PHYSICAL REVIEW A VOLUME 2, NUMBER 2 AUGUST 1970

Differential Cross Sections for the Elastic Scattering of 1- to 95-eV Electrons from Helium

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(Received 19 February 1970)

The extended-polarization-potential method has been used to calculate the differential cross sections for elastic electron-helium scattering. The results are presented in tabular form for collision energies between 1 and 95 eV and a selection of scattering angles from 0 to 180 deg. Total scattering cross sections are also given for all energies considered.

I. INTRODUCTION

In a recent paper,[1] the extended-polarization-potential method of Callaway *et al.*[2] was used to calculate the differential cross sections for elastic electron-helium scattering for energies between 100 and 500 eV. These calculations were found to be in good agreement with accurate experimental data.[3] A similarly good agreement with experiment was obtained in the original extended-polarization-potential paper[2] for the total scattering cross section from zero up to about 20 eV.

The main purpose of the present work is to fill the "gap" between 20 and 100 eV with predictions for the cross section using a formalism proven to give good results at either end. In addition, since a greater quantity of information is contained in the differential cross section, we have used the extended-polarization-potential method to calculate accurately the differential cross section through an energy range from 1 eV up to our previous calculations at 100 eV and higher. A quantitative comparison of our differential cross section with experiment is not possible in the energy range 20–100 eV since there are no accurate experimental data available at present. Some experimental data are available below 20 eV and comparison of this with our calculations has already been made by Gibson and Dolder[4] and Bransden and McDowell.[5] We will thus present here only a cursory comparison of our calculations with experimental data. This is given in Sec. III following a brief description of the details of our calculations.

II. CALCULATIONS

The extended-polarization-potential formalism is thoroughly discussed in Ref. 2. In brief, it consists of approximating the mutual distortion interaction by the sum of the adiabatic polarization potential plus the leading nonadiabatic correction which is termed the distortion potential. This sum, which is called the extended polarization potential, is then added to the direct part of a Hartree-Fock single-channel scattering equation for the wave function of the scattered electron. This single-channel equation is solved for the scattering wave function from which the scattering phase shifts are extracted.

The calculations for the present work parallel closely those used for the higher-energy calculations in Ref. 1. To obtain accurate differential cross sections, a great many partial-wave phase shifts are required. Because of the presence of the centrifugal potential, exchange effects become negligible for the higher partial waves. Thus, we solved the single-channel scattering equation (numerically) for only the lower partial waves and then estimated the higher partial waves by use of the Born and semiclassical approximations as discussed in Ref. 1. In particular, for energies up to 10 eV, we obtain the first eight phase shifts ($l=0$ to 7) from the scattering equation and then used the approximations for the next forty-three ($l=8$ to 50) phases. For energies greater than 10 eV, we solved the scattering equation for the first eleven phase shifts ($l=0$ to 10) and then approximated the remaining ($l=11$ to 50) phases. However, to obtain accurate differential cross sections at zero scattering angle (converged to at least 1%) we again found that more than 51 phase shifts were required for energies greater than 6 eV and in these cases, we included all (estimated) phase shifts through $l=10\,007$.

III. RESULTS AND DISCUSSION

The phase shifts calculated by the extended-polarization-potential method were used in the standard formulas to obtain the differential and total scattering cross sections for electron-helium scattering.[6] The results of these calculations are listed in Table I ($a_0=\hbar^2/me^2$, the atomic unit of length).

The available experimental data concerning the differential cross section for helium at low energies are contained in the following five publications: (a) Ramsauer and Kollath,[7] who considered an energy range from 1.8 to 19.2 eV; (b) Bullard and Massey,[8] 4 to 50 eV; (c) Hughes, McMillen, and Webb,[9] 25 on to 700 eV; (d) Westin[10] for energies between 49 and 155 eV but only for scattering angles between 90 and 180 deg; and (e) the most recent, by Gibson and Dolder[4] for energies from 3.1 to 19.1 eV. Of these five, only (a), (d), and (e) report absolute cross sections while (b) and (c) give relative values.

In Figs. 1–3, we have plotted our calculations in comparison with experimental data for three representative energies. The relative data of Bullard and Massey in Figs. 1 and 3 have been normalized so as to match our calculations at 90 deg. The relative data of Hughes, McMillen, and Webb in Fig. 3 have been normalized to give a good fit to our calculations for angles greater than 60 deg.

The measurements by Ramsauer and Kollath[7] have long been suspect since these display excessive forward scattering at low energies (Fig. 1). A recent phase-shift analysis of their data by Hoeper, Franzen, and Gupta[11] has shown that the effective higher partial-wave phases would have to be negative to reproduce the low-energy data. However, the higher partial waves are dominated by the polarization interaction, and since this is an attractive interaction the phases should be positive. Thus, the Ramsauer-Kollath data should be discarded in any serious analysis.

Our calculations are in qualitative agreement, in general, with the (suitably normalized) data of Bullard and Massey[8] and Hughes, McMillen, and Webb.[9] The latter data have previously been anal-

TABLE I. Differential and total cross sections for elastic electron-helium scattering.

E(eV) =	1	2	3	4	5	6	7	8	9
$\sigma(a_0^2)$ =	20.77	20.49	19.89	19.20	18.50	17.81	17.14	16.49	15.87
θ(deg)	$d\sigma/d\Omega$ (a_0^2/sr)								
0	0.791	0.625	0.612	0.688	0.816	0.970	1.135	1.301	1.462
5	0.832	0.662	0.636	0.695	0.803	0.936	1.081	1.227	1.368
7.5	0.853	0.681	0.649	0.700	0.798	0.922	1.057	1.193	1.324
10	0.874	0.701	0.663	0.706	0.795	0.910	1.035	1.162	1.284
15	0.918	0.744	0.695	0.722	0.793	0.889	0.996	1.104	1.209
20	0.963	0.789	0.731	0.743	0.798	0.876	0.964	1.055	1.144
25	1.010	0.838	0.770	0.769	0.808	0.869	0.941	1.015	1.088
30	1.058	0.888	0.814	0.800	0.824	0.869	0.925	0.984	1.042
45	1.209	1.057	0.968	0.923	0.908	0.912	0.925	0.943	0.962
60	1.367	1.245	1.154	1.089	1.045	1.015	0.994	0.978	0.964
90	1.686	1.653	1.587	1.512	1.436	1.362	1.292	1.225	1.162
115	1.925	1.978	1.952	1.888	1.805	1.713	1.617	1.521	1.428
135	2.077	2.193	2.200	2.149	2.067	1.966	1.857	1.744	1.633
155	2.182	2.342	2.374	2.335	2.255	2.150	2.032	1.909	1.786
180	2.230	2.412	2.456	2.423	2.344	2.237	2.116	1.988	1.858
E(eV) =	10	15	20	25	30	35	40	45	50
$\sigma(a_0^2)$ =	15.28	12.66	10.62	9.04	7.79	6.80	6.00	5.35	4.81
θ(deg)	$d\sigma/d\Omega$ (a_0^2/sr)								
0	1.614	2.203	2.551	2.748	2.858	2.918	2.950	2.963	2.969
5	1.501	2.005	2.284	2.423	2.486	2.505	2.499	2.480	2.453
7.5	1.448	1.912	2.159	2.274	2.316	2.318	2.299	2.267	2.229
10	1.399	1.827	2.047	2.140	2.165	2.153	2.122	2.081	2.035
15	1.307	1.664	1.832	1.888	1.884	1.849	1.800	1.744	1.686
20	1.226	1.520	1.644	1.668	1.642	1.592	1.531	1.467	1.403
25	1.156	1.392	1.478	1.477	1.435	1.374	1.306	1.238	1.172
30	1.096	1.278	1.331	1.311	1.257	1.189	1.118	1.048	0.983
45	0.980	1.025	1.001	0.940	0.867	0.792	0.723	0.659	0.602
60	0.952	0.887	0.806	0.720	0.639	0.566	0.503	0.449	0.402
90	1.103	0.860	0.685	0.555	0.458	0.383	0.324	0.278	0.240
115	1.339	0.971	0.719	0.548	0.429	0.344	0.281	0.233	0.197
135	1.526	1.075	0.770	0.568	0.431	0.336	0.269	0.219	0.182
155	1.665	1.157	0.812	0.587	0.438	0.336	0.265	0.213	0.175
180	1.732	1.196	0.833	0.597	0.441	0.336	0.263	0.210	0.172

TABLE I. (*continued*)

E(eV) =	55	60	65	70	75	80	85	90	95
$\sigma(a_0^2)$ =	4.36	3.97	3.64	3.35	3.10	2.89	2.69	2.52	2.37
θ (deg)				$d\sigma/d\Omega$ (a_0^2/sr)					
0	2.966	2.958	2.947	2.934	2.921	2.907	2.893	2.879	2.866
5	2.422	2.389	2.355	2.321	2.287	2.254	2.222	2.191	2.161
7.5	2.188	2.147	2.105	2.064	2.024	1.985	1.948	1.912	1.877
10	1.987	1.939	1.892	1.846	1.802	1.760	1.719	1.681	1.644
15	1.629	1.573	1.520	1.468	1.420	1.374	1.330	1.289	1.250
20	1.341	1.282	1.227	1.174	1.126	1.080	1.037	0.997	0.959
25	1.109	1.051	0.996	0.946	0.899	0.856	0.815	0.778	0.743
30	0.921	0.865	0.813	0.766	0.722	0.682	0.646	0.612	0.581
45	0.551	0.506	0.466	0.430	0.398	0.370	0.344	0.322	0.301
60	0.361	0.326	0.296	0.269	0.246	0.226	0.208	0.192	0.178
90	0.210	0.184	0.163	0.146	0.130	0.118	0.106	0.097	0.088
115	0.168	0.145	0.126	0.110	0.098	0.087	0.077	0.070	0.063
135	0.153	0.130	0.112	0.097	0.085	0.075	0.066	0.059	0.053
155	0.145	0.123	0.105	0.090	0.079	0.069	0.061	0.054	0.048
180	0.142	0.120	0.102	0.088	0.076	0.066	0.059	0.052	0.046

yzed and compared with a similar set of calculations by Khare and Moiseiwitsch.[12] In these calculations, only the adiabatic dipole polarization potential was included and only the first eight ($l=0$ to 7) partial-wave phase shifts calculated. As a consequence, Khare and Moiseiwitsch's calculations tend to underestimate the forward scattering while the Hughes, McMillen, and Webb data seem to again show excessive forward scattering.

The agreement between our calculations and the absolute data of Westin[10] and Gibson and Dolder[4] is quite good. There are relatively few points of comparison with Westin's data and the relative accuracy of these data has not been assessed. Our calculations agree best with the small-angle data of Gibson and Dolder but tend to be somewhat higher than these data, in general, especially at larger angles. A more quantitative comparison is given in Ref. 4.

Finally, a recent phase-shift analysis by Bransden and McDowell[5] has yielded estimates for the phases which are consistent with a wide range of experimental data and the dispersion relation. Our s- and p-wave phase shifts compare very favorably with this analysis while our d-wave phase shift is somewhat out of line.

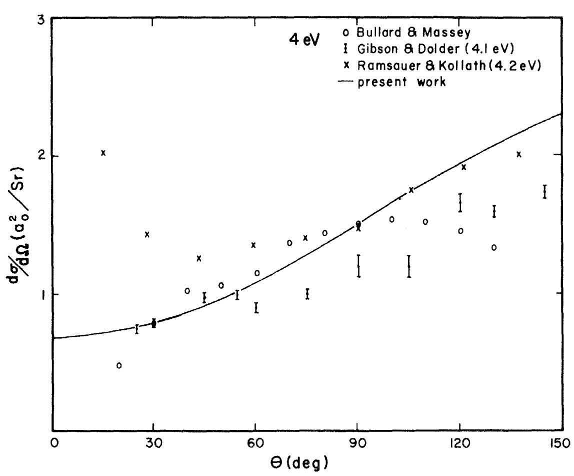

FIG. 1. Differential cross sections for the scattering of 4-eV electrons from helium. The solid curve is the result of the present extended-polarization-potential calculations. The experimental data are from Bullard and Massey (Ref. 8), Gibson and Dolder (Ref. 4), and Ramsauer and Kollath (Ref. 7).

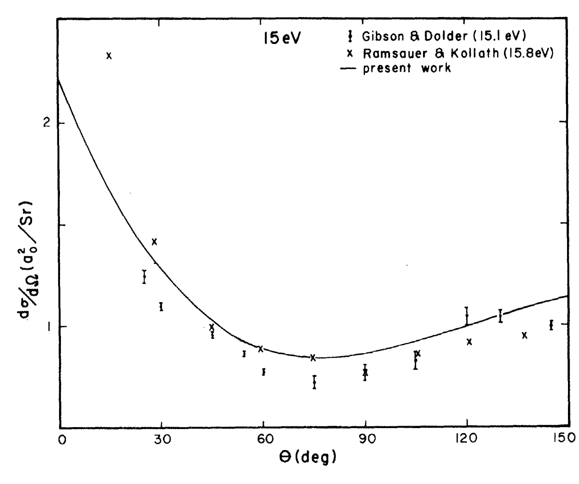

FIG. 2. Diffential cross sections for the scattering of 15-eV electrons from helium. (See caption to Fig. 1 for identification of the data.)

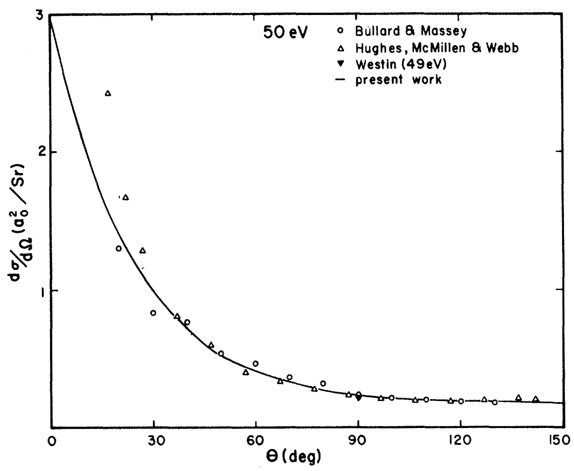

FIG. 3. Differential cross sections for the elastic scattering of 50-eV electrons from helium. The experimental point by Westin is from Ref. 10. (See caption to Fig. 1 for identification of the remaining data.)

In conclusion, we feel that our calculated differential cross sections give a good over-all estimate of the elastic electron-helium differential cross section. A reliable estimate of the error can not be given at this date since there are some apparent normalization problems in the most recent (and accurate) experimental data of Gibson and Dolder.[13] However, we feel that an accuracy estimate of ±5% is a reasonable guess at the discrepancies between our results and the actual cross sections.

ACKNOWLEDGMENTS

The authors would like to thank Dr. K. Dolder and Professor M.R.C. McDowell for useful communications on their work.

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[6]A supplement to this paper listing our calculated phases shifts for $l = 0$ to 50 at all energies considered between 1 and 500 eV will be sent to anyone interested upon receipt by the authors of a written request.

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[13]See, for example, Fig. 7(c) of Ref. 5 where the plot of Gibson and Dolder's data at 90° versus energy shows an anomalous dip between 5 and 10 eV.