that the $\vec{p}\cdot\vec{A}$ correction is never the largest one. This is of course only true when $E_B \ll \hbar\omega_1$. For $\hbar\omega_1 \sim E_B$ a resonance can occur in the $\vec{p}\cdot\vec{A}$ terms which will make it dominant.

A final remark concerning the $\vec{p}\cdot\vec{A}$ contribution to the Rayleigh and Raman terms discussed in Sec. IV is warranted. The same conditions hold for those transitions. If $ka \leq 1$ but $\omega_1,\ \omega_2 \gg E_B$, the pure $A^2$ terms will dominate the Raman and Rayleigh cross sections. If, however, $\omega_1\ \omega_2 \sim E_B$, then the $\vec{p}\cdot\vec{A}$ terms will make the dominant contribution as they do in scattering experiments performed using lasers operating at visible wavelengths.

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# Relative Orbital and Magnetic Substate Amplitudes in Single-Foil Excitation of Fast Hydrogen Atoms*

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Within the limits set by our experimental apparatus, large-amplitude zero-field oscillations of H fine-structure amplitudes treated in a recent letter of Macek were not observed in polarization–versus–flight-path measurements on $H_\beta$, $H_\gamma$, and $Ly_\alpha$ emissions from suddenly excited H atoms (50- and 150-keV incident protons). Random initial phases for orbital and magnetic substates and approximately equal magnetic substate populations are indicated, in contrast to binary electron capture in gases. A successful test of the theoretical polarization of 2$s$ Stark quench radiation was also made.

## INTRODUCTION

Several interesting suggestions for exploiting coherent emission effects stemming from zero-field oscillations of H fine-structure (fs) amplitudes in fast-beam experiments were made in a recent letter of Macek.[1] Briefly, atoms are impulsively excited (~ $10^{-14}$-sec duration passage through a foil) into a mixture of coherent fs levels. Oscillations in the intensity of electric dipole lines of fixed polarization which are subsequently emitted then occur.

While the total intensity of a field-free electric dipole transition from a coherent mixture of fs levels $|JM_J\rangle$ belonging to some principal quantum state $n$ to a group of final states $n'$ does not oscillate in time, Macek shows that in principle the intensity of each polarization $p = 1, 0, -1$ undergoes multiperiodic oscillations. They sum to a nonoscillatory total, in contrast to cases where a Stark field is present.[2] The frequencies which occur are just the zero-field fs level-separation frequencies. Under plausible random-phase conditions on the initial fs amplitudes, to be discussed below, these oscillations have significant magnitude only if there is considerable departure from an isotropic distribution of initial magnetic substate population probabilities. Such departures have been indicated in theoretical[3] and experimental[4,5] binary electron-capture investigations.

This anisotropy requirement raised the possibility of an experimental test of our conjecture that some preferential alignment of initial-state angular momenta perpendicular to the beam direction might occur as a result of the large angular momentum of the incident proton in that plane.[2,6] This alignment was one of two alternatives which we invoked in explaining the appearance of just a few prominent frequencies in the total intensities of various Stark-perturbed Balmer lines under equivalent collision conditions. The other was that *d* states were somewhat preferentially populated under the prevailing collision conditions (50–150-keV protons, 10-$\mu$g/cm$^2$ C foils). The study of the polarization of the lines as a function of flight path could yield the information about the initial distribution of magnetic substates required to distinguish between these alternatives.

Within the limits set by our experimental apparatus, the work described here serves the dual purpose of searching for oscillations of significant amplitude which might be exploited in experiments like ours and of testing the alignment hypothesis. It was also convenient to test our Lyman$_\alpha$ (Ly$_\alpha$) polarization apparatus by using it to measure the polarization of Ly$_\alpha$ quench radiation from the Stark-perturbed 2*s* state as a function of electric field. Within the accuracy of our data we were able to check the theory of this polarization and make a rough comparison with the interesting experiments of Fite, Kauppila, and Ott,[7] who used substantially lower electric fields and low-energy beams.

## EXPERIMENTAL ARRANGEMENT AND RESULTS

As shown schematically in Fig. 1, a beam of protons (~1 $\mu$A) from a 0–200-keV accelerator was collimated to 3.2-mm diameter and passed through an aluminum target chamber. This chamber was surrounded by three mutually perpendicular sets of Helmholtz coils which reduced all magnetic field components to less than 0.05 G, a matter of importance since fields of $\lesssim 1$ G are known[2] to induce observable Stark oscillations. Failure to consider the importance of small stray magnetic fields may have been responsible for some confusion in the interpretation of early experiments of this type such as those of Bashkin and Beauchemin,[2] in which oscillations were observed in apparently electric-field-free circumstances.

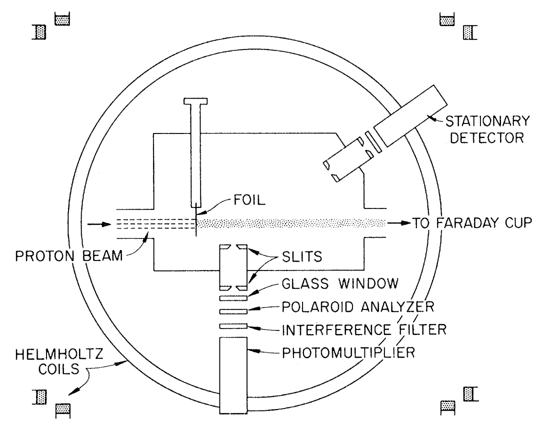


FIG. 1. Schematic diagram of the apparatus for the Balmer polarization measurements.

These authors correctly noted that electric fields due to beam space charge and collision effects due to residual background gas were largely negligible (the beam particles being ~$10^{-2}$ cm apart on the average and the residual gas pressure ~$10^{-6}$ Torr). Contact potentials and foil-charging effects were considered by them but rejected as likely candidates for producing these oscillations since it was difficult to see how fields $\gtrsim 10$ V/cm could occur. The $\vec{v}\times\vec{B}$ electric field can easily reach this magnitude, however, for hydrogen beams of a few hundred keV per amu and magnetic fields ~1 G, as was exploited in our earlier work on Stark-perturbed Balmer lines.[2] In these experiments as well no indication of foil-charging or collision effects was found. In the present experiments, in which magnetic fields were much reduced, it might be imagined that slight foil-charging effects might be important. This possibility is not borne out by our earlier observations on the high-series members $H_\delta$ and $H_\epsilon$. The important parameter in the Stark matrix elements of the perturbation Hamiltonian is not the field $F$ itself, but rather approximately $n^2F$, where $n$ is the principle quantum number; moreover, for small electric fields $F$, it is the square of this matrix element which governs the mixing. In the earlier work on the $H_\delta$ and $H_\epsilon$ lines, for which the $n^2$ factor is greater than for the $H_\beta$ and $H_\gamma$ lines studied in the present work, the photographic traces showed no distortion near the foil which might be attributed to foil-charging effects. We concluded that such effects were too small to be significant in the present experiments.

The proton beam was passed through a 10-$\mu$g/cm$^2$ carbon foil and collected by a Faraday cup located downstream from the foil. The luminous beam from excited hydrogen formed in the foil was viewed by a detection system which could be moved parallel to the beam. A stationary monitor detector measured the light emitted by a 2-cm length of the beam downstream from the foil. In the case of the Balmer-line measurements, the movable detection system consisted of two collimator slits, a rotatable Polaroid analyzer, an appropriate interference filter, and an EMI 6256 S photomultiplier (PM) tube. The current from the PM tube was amplified and then integrated. The stationary detection system was the same except it did not include an analyzer or filter. The signal from this monitor was used to normalize the output of the moving detector. For the $Ly_\alpha$ experiment, a rotatable stacked-plate (lithium fluoride) polarization analyzer[8] was used in conjunction with a $Ly_\alpha$-sensitive GM tube filled with iodine and helium whose characteristics have been described in detail by Sellin.[9] In both experiments the output from both detectors was corrected for dark current and for signal arising from collisional excitation of residual gas in the chamber. Because of the finite beam-length segment sampled (1.6 mm for $Ly_\alpha$ and 6.4 mm for the Balmer series), the upper limit of frequency resolution for any periodic phenomenon in the beam intensity was 2.0 and 3.4 GHz at 50 and 150 keV, respectively, for $Ly_\alpha$ and 0.49 and 0.86 GHz for the Balmer series. For some of the $H_\beta$ data a revised collimator system extended this upper limit to 1.7 GHz. In addition to the limit imposed by frequency resolution there is a limit imposed by the minimum-amplitude oscillations that can clearly be identified. Some direct information on this point is available from our earlier work on the Stark-perturbed $Lyman_\alpha$ line,[2] done with identical collimating geometry. From the lower curve of Fig. 2 of the second paper, we note that field-induced oscillations of amplitude $\sim\pm 3\%$ of the mean intensity are clearly visible on a trace whose oscillation frequency $\nu$ is 1.22 GHz as measured at 150-keV beam energy. If as seems reasonable one assumes that the same spatial wave-form aspect ratio for the oscillatory component can be detected at other frequency-energy combinations as well (the spatial peak separation is proportional to $E^{1/2}/\nu$), then the amplitude resolution limits for the present experiments would be $\lesssim 5\%$ ($\lesssim 10\%$ in the fractional polarization).

Outside of these experimental resolution limits, no periodic $H_\beta$ or $H_\gamma$ intensity oscillations were observed in our zero-field experiments, for either polarization. Intensity of the light polarized parallel and perpendicular to the beam direction and emerging at 90° to it was measured at each of a number of positions of the movable detector. The linear polarization fraction $P \equiv (I_\parallel - I_\perp)/(I_\parallel + I_\perp)$ at

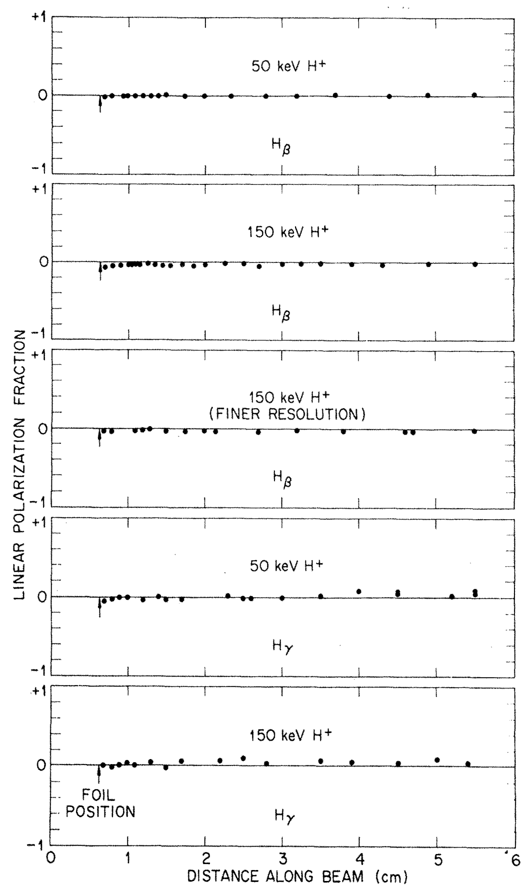


FIG. 2. Linear-polarization fractions of the $H_\beta$ and $H_\gamma$ lines as a function of distance along the beam.

each observation point along the beam is shown in Fig. 2. This plot shows that within experimental resolution $|P|$ was always $\lesssim 0.07$, and that no oscillations leading to values of $|P|$ larger than the 0.1 amplitude resolution limit were observed. It is possible that oscillations leading to $P \lesssim 0.1$ occurred but were not observed. The polarization fraction is also less sensitive to reflection effects which might yield misleading bumps on intensity-versus-distance curves. Polarization fraction data on the $Ly_\alpha$ line at zero field was also taken and is included in Table I.

The polarization of $Ly_\alpha$ quench radiation from perburbed 2$s$ atoms in the beam was measured over the range 100 to 1000 V/cm in 100-V/cm steps. For these measurements the foil was moved 50 cm upstream from the chamber. In this arrangement the allowed states had substantially decayed away by the time they reached the observation region, in which a vertical electrostatic field was introduced by a pair of plates straddling the

TABLE I. Summary of level splittings, experimental resolution, and zero-field polarization fraction data.

| Line | Beam energy (keV) | $P$ | Approximate $J'$-$J$ splittings | (GHz) | Experimental resolution (GHz) | Observable |
|---|---|---|---|---|---|---|
| $H_\beta$ | 150 | Fig. 2 | $\frac{7}{2}-\frac{5}{2}$ | 0.23 | 1.72 | Yes |
| | | | $\frac{5}{2}-\frac{3}{2}$ | 0.46 | | Yes |
| | | | $\frac{3}{2}-s_{1/2}$ | 1.24[a] | | Yes |
| | | | $s_{1/2}-p_{1/2}$ | 0.13 | | Yes |
| $H_\beta$ | 150 | Fig. 2 | $\frac{7}{2}-\frac{5}{2}$ | 0.23 | 0.86 | Yes |
| | | | $\frac{5}{2}-\frac{3}{2}$ | 0.46 | | Yes |
| | | | $\frac{3}{2}-s_{1/2}$ | 1.24 | | No |
| | | | $s_{1/2}-p_{1/2}$ | 0.13 | | Yes |
| $H_\beta$ | 50 | Fig. 2 | $\frac{7}{2}-\frac{5}{2}$ | 0.23 | 0.49 | Yes |
| | | | $\frac{5}{2}-\frac{3}{2}$ | 0.46 | | Yes |
| | | | $\frac{3}{2}-s_{1/2}$ | 1.24 | | No |
| | | | $s_{1/2}-p_{1/2}$ | 0.13 | | Yes |
| $H_\gamma$ | 150 | Fig. 2 | $\frac{9}{2}-\frac{7}{2}$ | 0.07 | 0.86 | Yes |
| | | | $\frac{7}{2}-\frac{5}{2}$ | 0.12 | | Yes |
| | | | $\frac{5}{2}-\frac{3}{2}$ | 0.23 | | Yes |
| | | | $\frac{3}{2}-s_{1/2}$ | 0.63 | | Yes |
| | | | $s_{1/2}-p_{1/2}$ | 0.07 | | Yes |
| $H_\gamma$ | 50 | Fig. 2 | $\frac{9}{2}-\frac{7}{2}$ | 0.07 | 0.49 | Yes |
| | | | $\frac{7}{2}-\frac{5}{2}$ | 0.12 | | Yes |
| | | | $\frac{5}{2}-\frac{3}{2}$ | 0.23 | | Yes |
| | | | $\frac{3}{2}-s_{1/2}$ | 0.63 | | No |
| | | | $s_{1/2}-p_{1/2}$ | 0.07 | | Yes |
| $Ly_\alpha$ | 50, 150 | $0\pm10\%$[b] at $Z=0$ | $\frac{3}{2}-s_{1/2}$ | 9.91 | 1.96, 3.44 | No |
| | | $Z=1$, $Z=2$ | $s_{1/2}-p_{1/2}$ | 1.06 | | Yes |

[a]Lamb-shift estimate included. [b]$Z$ = distance downstream from foil in cm.

horizontal H beam. The viewing geometry remained the same as in Fig. 1. The results are plotted in Fig. 3 together with the results of the theoretical calculation discussed below. Also plotted is the low-field result of Fite, Kauppila, and Ott,[7] who made a single measurement of higher accuracy and noted that there appeared to be an unexplained discrepancy between theory and experiment. We note that our cruder but more extensive measurements are within the estimated errors and that the apparent systematic deviation from theory is opposite to that of Fite, Kauppila, and Ott at our lowest fields. Though the noted discrepancy is still unresolved, our results do not corroborate a failure of the theory of 2$s$ Stark quench radiation polarization. We note that there is a tendency for our polarization measurements to have systematically smaller absolute values than predicted by theory, a circumstance common in polarization experiments in which there are nearly always some depolarizing influences. Among the sources in the present instance are (a) the finite spread in observation angle about 90° to the beam direction produced by the finite collimating geometry; (b) the possible underestimate of the few percent correction required to take account of the residual gas excitation, which would cancel out of the numerator of $P$ but not out of the denominator (the residual gas excitation was

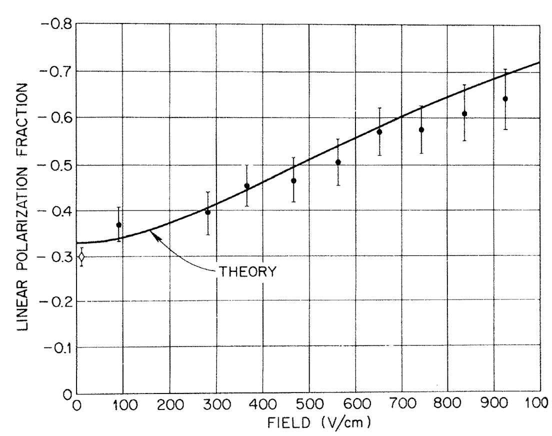


FIG. 3. Linear polarization fraction of the 2$s$-1$s$ $Ly_\alpha$ quench radiation as a function of electric field strength. The open triangle point is from Ref. 7.

measured with the foil out); (c) possible unpolarized cascade contributions to the detected radiation. For the first few cm of travel downstream from the foil, cascade contributions can amount to at most a few percent of the total intensity but tend to lower the measured polarization proportionately. The error flags in Fig. 3 are derived from estimates of the size of the correction such effects could lead to as well as from counting statistics. In view of the uncertainty in the exact size of the systematic depolarizing effects, they have not been used to adjust the measured values, but have been incorporated approximately in the error flags. In addition the counting statistical error is an increasing function of $F$, since the atoms have had to pass through a fringe field before reaching the uniform field prevailing in the viewing region. As the field is increased, an increasing fraction of the incident atoms are quenched before they reach the viewing region, resulting in a smaller signal strength. Attempts to compensate for this by increasing the beam current resulted in an unacceptable rate of foil damage. Since it is believed that purely systematic effects lowered the measured polarization, no statistical goodness of fit criterion, which would be a valid test of the theoretical formulas in the absence of such systematic effects, was applied to the data in Fig. 3.

## DISCUSSION

Although our search for oscillations of large amplitude was unsuccessful, it is our hope that some other set of collision conditions (other energies, projectiles, or targets) might give substantial polarization oscillations which could be exploited in atomic fine-structure experiments. Because of the large numbers of atomic levels involved and the tensor nature of the radiative dipole matrix elements, it is not a simple matter to directly interpret a polarization result in terms of the original collision-generated state amplitudes. Even for our small polarization results, we found it necessary to unravel the various sums that occur in expressions for the light intensity in order to interpret the results. It seems of value to set down the theoretical results in a form that can be directly used by experimenters. Nothing basically new to the quantum theory of radiation is contained in our formulas; but we hope other experimenters will find various expressions useful in interpreting their own polarization data.

We consider the dipole transition probability for a radiative transition of polarization $p$ between a coherent mixture of fs levels of some initial state $n$ and some state $n'$, summed over final fs states. We wish to display explicitly the dependence of the transition rate on the initial collision-generated amplitudes of the various fs eigenstates in the mixture. Upper case letters will be used to describe the orbital and magnetic quantum numbers of the initial state and lower case letters those of the final state. Since the collision process is dominated by Coulomb forces, we neglect spin-orbit forces in the formation of the initial state, which then can be described with the wave function

$$\psi(t=0)=\sum_{LM'_LM'_S}\beta(L,M'_L,M'_S)u(L,M'_L,M'_S)\ , \qquad (1)$$

where $\beta(L,M'_L,M'_S)$ is the initial amplitude of the spatial eigenstate $u(L,M'_L,M'_S)$, which, of course, is an eigenstate of the Coulomb Hamiltonian but not of the spin-orbit interaction.

We point out the existence of two quite distinct types of coherence. One stems from the fact that the state $u(L,M'_L,M'_S)$ is a superposition of two states of the same $M_J$, each of which develops in time as $\exp(-i\omega_J t)$, where $\omega_J$ is the $J$th-level energy divided by $\hbar$; the occurrence of cross terms in the product of these amplitudes in the transition probability leads to interferences even if the relative phases of the constants $\beta(L,M'_L,M'_S)$ are random (the quantization axis is chosen to lie along the beam direction). The second type of coherence arises if there are specific nonrandom-phase relations among the constants $\beta(L,M'_L,M'_S)$. Such phase relations lead to interferences among $J$ states belonging to various $L$ values, in contrast to the first type of coherence which involves only the $J$ states of a single $L$. Macek does not make this distinction explicit, and as a result there appears to be a slight oversight in his illustrative treatment of the electron impact Lγ polarization problem,[1] as discussed below.

The dipole transition probability for a line of polarization $p$ is proportional to the absolute square of the dipole matrix element summed over final states. The calculation of the time dependence of this matrix element can be done conveniently by first expressing the initial-state wave function in the $JM_J$ representation, using the spin-$\frac{1}{2}$ Clebsch-Gordan coefficients[10]:

$$\psi=\sum_{LJM_J}b(L,J,M_J)u(L,J,M_J)\times\exp(-i\omega_J t-\tfrac{1}{2}\gamma_{LJ}t)\ , \qquad (2)$$

where $b(L,J,M_J)=\sum_{M'_LM'_S}\beta(L,M'_L,M'_S)\times C(LSJ;M'_LM'_SM_J)\ , \qquad (3)$

and the over-all radiative decay of each $L$ state has been accounted for by the introduction of the usual phenomenological damping constants. Equation (2) is essentially the same equation as Macek's

Eq. (1).

The transition probability at time $t$ is proportional to the absolute squared dipole matrix element using Eq. (2) to describe the initial state. The matrix element calculation is simplified by reexpressing $u(L,J,M_J)$ as a linear combination of $u(L,M_L,M_S)$ states, using once again the spin-$\frac{1}{2}$ Clebsch-Gordan coefficients:

$$\alpha(L,M_L,M_S;t)=\sum_J b(L,J,M_L+M_S) \times\exp(-i\omega_J t-\tfrac{1}{2}\gamma_{LJ}t) \times C(LSJ;M_L M_S M_L+M_S) \quad ,(4)$$

where $\alpha(L,M_L,M_S;t)$ is the total coefficient of $u(L,M_L,M_S)$ in the expansion of $\psi(t)$, including time dependence $[\alpha(L,M_L,M_S;0)\equiv\beta(L,M_L,M_S)]$. The absolute squared transition matrix element for polarization $p$, summed over final states, becomes

$$\sum_{lm_lm_s}\left|\sum_{LM_LM_S}\alpha(L,M_L,M_S;t) \times(n'lm_lm_s|X^p|nLM_LM_S)\right|^2 , \qquad (5)$$

where $X^1=-(x+iy)/\sqrt{2}$, $X^0=z$ ,

and $X^{-1}=(x-iy)/\sqrt{2}$ .

The Wigner-Eckart theorem permits the factorization of this expression

$$\sum_{lm_lm_s}\left|\sum_{LM_LM_S}\alpha(L,M_L,M_S;t) \times(n'l\|X\|nL)\,C(L1l;M_Lpm_l)\right|^2 , \qquad (6)$$

where $(n'l\|X\|nL)$ is the usual reduced matrix element for the transition. The above expression vanishes unless $M_S=m_s$ and $M_L=m_l-p$. Equation (6) can be written in terms of the initial amplitudes $\beta(L,M_L',M_S')$ by means of Eqs. (3) and (4) to derive a squared transition matrix element

$$\sum_{lm_lm_s}\Big|\sum_L(n'l\|X\|nL)\,C(L1l;M_Lpm_l) \times\sum_{M_L'M_S'}\beta(L,M_L',M_S')\sum_J C(LSJ;M_L'M_S'\,M_L+M_S) \times C(LSJ;M_LM_S\,M_L+M_S)\exp(-i\omega_Jt-\tfrac{1}{2}\gamma_{LJ}t)\Big|^2 , \quad (7)$$

bearing in mind the conditions on $M_L$ and $M_S$. The total transition probability $A_p$ for radiation of polarization $p$ is proportional to this quantity. The angular distribution of the radiation has the usual form for electric dipole radiation.[11] Taking the beam direction as the $z$ axis and the direction of view as the $y$ axis, the linear polarization fraction becomes $P\equiv(I_\parallel-I_\perp)/(I_\parallel+I_\perp)=(|z|^2-|x|^2)/(|z|^2+|x|^2)$, which is evaluated using Eq. (7).

Coherency of the first type stems from cross terms in the absolute square involving the sum over $J$ for a single $L$; that of the second arises from cross terms of variable $L$. Those of the second will not occur if the phase differences among the various $\beta(L,M_L',M_S')$ are random from collision to collision; only $J$ states of a given $L$ then give oscillatory behavior. If in addition to this random-phase condition there exists an isotropic distribution of magnetic substates [$|\beta(L,M_L',M_S')|^2$ independent of $M_L'$ and of $M_S'$], then those of the first type will also vanish.

To summarize, intensity oscillations of a line of fixed polarization, when averaged over all collisions, will not be observed if the various $\beta(L,M_L',M_S')$ have random phases and if $|\beta(L,M_L',M_S')|^2$ is independent of magnetic quantum numbers. It should be noted, as Macek pointed out, that observable oscillations can occur involving an $L$ state for which $|\beta(L,M_L',M_S')|^2$ is independent of magnetic quantum numbers, if there is a definite nonrandom-phase relation between $\beta(L,M_L',M_S')$ and some amplitude describing a state of different orbital angular momentum.

It is helpful to consider the same concrete example as Macek for illustrative purposes. Consider the $Ly_\alpha$ radiation from coherently excited $2p$ states (say in an electron impact experiment), more specifically $Ly_\alpha$ polarized along the beam axis ($z$ direction). The sums over $M_L'$, $M_S'$ and $J$ can be written using the Clebsch-Gordan coefficients in the form given by Rose.[10] For $m_s=+\frac{1}{2}$ ($m_l=0$ since $l=0$), one gets

$$\exp(-\tfrac{1}{2}\gamma_pt)\{[\tfrac{1}{3}\beta(1,0,\tfrac{1}{2})][2\exp(-i\omega_{3/2}t)+\exp(-i\omega_{1/2}t)]+[\tfrac{1}{3}\beta(1,1,-\tfrac{1}{2})]\times[\sqrt{2}\exp(-i\omega_{3/2}t)-\sqrt{2}\exp(-i\omega_{1/2}t)]\} \quad . \quad (8)$$

Essentially the same equation holds for $m_s=-\frac{1}{2}$. The sums over $L$ and $l$ reduce to one term and give just a constant factor $\frac{1}{3}|(1s\|X\|2p)|^2$. Equation (8) then provides, apart from this constant factor, all of the information concerning amplitudes and phases of the various oscillatory terms which result from taking the absolute square. If the phase between $\beta(1,0,\frac{1}{2})$ and $\beta(1,1,-\frac{1}{2})$ is random, and if one defines cross sections $\sigma(|M_L'|)\equiv|\beta(L,M_L',M_S')|^2$ which are assumed to be independent of $M_S=m_s$ and of the sign of $M_L$, then apart from constants Eq. (7) becomes

$$\exp(-\gamma_pt)[5\sigma_0+4\sigma_1+4(\sigma_0-\sigma_1)\cos(\omega_{3/2}t-\omega_{1/2}t)]/9 \quad , \quad (9)$$

which agrees exactly with Eq. (4) of Ref. 1. Note that this expression predicts no oscillations for $\sigma_0=\sigma_1$, i.e., for an isotropic distribution of magnetic substates. It appears then that Macek's expression assumes random phases between $\beta(1,0,\frac{1}{2})$ and $\beta(1,1,-\frac{1}{2})$. A suitably chosen-phase relation, such as $\beta(1,1,-\frac{1}{2})=i\beta(1,0,\frac{1}{2})$ yields a quite different result of the form $c_1+c_2\sin\delta\omega t$. On the other hand, he states that even for an isotropic distribution of magnetic substates of given $L$ [$\sigma(|M_L'|)$ independent of $M_L'$], interference of these substates with those of some other orbital angular momentum can result; but this appears to require a coherency of phase with the amplitudes for these other orbital-angular-momentum states, which is in conflict with the assumptions giving his Eq. (4), i.e., our Eq. (9).

Within the resolution and systematic error limits of our experiments, the polarization of the $H_\beta$, $H_\gamma$, and $Ly_\alpha$ lines at zero field all were small (< 0.1). This is not to say that polarizations at substantially different energies, with different projectiles, or with different foil targets would also be small. Indeed, since the evidence from binary electron capture studies in gases[3–5] is that polarization is quite a common occurrence, one hopes that some other collision situation will allow the preferential creation of physically interesting fs states. In Table I we summarize the limits of resolution for which the nonoscillatory polarization results apply together with the fs splittings of the $J$ levels from one another. Frequency splittings greater than the experimental limit would have gone undetected. The bulk of the splittings are smaller, including all but one of those separating the two $J$ levels of the same $L$. Within these limits, then, it appears that the initial phases of both orbital and magnetic substates are random, and that the distribution of magnetic substates is substantially isotropic. Anisotropies of < 10% could still be accommodated by the amplitude resolution limits.

The calculation of $Ly_\alpha$ polarization involves matrix elements which are very similar to those in Eq. (7); we are now dealing with a Stark-perturbed $s_{1/2}^{\pm 1/2}$ state, which has admixtures in it of the $p_{1/2}^{\pm 1/2}$ and $p_{3/2}^{\pm 1/2}$ states which have a definite amplitude and phase relative to the $s_{1/2}^{\pm 1/2}$ part of the wave function. (The superscript denotes $M_J$.) These admixtures were calculated by the method we described fully in Ref. 2, to which the reader is referred for details. One finds $P = (|x|^2 - |z|^2)/(|x|^2 + |z|^2)$ to be

$$P = \{[(\sqrt{6})f + \sqrt{3}\,]^2 - [-(\sqrt{\tfrac{3}{2}})f + \sqrt{3}\,]^2\}/\{[(\sqrt{6})f + \sqrt{3}\,]^2 + [-(\sqrt{\tfrac{3}{2}})f + \sqrt{3}\,]^2\}\ , \qquad (10)$$

where $f = -\sqrt{2}\,[(1.058 + \Delta)/(9.911 - \Delta)]$, $\Delta$ is the calculated Stark shift of the $s_{1/2}$ level, and the values 1.058 and 9.911 GHz have been used for the zero-field $s_{1/2} - p_{1/2}$ and $s_{1/2} - p_{3/2}$ splittings.[12] These results are plotted as a function of field in Fig. 3 along with the experimental results.

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