^\dagger a\}, \tag{2.10a}$$

$$\alpha(t) \equiv \mathrm{Tr}\{\rho_a(t)a\}\,, \tag{2.10b}$$

$$\alpha^*(t) \equiv \mathrm{Tr}\{\rho_a(t)a^\dagger\}, \tag{2.10c}$$

$$\bar{m}(t) \equiv \mathrm{Tr}\{\rho_a(t)\, a a^\dagger\}, \tag{2.10d}$$

obey the equations

$$\frac{d}{dt}\,\bar{n}(t) = i\lambda\mathcal{E}(t)\alpha^*(t) - i\lambda^*\mathcal{E}^*(t)\alpha(t) - \kappa(1-\bar{n}_T)\bar{n}(t) + \kappa\bar{n}_T\bar{m}(t)\,, \tag{2.11a}$$

$$\frac{d}{dt}\,\alpha(t) = -i\lambda\mathcal{E}(t)\,[\bar{n}(t) - \bar{m}(t)] - (\kappa + i\omega_0)\,\alpha(t)\,, \tag{2.11b}$$

$$\frac{d}{dt}\,\alpha^*(t) = i\lambda^*\mathcal{E}^*(t)[\bar{n}(t) - \bar{m}(t)] - (\kappa - i\omega_0)\,\alpha^*(t)\,, \tag{2.11c}$$

$$\frac{d}{dt}\,\bar{m}(t) = -\frac{d}{dt}\,\bar{n}(t)\;. \tag{2.11d}$$

In the case in which the incident field is the harmonic function

$$\mathcal{E}(t) = \mathcal{E}_0 e^{-i\omega t} \tag{2.12}$$

the excitation probability $\bar{n}(t)$ and the dipole moment amplitude $\alpha(t)$ have the equilibrium values[5]

$$\bar{n}_\infty = \{\tfrac{1}{2}\Omega^2 + \bar{n}_T[\kappa^2 + (\omega-\omega_0)^2]\}/[\Omega^2 + \kappa^2 + (\omega-\omega_0)^2]\,, \tag{2.13a}$$

$$\alpha_\infty(t) = e^{-i\omega t}\,\frac{i\lambda\mathcal{E}_0[\kappa + i(\omega-\omega_0)]}{\Omega^2 + \kappa^2 + (\omega-\omega_0)^2}\,(1-2\bar{n}_T), \tag{2.13b}$$

in which the parameter Ω is defined as

$$\Omega \equiv 2|\lambda\mathcal{E}_0|\;. \tag{2.14}$$

The excitation produced by the driving field is given by the expression

$$\Delta\bar{n}_\infty \equiv \bar{n}_\infty - \bar{n}_T \equiv [\tfrac{1}{2}\Omega^2(1-2\bar{n}_T)]/[\Omega^2 + \kappa^2 + (\omega-\omega_0)^2]\,, \tag{2.15}$$

which is essentially the absorption line-shape function for our model, since the mean rate at which the atom absorbs energy from the incident field is equal to the collision rate κ times the mean loss in internal energy $\hbar\omega_0\Delta\bar{n}_\infty$ of the atom during a collision. It should be noted in this connection that since by assumption the collision rate is much greater than the natural decay rate $\kappa_0 = |\mu|^2\omega_0^3/3\pi\hbar c^3$, the mean total power $\kappa_0\hbar\omega_0\Delta\bar{n}_\infty$ of the scattered field (which is equal to total radiated power $\kappa_0\hbar\omega_0\bar{n}_\infty$ minus thermally radiated power $\kappa_0\hbar\omega_0\bar{n}_T$) represents only a very small fraction of the energy loss of the incident beam. The contribution of the scattering process to the attenuation of the incident beam has been omitted from our analysis, which does not include terms representing radiative damping in the equations of motion for the atomic density operator.

In the limit in which the driving field is weak enough $[\Omega^2 \ll \kappa^2 + (\omega-\omega_0)^2]$ and the temperature low enough ($\bar{n}_T \ll 1$) so that saturation effects are unimportant ($\bar{n}_\infty \ll 1$), the response of the atom to the driving field is essentially linear, and the equilibrium expectation values of $\bar{n}$ and α are given by the relations

$$\bar{n}_\infty = \bar{n}_T + \tfrac{1}{2}\Omega^2/[\kappa^2 + (\omega-\omega_0)^2] \tag{2.16a}$$

and $$\alpha_\infty(t) = e^{-i\omega t}\, i\lambda\mathcal{E}_0/[\kappa - i(\omega-\omega_0)]\,, \tag{2.16b}$$

for $\Omega^2 \ll \kappa^2 + (\omega-\omega_0)^2$ and $\bar{n}_T \ll 1$.

It is worth noting that these expressions are valid for arbitrary temperatures and field strengths in the case in which the atoms in question are harmonic oscillators rather than two-level systems, if the operators a and $a^\dagger$ in the definitions (2.10a) and (2.10b) are taken to be the familiar oscillator lowering and raising operators, and $\bar{n}_T$ is taken to be the thermal expectation value of $a^\dagger a$. For the case of zero temperature, the expectation values in Eqs. (2.16) satisfy the relation

$$\bar{n}_\infty = 2|\alpha_\infty|^2\,, \tag{2.17}$$

and the oscillating atomic dipole moment is thus the sum of a coherent and a fluctuating or incoherent component, each with the same mean intensity $|\mu|^2\,|\alpha_\infty|^2$.

III. POWER SPECTRUM OF THE RADIATED FIELD

The power spectrum of the field radiated by a two-level atom may be shown to be equal (in the dipole approximation) to some simple factors times the function

$$\tilde{g}(\nu) = \int_{-\infty}^{\infty} d\tau\, e^{i\nu\tau} g(\tau), \tag{3.1}$$

where $g(\tau)$ is the atomic correlation function

$$g(\tau) = \langle a^\dagger(t')a(t'+\tau)\rangle\,, \tag{3.2}$$

which in equilibrium is independent of t'. In Ref. 10 it is shown that the function $g(\tau)$ may be obtained by first solving the coupled equations for the atomic expectation values defined by Eqs. (2.10) in terms of their values at some initial time t', and expressing the solution for $\alpha(t'+\tau)$ in the form

$$\alpha(t'+\tau) = \mathfrak{U}_{\alpha n}(\tau;t')\bar{n}(t') + \mathfrak{U}_{\alpha\alpha}(\tau;t')\alpha(t') + \mathfrak{U}_{\alpha\alpha^*}(\tau;t')\alpha^*(t') + \mathfrak{U}_{\alpha m}(\tau;t')\bar{m}(t'), \tag{3.3}$$

where $\tau > 0$. The function $g(\tau)$ may then be expressed (in equilibrium) as

$$g(\tau) = \mathfrak{U}_{\alpha\alpha}(\tau)\bar{n}_\infty + \mathfrak{U}'_{\alpha m}(\tau)e^{-i\omega t'}\,\alpha_\infty^*(t'), \tag{3.4}$$

where use has been made of the fact that for a harmonically oscillating driving field, $\mathfrak{U}_{\alpha\alpha}(\tau;t')$ is independent of t', while $\mathfrak{U}_{\alpha m}(\tau;t')$ has the form

$$\mathfrak{U}_{\alpha m}(\tau;t') = \mathfrak{U}'_{\alpha m}(\tau)e^{-i\omega t'}\;. \tag{3.5}$$

The functions $\mathfrak{U}_{\alpha\alpha}(\tau)$ and $\mathfrak{U}'_{\alpha m}(\tau)$ may be found directly for the model we are considering by solving the linear Eqs. (2.11) for the case $\mathcal{E}(t) = \mathcal{E}_0 e^{-i\omega t}$, and identifying the coefficients of $\alpha(t')$ and $e^{-i\omega t'}\,\bar{m}(t')$, respectively, in the solution for $\alpha(t'+\tau)$. It is convenient to introduce complex parameters z, s, and s_- by means of the definitions

$$z \equiv \kappa + i\,\Delta\omega\,, \tag{3.6a}$$

$$s_\pm \equiv -\kappa \pm i\,\Omega'\,, \tag{3.6b}$$

where $\Delta\omega$ is the frequency difference

$$\Delta\omega \equiv \omega - \omega_0\,, \tag{3.7}$$

and Ω' is the Rabi[15] frequency of population inversion

$$\Omega' \equiv [\Omega^2 + (\Delta\omega)^2]^{1/2} \quad . \tag{3.8}$$

We find that the Laplace transform functions

$$\hat{\mathfrak{U}}_{\alpha\alpha}(s) \equiv \int_0^\infty d\tau e^{-s\tau}\, \mathfrak{U}_{\alpha\alpha}(\tau), \tag{3.9a}$$

$$\hat{\mathfrak{U}}'_{\alpha m}(s) \equiv \int_0^\infty d\tau e^{-s\tau}\, \mathfrak{U}'_{\alpha m}(\tau), \tag{3.9b}$$

may be expressed in terms of these parameters as

$$\hat{\mathfrak{V}}_{\alpha\alpha}(s - i\omega) = \frac{(s+\kappa)(s+z) + \frac{1}{2}\Omega^2}{(s+\kappa)(s-s_+)(s-s_-)} , \tag{3.10a}$$

$$\hat{\mathfrak{U}}'_{\alpha m}(s - i\omega) = i\lambda\mathcal{E}_0 \frac{(s+z)[s+\kappa(1-2\bar{n}_T)]}{s(s+\kappa)(s-s_+)(s-s_-)} . \tag{3.10b}$$

By making use of these relations and Eq. (2.13b) in Eq. (3.4), we find that the Laplace transform of the atomic correlation function

$$\hat{g}(s) \equiv \int_0^\infty d\tau e^{-s\tau}\, g(\tau) \tag{3.11}$$

is given by

$$\hat{g}(s - i\omega) = \bar{n}_\infty \left(\frac{(s+\kappa)(s+z) + \frac{1}{2}\Omega^2}{(s+\kappa)(s-s_+)(s-s_-)} \right) + \tfrac{1}{2} z^* \Delta\bar{n}_\infty \left(\frac{(s+z)[s+\kappa(1-2\bar{n}_T)]}{s(s+\kappa)(s-s_+)(s-s_-)} \right), \tag{3.12}$$

in which $\bar{n}_\infty$ and $\Delta\bar{n}_\infty$ are given by Eqs. (2.13a) and (2.15), respectively. The time-dependent correlation function $g(\tau)$ may be evaluated directly by inverting the Laplace transform (3.12) to find its value for $\tau > 0$, and then using the Hermiticity relation $g(-\tau) = g^*(\tau)$ to evaluate it at negative times. We find

$$g(\tau) = |\alpha_\infty|^2 e^{-i\omega\tau} + C_0 e^{-i\omega\tau - \kappa\tau} + C_+ e^{-i\omega\tau + s_+\tau} + C_- e^{-i\omega\tau + s_-\tau}, \qquad \text{for} \quad \tau > 0 \tag{3.13a}$$

$$g(\tau) = |\alpha_\infty|^2 e^{-i\omega\tau} + C_0^* e^{-i\omega\tau + \kappa\tau} + C_+^* e^{-i\omega\tau - s_-\tau} + C_-^* e^{-i\omega\tau - s_+\tau}, \qquad \text{for} \quad \tau < 0 \tag{3.13b}$$

where $|\alpha_\infty|$ is the modulus of the right-hand side of Eq. (2.13b), and the coefficients C_0, C_+, and C_- are defined as

$$C_0 = \tfrac{1}{2}\bar{n}_\infty \Omega^2/\Omega'^2 + \Delta\bar{n}_\infty \bar{n}_T z^* i\Delta\omega/\Omega'^2 , \tag{3.14}$$

and

$$C_\pm = \frac{1}{4}\, \frac{\Omega' \pm \Delta\omega}{\Omega'} \left[\bar{n}_\infty \left(\frac{\Omega' \pm \Delta\omega}{\Omega'} \right) + \Delta\bar{n}_\infty \left(1 \pm \frac{2i\kappa\bar{n}_T}{\Omega'} \right) \frac{z^*}{s_\pm} \right] . \tag{3.15}$$

The first term on the right-hand side of Eqs. (3.13) represents the coherent harmonically oscillating component of the atomic dipole moment. It is convenient to separate out the remaining incoherent component by means of the definition

$$g(\tau) \equiv |\alpha_\infty|^2 e^{-i\omega\tau} + g_{\text{inc}}(\tau) . \tag{3.16}$$

The Laplace transform of $g_{\text{inc}}(\tau)$ is then

$$\hat{g}_{\text{inc}}(s) = \hat{g}(s) - |\alpha_\infty|^2/(s + i\omega) , \tag{3.17}$$

and the spectral correlation function defined by Eq. (3.1) is given by the relation

$$\tilde{g}(\nu) = 2\pi|\alpha_\infty|^2 \delta(\nu - \omega) + \tilde{g}_{\text{inc}}(\nu) = 2\pi|\alpha_\infty|^2 \delta(\nu - \omega) + 2\,\text{Re}[\hat{g}_{\text{inc}}(-i\nu)] , \tag{3.18}$$

where $\hat{g}_{\text{inc}}(s)$ is defined by Eqs. (3.17) and (3.12). Either by making use of these relations or by directly evaluating the Fourier transform of Eqs. (3.13) we find that $\tilde{g}(\nu)$ may be expressed in the form

$$\tilde{g}(\nu) = 2\pi|\alpha_\infty|^2 \delta(\nu - \omega) + \frac{M_0 - (\nu - \omega)N_0}{(\nu - \omega)^2 + \kappa^2} + \frac{M_+ - (\nu - \omega + \Omega')N_+}{(\nu - \omega + \Omega')^2 + \kappa^2} + \frac{M_- - (\nu - \omega - \Omega')N_-}{(\nu - \omega - \Omega')^2 + \kappa^2} , \tag{3.19}$$

in which the parameters M_0, N_0, $M_\pm$, and $N_\pm$ are defined by the relations

$$M_0 = \kappa\bar{n}_\infty \Omega^2/\Omega'^2 + 2\kappa\,\Delta\bar{n}_\infty \bar{n}_T (\Delta\omega)^2/\Omega'^2 , \tag{3.20a}$$

$$N_0 = 2\kappa\,\Delta\bar{n}_\infty \bar{n}_T \Delta\omega/\Omega'^2 , \tag{3.20b}$$

$$M_\pm = \tfrac{1}{2}\kappa \left(\frac{\Omega' \pm \Delta\omega}{\Omega'} \right) \left\{ \bar{n}_\infty \left(\frac{\Omega' \pm \Delta\omega}{\Omega'} \right) - \frac{\Delta\bar{n}_\infty}{\Omega'^2 + \kappa^2} \left[\pm\Omega'\Delta\omega + \kappa^2 \left(1 - \frac{2\bar{n}_T(\Omega' \mp \Delta\omega)}{\Omega'} \right) \right] \right\} , \tag{3.20c}$$

$$N_\pm = \frac{-\frac{1}{2}\kappa\,\Delta\bar{n}_\infty}{\Omega'(\Omega'^2 + \kappa^2)} \left(\pm\Omega^2 + \frac{2\bar{n}_T(\Omega' \pm \Delta\omega)(\Omega'\Delta\omega \pm \kappa^2)}{\Omega'} \right) . \tag{3.20d}$$

The spectral atomic correlation function $\tilde{g}(\nu)$, which is proportional to the power spectrum of the field radiated by the atom, is thus given by Eqs. (3.19) and (3.20), in terms of the collision rate κ, the mean thermal excitation probability $\bar{n}_T$, and the parameters defined by Eqs. (2.13) – (2.15) and (3.6) – (3.8).

The expression (3.19) for the spectral density $\tilde{g}(\nu)$ resembles the result which has been found[16] for the case in which the atomic relaxation mechanism is radiative rather than collisional. An important difference between the two cases is that for radiative damping $N_0 = 0$, $M_+ = M_-$, and $N_+ = -N_-$, and hence $\tilde{g}(\nu)$ is a symmetric function centered at the driving frequency ω, i.e., $\tilde{g}(\omega + \nu) = \tilde{g}(\omega - \nu)$.

No such symmetry relation is satisfied in the case of collisional relaxation.

In the absence of a driving field ($\Omega = 0$) we have $\Omega' = \Delta\omega \equiv \omega - \omega_0$, and the only nonvanishing term among the parameters defined by Eqs. (2.13b) and (3.20) is $M_+ = 2\kappa \bar{n}_T$. The spectral density is then simply the Lorentzian function

$$\tilde{g}(\nu) = 2\kappa\bar{n}_T / [(\nu - \omega_0)^2 + \kappa^2], \quad (\Omega = 0) \qquad (3.21)$$

which is the familiar thermal spontaneous emission field.

In the low-excitation limit described by Eqs. (2.16), we find with the aid of Eqs. (3.19), (3.20), and (2.16), that the function $\tilde{g}(\nu)$ is well approximated by the expression

$$\tilde{g}(\nu) = \frac{\frac{1}{4}\Omega^2}{(\omega - \omega_0)^2 + \kappa^2} \cdot 2\pi\delta(\nu - \omega) + \left(\frac{\frac{1}{4}\Omega^2}{(\omega - \omega_0)^2 + \kappa^2} + \bar{n}_T\right) \frac{2\kappa}{(\nu - \omega_0)^2 + \kappa^2} \qquad (3.22)$$

$$\text{for} \quad \Omega \ll |z| \quad \text{and} \quad \bar{n}_T \ll 1,$$

which, like Eqs. (2.16), may be shown in the case of harmonic oscillators to be valid for arbitrary field strengths and temperatures. In addition to the thermal spontaneous emission field proportional to $\bar{n}_T$, the spectral density given by Eq. (3.22) consists of two components of equal integrated intensity, one a coherent component at the driving frequency ω, and the other a Lorentzian function of width κ, centered at the atomic resonance frequency ω_0. These two terms originate from the inhomogeneous and the homogeneous parts, respectively, of the solution for the atomic dipole moment as it is driven between collisions by the incident field.

The limit of high saturation ($\bar{n}_\infty = \frac{1}{2}$) is achieved either at very high temperatures ($\bar{n}_T = \frac{1}{2}$) or for very intense driving fields ($\Omega \gg |z|$). In either case the spectral density $\tilde{g}(\nu)$ may be approximated by a superposition of three Lorentzian functions of width κ, one centered at the driving frequency ω, and one at each of the displaced frequencies $\omega + \Omega'$ and $\omega - \Omega'$. In the high temperature limit, we find

$$\tilde{g}(\nu) = \frac{\frac{1}{2}\kappa\,\Omega^2/\Omega'^2}{(\nu - \omega)^2 + \kappa^2} + \frac{\frac{1}{4}\kappa(\Omega' + \Delta\omega)^2/\Omega'^2}{(\nu - \omega + \Omega')^2 + \kappa^2} + \frac{\frac{1}{4}\kappa(\Omega' - \Delta\omega)^2/\Omega'^2}{(\nu - \omega - \Omega')^2 + \kappa^2} \quad \text{for} \quad \bar{n}_T = \tfrac{1}{2}, \qquad (3.23)$$

while in the limit of very intense driving fields,[11] $\Omega' \to \Omega$, and we find

$$\tilde{g}(\nu) = \frac{\frac{1}{2}\kappa}{(\nu - \omega)^2 + \kappa^2} + \frac{\frac{1}{4}\kappa}{(\nu - \omega + \Omega)^2 + \kappa^2} + \frac{\frac{1}{4}\kappa}{(\nu - \omega - \Omega)^2 + \kappa^2}$$

$$\text{for} \quad \Omega^2 \gg (\omega - \omega_0)^2 + \kappa^2, \qquad (3.24)$$

a result which depends upon temperature only through the temperature dependence of the collision rate κ.

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