Coupling between Pressure and Temperature Waves in Liquid Helium†

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When the thermal expansion coefficient (α) is retained in the linearized hydrodynamic equations of the two-fluid model of superfluid liquid helium, pressure and temperature waves are not independent. Thus a periodically varying temperature source produces not only temperature waves in liquid helium, but also two pressure waves p_1' and p_2', which propagate at the velocity of first and second sound, respectively. Similarly, a vibrating diaphragm produces not only pressure waves but also two temperature waves T_1' and T_2', which also propagate at the velocity of first and second sound, respectively. Lifshitz has shown that the amplitudes of these cross modes should be proportional to α. An investigation of this coupling has been made by observing and studying the two pressure waves p_1' and p_2' produced by a heater and the temperature wave T_2' produced by a capacitor microphone. The temperature dependence of the amplitude of these waves has been studied using both pulse and standing-wave techniques in the temperature range from 1.2 K to the λ point. Good agreement with theory has resulted only for the p_1' mode.

I. INTRODUCTION

In seeking plane-wave solutions to the linearized hydrodynamic equations based on the two-fluid model of liquid ^{4}He, it is customary to neglect terms containing the thermal expansion coefficient α. In this case one arrives at the well-known result[1] that there exist two independent wave solutions corresponding to two different propagation velocities. One solution represents first sound, in which pressure and density fluctuations propagate with the velocity $u_1 = [(\partial p/\partial\rho)_S]^{1/2}$, where p is the pressure, ρ is the density, and S is the specific entropy. In the other solution, which is called second sound, temperature and entropy fluctuations propagate with the velocity $u_2 = [(TS^2/C)(\rho_s/\rho_n)]^{1/2}$, where T is the absolute temperature, C is the specific heat, and ρ_s and ρ_n are the density of the superfluid and normal fluid components, respectively. From a microscopic viewpoint, second sound is density fluctuations in the quasiparticle or excitation gas of the system.

When small terms containing the thermal expansion coefficient are retained in solving the hydrodynamic equations, these two wave modes are no longer independent, that is, there exist both pressure and temperature fluctuations which propagate with each velocity u_1 and u_2. The theoretical treatment of this coupling is due to Lifshitz.[2] As already indicated, the starting point is the set of linearized two-fluid hydrodynamic equations in the following form[3]:

$$\frac{\partial\rho}{\partial t} + \vec{\nabla}\cdot\vec{j} = 0 \tag{1a}$$

$$\frac{\partial(\rho S)}{\partial t} + \rho S\vec{\nabla}\cdot\vec{v}_n = 0, \tag{1b}$$

$$\frac{\partial \vec{j}}{\partial t} + \vec{\nabla}p = 0, \tag{1c}$$

$$\frac{\partial\vec{v}_s}{\partial t} + \left(\frac{1}{\rho}\right)\vec{\nabla}p - S\vec{\nabla}T = 0. \tag{1d}$$

In these equations v_s and v_n are the velocities of the superfluid and normal fluid components, respectively. The mass flux $\vec{j}$ is given by

$$\vec{j} = \rho_s\vec{v}_s + \rho_n\vec{v}_n \quad . \tag{2}$$

No dissipative terms are included in these equations. Lifshitz[2] used these equations to calculate the amplitudes of plane-wave pressure and temperature fluctuations produced in liquid He by various types of transmitters. The relative amplitudes of these pressure and temperature fluctuations depend upon the boundary conditions at the transmitter. For instance, consider a plane surface whose temperature is varying as $T = T_0 e^{i\omega t}$. The boundary conditions in this case require that the normal component of the mass flux vanishes at the surface and also that the temperature of the helium adjacent to the surface equals that of the surface. Under these conditions the amplitudes of the pressure and temperature waves produced in the liquid He are

$$p_1' = (\alpha\rho u_1 u_2)T_0 , \tag{3a}$$

$$T_1' = (\alpha^2 T u_1 u_2/C)T_0 , \tag{3b}$$

$$p_2' = (-\alpha\rho u_2^2)T_0 , \tag{3c}$$

$$T_2' = (1-\alpha^2 T u_1 u_2/C)T_0 . \tag{3d}$$

The notation here is that a primed quantity represents the amplitude of the fluctuation of that quantity from its equilibrium value and the subscript 1 or 2 indicates that the fluctuation propagates at the velocity u_1 or u_2, respectively. In this case, the predominant mode is T_2' which is the only one which, to lowest order, is not proportional to α. Thus this situation is more favorable to the production of second sound.

The second situation which we want to consider is that of a plane solid surface vibrating perpendicular to itself with velocity given by $u = u_0 e^{i\omega t}$. The boundary conditions in this case require that the normal components of the velocities v_s and v_n of the superfluid and normal fluid adjacent to the surface equal that of the surface. Under these conditions the amplitudes of the pressure and temperature waves produced in the liquid He are

$$p_1' = (\rho u_1 - \alpha\rho u_1 u_2^2/S)u_0 \ , \qquad (4a)$$

$$T_1' = (\alpha T u_1/C)u_0 \ , \qquad (4b)$$

$$p_2' = (\alpha^2 \rho T u_2^3/C)u_0 \ , \qquad (4c)$$

$$T_2' = (\alpha T u_2/C)u_0 \ . \qquad (4d)$$

This situation is more favorable to the production of first sound, p_1' being the only mode which, to lowest order, is not proportional to α.

We present here an experimental investigation of the smaller wave amplitudes which are proportional to α in the two situations described above. Specifically, we have observed and investigated the temperature dependence of the two pressure waves p_1' and p_2', produced by a heater, and the temperature wave T_2', produced by an ordinary microphone which corresponds to the second situation discussed above.

The pressure waves produced in liquid He by a periodically heated resistor have been studied by other investigators. Jacucci and Signorelli[4] have optically observed p_2' by the Debye-Sears effect. Eynatten *et al.*[5] have also observed this wave mode with a magnetic microphone. Finally, we recently became aware of the work of Hofmann *et al.*[6] in which they observed and studied both p_1' and p_2'. None of the above results agree with each other or with the results predicted by Lifshitz's theory. The results of our independent measurements of p_1' and p_2' are in substantial agreement with those of Hofmann *et al.*[6]

II. EXPERIMENTAL

A. Measurement of p_1' and p_2' Produced by a Heater

Figure 1 shows the experimental chamber used to investigate the two pressure waves produced by a heater. It consists of a cylindrical nylon chamber approximately 5.7 cm long with 2.0 cm i.d. One end of the chamber is closed by a 1000 Ω per square carbon disk resistor[7] which serves as the heater. The other end is closed by a capacitor microphone which consists of a 0.000 25-in.-thick Mylar[8] diaphragm with a conductive aluminum film on one side, stretched over a brass back plate. The back plate

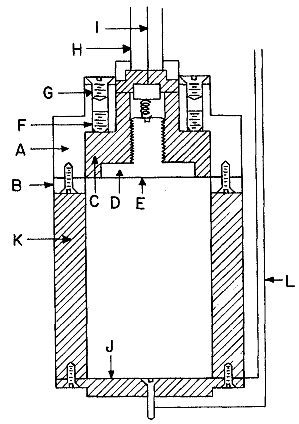

FIG. 1. Experimental chamber. A: Outer brass housing; B: brass ring which holds the Mylar diaphragm down; C: nylon insulator; D: brass back plate; E: aluminized M Mylar diaphragm; F: set screws to adjust the tension in the diaphragm; G: screws holding the whole chamber assembly to the support rod; H: stainless-steel support rod; I: copper electrical lead which is connected to the back plate via a metal spring; J: carbon disk; K: nylon tube; L: electrical leads to carbon disk.

and aluminum film form the two plates of a capacitor with the Mylar dielectric in between. Small grooves were made on the flat surface of the brass ring which holds the Mylar diaphragm down. This provided passages for liquid He to fill the chamber. The chamber is immersed in a bath of liquid He and supported by a 0.25-in.-o.d. thin-walled stainless-steel tube which also serves as the outer conductor of a coaxial cable. The inner conductor is a thin copper wire which is connected to the back plate of the microphone via a metal spring.

A dc bias voltage of 270 V was applied between the back plate and aluminum film. The ac output voltage of the microphone, resulting from pressure waves impinging on the diaphragm, was amplified by a variable Q frequency selective amplifier and then displayed on an oscilloscope.

Both pulse and standing-wave techniques were used to study the pressure waves p_1' and p_2' produced by the heater.

In the pulse technique the input to the carbon disk resistor was a pulse of current consisting of two cycles of a 5.0-kHz signal. A heat pulse consisting of 4 cycles of a 10-kHz wave was then transmitted into the liquid He. The input power to the heater during a pulse was 25 mW/cm^2. This was the maximum power input which could be used in order to remain in the linear region of received signal versus input power.

The output signal from the microphone was amplified with the frequency selective amplifier tuned to 10 kHz and the Q value set at 1. At this low Q the received pulse was distorted only slightly, and

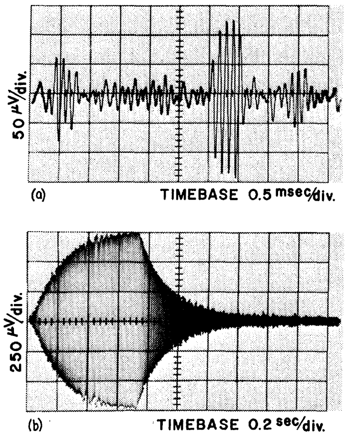

FIG. 2. (a) Oscilloscope trace of a typical received signal in the pulse measurements. The input to the carbon disk transmitter was 2 cycles of 5-kHz sinusoidal current. The temperature was 1.25 K. (b) Oscilloscope trace from a standing-wave measurement of p_2'. The input to the carbon disk was 64 cycles of 90.0-Hz sinusoidal current. The temperature was 1.70 K.

enough filtering was obtained to eliminate most low-frequency noise due to vibrations. The oscilloscope trace was triggered simultaneously with the application of the input signal. Figure 2(a) is an example of the type of signal which was received. The time base on this photograph is 0.5 msec per large division and the vertical scale is 50 μV per large division. The times of flight of the first and second received pressure pulses verify that they propagated at the velocities of first and second sound, respectively. Subsequent reflections of the first pulse were reduced considerably by drilling many small holes through the carbon disk transmitter and backing. This was necessary because otherwise the reflections of p_1' interfered with the initial p_2' pulse. The smaller pulse appearing after the p_2' pulse is due to first-sound or p_1' waves being produced at the receiver diaphragm when it is caused to vibrate by the p_2' pulse.[6]

With the standing-wave technique, the carbon disk transmitter was driven continuously with ac current and the frequency varied until a standing-wave resonance of either p_1' or p_2' was observed. In this case a solid transmitter with no holes drilled in it was used. Usually the fundamental resonance, corresponding to a wavelength equal to twice the length of the cavity, was used. As 1.5 K, for waves propagating at u_2 this corresponded to a frequency of approximately 200 Hz and for waves propagating at u_1 the frequency was about 2 kHz.

It turned out that this method was satisfactory for making quantitative measurements of p_2' but not for p_1'. This is because other resonances, which are believed to be higher harmonics of p_2', were observed at frequencies very close to the frequency of the fundamental resonance of p_1'. At some temperatures it was difficult to distinguish these resonances from one another. This also made measurements of the Q of the cavity for the p_1' resonance impossible. As will be discussed below the measurement of the Q of the cavity at each resonance is necessary in this technique.

When the frequency of a sound source in a small enclosure equals one of the normal-mode frequencies of the enclosure, the amplitude of the sound pressure will be directly proportional to the output of the source and inversely proportional to a damping constant k which is due to absorption.[9] When the driving frequency does not coincide with, but is close to, the normal frequency, the sound pressure builds up according to a standard resonance curve. The width of this curve at the points where the amplitude is down by $1/\sqrt{2}$ from its value at the normal frequency is proportional to k.

In the standing-wave technique we, therefore, measured the amplitude of the fundamental resonance at each temperature and also the width of the frequency response curve around each resonance. In this way we were able to study the temperature dependence of the output of the source. The width of a resonance curve was obtained by measuring the amplitude at points off resonance and simultaneously measuring the frequency of the driving oscillator with a frequency counter. This method of taking into account the damping constant k was rather tedious. A less time-consuming method will now be described.

In this method the driving oscillator was tuned to the resonant frequency but then gated on and off as in the case of the pulse technique. Now, however, many more cycles were included so that the resonance could build up to its maximum value before the oscillator was turned off. Figure 2(b) shows the response of the cavity to this type of driving current. The resonance dies out with a characteristic exponential envelope e^{-kt}. From this type of photograph we can then measure the damping constant k and also the amplitude.

The signal shown in Fig. 2(b) was the result of amplification of the microphone output by the frequency selective amplifier tuned to the resonant frequency and with the Q value set at 1. Higher Q values could not be used in this case because of the longer time constants associated with the filtering circuitry of the amplifier. However, in the first method in which the width of the resonance curve was measured, the Q of the amplifier could be set at 100. In that case virtually all of the noise was eliminated.

B. Measurement of T_2' Produced by a Microphone

Initially we attempted to observe the temperature waves produced by a microphone by merely interchanging the transmitter and receiver in the apparatus already described (Fig. 1). The capacitor microphone was driven with ac voltage and the carbon disk resistor was used in the usual manner as a second-sound receiver. The results were negative because this carbon disk receiver was not sensitive enough to detect the small temperature fluctuations. We succeeded in observing the second-sound temperature fluctuations T_2', produced by the capacitor microphone with a new type of second-sound receiver.

This receiver is identical to the capacitor microphone already described except that the aluminized Mylar diaphragm is replaced by a diaphragm made from a porous filter material[10] on one side of which we vacuum deposited a conducting layer of aluminum. The superfluid component of the liquid He can flow through this porous diaphragm unimpeded while the normal fluid motion is impeded. In the first approximation, second sound consists of the superfluid and normal-fluid components oscillating 180° out of phase in such a way that the net density or pressure fluctuation is zero.[1] Thus a second-sound wave impinging on the porous diaphragm will exert a force on it.

This type of transducer has been used to study the velocity of second sound close to the λ point[11] and also to study second sound in ^3He-^4He mixtures below 0.3 K.[12] In these experiments the principal advantage of this transducer was that it was an efficient transmitter and receiver of second sound without introducing heating. A detailed study of these transducers has recently been made by Sherlock and Edwards.[13]

We conducted an experiment to test the sensitivity of these transducers. The details of this experiment will briefly be described in Sec. III B. The results were that, in addition to the above-mentioned

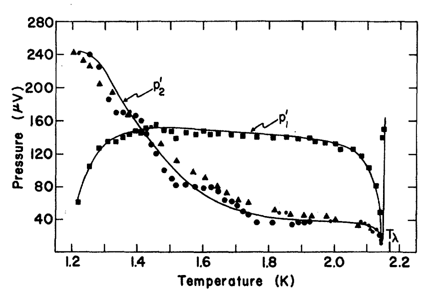

FIG. 3. Amplitude of the received signal from the capacitor microphone versus temperature. The symbols are: squares, p_1' pulse data; circles, p_2' pulse data; triangles, p_2' standing-wave data. The p_2' standing-wave data have been reduced by a constant factor so that they could be plotted on the same scale as the pulse data. The solid lines are smooth curves down through the data points.

desirable properties, these transducers are also much more sensitive to second sound temperature fluctuations than ordinary resistance thermometer receivers.

The measurements of T_2' were made in the experimental chamber previously described with the ordinary capacitor microphone replacing the carbon disk as transmitter and with the porous diaphragm transducer as receiver. Even with the increased sensitivity of this receiver, the T_2' signal could only be observed with the standing-wave technique. The microphone transmitter was driven with 30-V-peak ac voltage. In addition a 180-V dc bias was applied in order to increase the output. Again, the amplitude of the fundamental resonance and the width of the frequency response curve were measured as a function of temperature.

III. RESULTS

A. Pressure Waves p_1' and p_2' Produced by a Heater

Figure 3 shows a plot of p_1' data from the pulse measurements and p_2' data from both the pulse and standing-wave measurements. The ordinate is the amplitude of the ac output voltage of the microphone from the pulse experiments. The output signal from the standing-wave experiment was much larger than that from the pulse experiments. Thus in Fig. 3 the standing-wave data for p_2' have been reduced by a constant factor in order that they could be displayed on the same graph as the pulse data.

Before comparing these data with the theoretical expressions of Eqs. (3a) and (3b) we must consider the following: From the hydrodynamic Eqs. (1a)-(1d) and the relation $\dot{Q} = \rho S T v_n$, it can be shown[14] that the amplitude of a second-sound wave is given by

$$T_2' = \dot{Q}/\rho C u_2 , \tag{5}$$

where $\dot{Q}$ is the energy flux associated with the wave. Thus the carbon disk heater with constant input power will not produce temperature swings whose amplitude is independent of temperature in the adjacent liquid He. In our experiment $\dot{Q}$ is independent of temperature. Therefore T_0 in Eqs. (3a) and (3b) varies with temperature.

We verified Eq. (5) by performing an experiment in which we replaced the capacitor microphone of Fig. 1 with a 1000 Ω per square carbon disk resistor to act as a receiver of the second-sound waves produced by the transmitter. The input power was the same as was used in the measurements of p'_1 and p'_2, produced by the same transmitter. The temperature dependence of the amplitude of the received second-sound waves agreed reasonably well with Eq. (5).

Figures 4 and 5 show the results of dividing the p_1' and p_2' data, respectively, of Fig. 3 by the data of the experiment described in the preceding paragraph. Since the temperatures of the data points

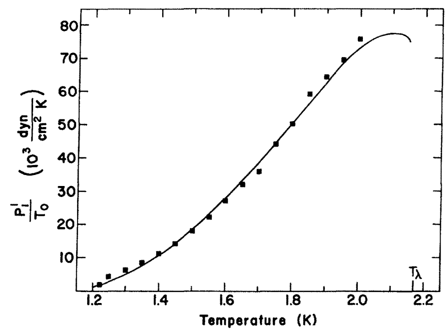

FIG. 4. Data points are the result of dividing the p_1' data of Fig. 3 by the second-sound amplitude data from the experiment described in Sec. III A. The solid line is the theoretical curve of p_1'/T_0 from Eq. (3a). The data have been normalized to agree with the theoretical curve at 1.4 K.

in the two experiments did not coincide, the points used for Figs. 4 and 5 were taken from smooth curves drawn through the data. In Fig. 4 the solid line is the theoretical curve of p_1'/T_0 given in Eq. (3a). The capacitor microphone was not calibrated, so the experimental data have been normalized to agree with theory at one point in order that the temperature dependence could be compared with the theory. Reasonably good agreement with the theory is indicated.

Figure 3 shows a sharp increase in p_1' just before the λ point was reached. Our temperature measurements were not sophisticated enough to determine with great accuracy at what temperature this sharp increase in p_1' occurred but we can say that it was definitely below the λ point. This phenomenon is not predicted by the Lifshitz theory.[2] We believe it to be associated with nonlinearities near the λ point.

In Fig. 5 the solid line is the theoretical curve of p_2'/T_0 given in Eq. (3b). The dashed line represents a correction to Eq. (3b) due to the viscosity of the normal fluid and the coefficient of second viscosity. This correction will be discussed in greater detail in Sec. IV A. Here the data have been normalized to agree with theory at the maximum in order that the temperature dependence could be compared with theory. In this case disagreement between the experimental data and the theory is apparent.

There is a more serious disagreement between theory and experiment. An inspection of Eqs. (3a) and (3b) shows that the ratio of p_2' to p_1' should be

$$p_2'/p_1' = u_2/u_1 \ . \qquad (6)$$

In the temperature range of our investigations $u_2 < 0.1u_1$, so that p_2' should be less than $0.1p_1'$. However, our measurements indicate that p_2' is the same order of magnitude as p_1', as may be seen from Fig. 3. In addition, the thermal expansion coefficient of liquid He changes sign at 1.17 K. Therefore, according to Eqs. (3a) and (3b), p_1' and p_2' should be zero at this point. Although we could not quite reach this temperature with our apparatus, our results on Figs. 3 and 5 would indicate that p_2' is not zero at this point although p_1' is.

B. Temperature Wave T_2' Produced by a Microphone

The second-sound waves produced by an ordinary microphone were observed by using the previously described porous diaphragm transducer as receiver. The sensitivity of this transducer was investigated by using it to receive second-sound waves produced by the carbon disk heater which had been used as the transmitter in the measurements of p_1' and p_2'. Again the input power was the same as that used in the investigation of p_1' and p_2'. The resulting output signal of this transducer was then compared with the output signal of the carbon disk receiver in the second-sound experiment described in Sec. III A.

The results showed that the porous diaphragm transducer was approximately 500 times more sensitive to second-sound temperature fluctuations than the carbon disk receiver. In addition, comparison of the temperature dependence of the output signal of the porous diaphragm transducer with Eq. (5) indicated that the sensitivity was only slightly temperature dependent.

Figure 6 shows the results of the standing-wave measurements of T_2' produced by the regular capacitor microphone and received by the porous diaphragm transducer. The solid line is the theoretical curve of T_2'/u_0 given by Eq. (4d). The dashed line is, as before, the theoretical curve of T_2'/u_0 which results when the contribution from the viscous forces at the chamber walls are taken into account. This

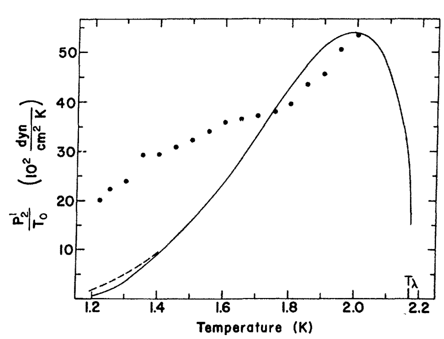

FIG. 5. Data points are the result of dividing the p_2' data of Fig. 3 by the second-sound amplitude data from the experiment described in Sec. III A. The solid line is the theoretical curve of p_2'/T_0 from Eq. (3b). The dashed line is a correction due to viscosity which is discussed in Sec. IV A. The data have been normalized to agree with the theoretical curve at the maximum near 2.0 K.

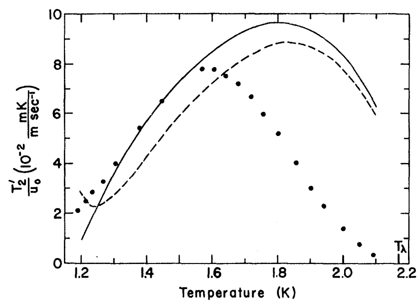

FIG. 6. Experimental data are the amplitudes of the signal from the porous diaphragm transducer. It was used as the receiver of T_2' waves produced by a regular capacitor microphone. The solid line is the theoretical curve of T_2'/u_0 from Eq. (4d). The dashed line is the theoretical curve from Eq. (8). The difference is a correction due to viscosity which is discussed in Sec. IV. The data have been normalized to agree with the theory at 1.4 K.

correction will be discussed in more detail in Sec. IV B. The magnitude of u_0 was not known, so the experimental data have again been normalized in order that the temperature dependence could be compared with the theory. As indicated there is a fair agreement at lower temperatures but a large discrepancy at the higher temperature.

In this experiment, plane-wave resonances corresponding to waves which propagate at the velocity of first sound u_1 were also observed. The amplitudes of these resonances were an order of magnitude larger than the T_2' resonances. Since the porous diaphragm receiver will also respond to pressure waves it is not clear whether these are resonances of p_1' of Eq. (4a), or of T_1' of Eq. (4b), or both. To answer this question we would need to know the relative sensitivity of the porous diaphragm transducer to pressure and temperature waves. The amplitude of these u_1 resonances varied little with temperature. This would seem to indicate that they are resonances of p_1' of Eq. (4a) since ρu_1 is relatively constant in temperature compared to the corresponding coefficient for T_1' given in Eq. (4b).

IV. DISCUSSION

A. Pressure Waves p_1' and p_2' Produced by a Heater

The results of the measurements of p_1' and p_2' produced by a heater indicate that there are contributions to p_2' in addition to that obtained by merely retaining the thermal expansion coefficient in the two-fluid hydrodynamic equations. We recall that the theoretical expressions, Eqs. (3a) and (3b), for p_1' and p_2' were obtained from the set of hydrodynamic Eqs. (1a)–(1d) in which no dissipative terms were included. We should like now to examine the effect on the coupling of retaining some of these dissipative terms.

Dingle,[15] using the method of Lifshitz,[2] has calculated the coupling coefficients when terms due to the viscosity η of the normal fluid are included in the hydrodynamic equations. He took into account both the viscous forces in the bulk liquid and also the forces which result when the liquid is confined in a cylindrical tube. The contribution to the coupling of the former is negligibly small compared to that due to α. This is also true for the contribution to the coupling due to the viscous forces at the tube walls except for p_2' waves at low frequencies and temperatures below about 1.4 K.

We have extended the calculations by also including in the hydrodynamic equations terms due to the coefficient of second viscosity ζ_2 and the thermal conductivity coefficient κ. The coefficient of second viscosity is an order of magnitude larger than η and contributes the largest term in the expression for the attenuation of first sound.[16] The attenuation of second sound is due mainly to the thermal conductivity coefficient.[16]

We find that the contribution to the coupling due to the thermal conductivity coefficient is negligibly small compared to that of α. This is true for both p_1' and p_2' at all temperatures in the range of our investigations. It is also true of the combined contribution of η and ζ_2 except for p_2' waves at temperatures below about 1.4 K.

When the viscous forces at the walls of the tube and the combined effect of η and ζ_2 in the bulk liquid are taken into account, Eqs. (3a) and (3b) take the following form:

$$\frac{p_1'}{T_0} = \left\{\alpha + \left(\frac{C}{ST}\right)\left[\left(\frac{i\omega}{\rho u_1^2}\right)\left(\frac{u_2}{u_1}\right)^2\left(\frac{4}{3\eta} + \zeta_2\right) + \left(\frac{2(i+1)}{\rho r}\right) \times \left(\frac{\eta\rho_n}{2\omega}\right)^{1/2}\left(\frac{u_2}{u_1}\right)^2\right]\right\}\rho u_1 u_2 , \tag{7a}$$

$$\frac{p_2'}{T_0} = \left\{\alpha + \left(\frac{C}{ST}\right)\left[\left(\frac{i\omega}{\rho u_2^2}\right)\left(\frac{4}{3\eta} + \zeta_2\right) + \left(\frac{2(i+1)}{\rho r}\right)\left(\frac{\eta\rho_n}{2\omega}\right)^{1/2}\right]\right\}\rho u_2^2 . \tag{7b}$$

In Eqs. (7a) and (7b), r is the radius of the chamber and ω is the angular frequency of the wave.

Both terms which are added to α in Eq. (7a) are negligibly small compared to α. However, in Eq. (7b) the first term $(Ci\omega/ST\rho u_2^2)(4/3\eta+\zeta_2)$ is the same order of magnitude as α for temperatures below about 1.4 K and at the higher frequency of 10 kHz used in our pulse measurements. Similarly the second term $\{C[2(i+1)]/ST\rho r\}(\eta\rho_n/2\omega)^{1/2}$ is the same magnitude as α below about 1.4 K and at the lower frequencies used in our standing-wave technique. Above about 1.4 K both these terms become small compared to α at the frequencies of our experiments.

The combined effects of η and ζ_2 do therefore provide a mechanism which results in the production of larger amplitude p_2' waves while not affecting the p_1' amplitude. This contribution increases with decreasing temperature and is not zero at the temperature where α vanishes. Thus the corrections due to this mechanism do shift the theoretical curves in the right direction, that is, toward better agreement with our experimental data. However, as can be seen in Fig. 5, the magnitude of these corrections is still much too small to bring about good agreement between our experimental p_2' data and the theory.

The values of η used in the above analysis were those measured by Tough *et al.*[17] and the values of ζ_2 are those calculated by Khalatnikov and Chernikova.[18]

Hofmann *et al.*[6] have also observed a larger amplitude p_2' wave produced by a heater than predicted by Lifshitz.[2] They suggest that this might be due to the microphone receiver being more sensitive to p_2' pressure waves than to p_1' pressure waves. This appears to us unlikely since both signals are propagated at the same frequency. The microphone sensitivity would therefore depend upon wavelength. We have at present not been able to devise a technique whereby we might check this speculation.

B. Temperature Wave T_2' Produced by a Microphone

There are a number of factors which might explain the discrepancy between the T_2' data and the theoretical expression of Eq. (4d). We have found that the correction to Eq. (4d) due to the effects of the normal fluid viscosity and the coefficient of second viscosity ζ_2 in the bulk liquid is negligible. However, the correction calculated by Dingle[15] due to the viscous forces at the walls of the tube must be considered. In this Eq. (4d) becomes

$$\frac{T_2'}{u_0} = \left[\alpha + \left(\frac{C}{ST}\right)\left(\frac{2(i+1)}{\rho r}\right)\left(\frac{\eta\rho_n}{2\omega}\right)^{1/2}\right]\frac{Tu_2}{C} . \quad (8)$$

In Fig. 6 the solid line is the theoretical curve of T_2'/u_0 from Eq. (4d) and the dashed line from Eq. (8). The correction, although not negligible, is much too small to explain the discrepancy.

Another possible source of the discrepancy is the fact that in Eqs. (4) and (8) it is assumed that u_0, the velocity amplitude of the transmitter diaphragm, is constant. This may not be the case. In general, for a driving force $F = F_0 e^{i\omega t}$, the velocity of the diaphragm will be [19]

$$u_0 = F_0/(Z_m + Z_r) . \quad (9)$$

In Eq. (9), Z_m is the mechanical impedance of the diaphragm and Z_r is the acoustic radiation impedance of the liquid He. The mechanical impedance of the diaphragm is given by

$$Z_m = R + i(\omega M - S/\omega) \doteq R + i\omega M[1 - (\omega_0/\omega)^2] , \quad (10)$$

where M is the mass of the diaphragm, S is the stiffness, R is the mechanical resistance which results in dissipation, and $\omega_0^2 = S/M$. The radiation impedance Z_r is proportional to ρu_1.

We do not have quantitative knowledge of the parameters of the microphone. However, assuming $\omega \ll \omega_0$ and neglecting R, Eq. (10) gives $Z_m = iM\omega_0^2/\omega$. If also $Z_m \gg Z_r$, then Eq. (9) shows that u_0 is proportional to ω. If these assumptions are correct, then u_0 is not in fact constant. However, the variation in frequency in the standing-wave measurements was not enough to result in this type of correction reducing the discrepancy between the T_2' data and the theory.

The T_2' resonant frequencies are much smaller than the fundamental resonance of a first-sound wave in the experimental chamber. Also ρu_1 is not strongly temperature dependent. Thus we would not expect the variation of the acoustic impedance Z_r in Eq. (9) to be large enough to explain the discrepancy.

A final consideration which might be the cause of the discrepancy between the T_2' data and the theoretical expression of Eq. (4d) is the fact that the porous diaphragm transducer will also be sensitive to pressure waves. This was already mentioned in Sec. III B where it was pointed out that resonances of waves propagating at u_1 were also observed in this experiment. The question is: Are there resonances of p_1' of Eq. (4a) or of T_1' of Eq. (4b)? Actually, a better form of the question is: What is the relative sensitivity of the porous diaphragm transducer to pressure and temperature waves?

The same question is involved in our observation of resonances corresponding to waves propagating at u_2. Could there be a significant contribution to the received signal due to the p_2' waves of Eq. (4c)? Since α is very small, Eqs. (4c) and (4d) would indicate that p_2', which is proportional to α^2, could be neglected in comparison to T_2', which is proportional to α. However, this may not be true if the porous diaphragm transducer is much more sensitive to pressure waves than to temperature waves.

We have not been able to arrive at a definite quantitative answer to these questions. However, certain results obtained in our investigations and also the results of the investigation by Sherlock and Edwards[13] would seem to indicate that the porous diaphragm transducer is a more efficient receiver of temperature waves than of pressure waves.

Recall the following two experiments which we performed in this investigation. First a regular capacitor microphone was used as the receiver in the experiment in which the transmitter was a periodically heated carbon disk resistor. In this case, two pressure waves, one propagating at u_1 and the other at u_2, were observed. In a succeeding experiment, described in Sec. III B, the same carbon disk transmitter was used but the receiver was now the porous diaphragm transducer. In this case only waves propagating at u_2 were observed. Moreover, the output signal from this wave was much larger than that from the u_2 wave of the first experiment.

In the first experiment the two waves were p_1' and p_2' of Eqs. (3a) and (3c) because the regular microphone is sensitive only to pressure waves. In the second experiment the received signal could be due to either p_2' or T_2' of Eqs. (3c) and (3d), respectively. However, if the contribution of p_2' to this signal were significant, then we should also have been able to observe p_1' waves in this experiment.

The fact that we did not observe any waves propagating at u_1 in the second experiment, but did observe very strong signals corresponding to waves propagating at u_2, would indicate that the porous diaphragm transducer is a more efficient receiver of temperature waves than of pressure waves. This conclusion is also indicated form the work of Sherlock and Edwards.[13] They used a configuration in which both transmitter and receiver were porous diaphragm transducers. They also observed only waves propagating at u_2.

These results indicate that the discrepancy between the T_2' data and the theoretical expression of Eq. (4d) is not due to a contribution from p_2' of Eq. (4c). However, it is still not clear what the relative contributions of p_1' and T_1' of Eqs. (4a) and (4b), respectively, are to the u_1 resonances which were observed. In this case p_1' is not proportional to α while T_1' is. Thus, these resonances could still be due mainly to p_1'. The fact that the amplitude of these resonances varied little with temperature would seem to indicate that this is the case.

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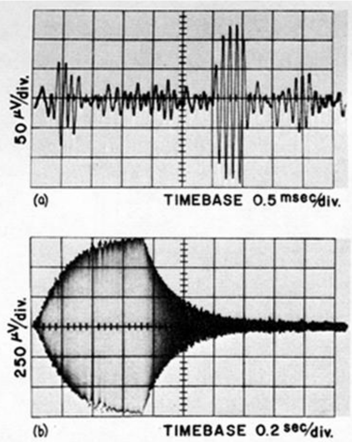

FIG. 2. (a) Oscilloscope trace of a typical received signal in the pulse measurements. The input to the carbon disk transmitter was 2 cycles of 5-kHz sinusoidal current. The temperature was 1.25 K. (b) Oscilloscope trace from a standing-wave measurement of p_2'. The input to the carbon disk was 64 cycles of 90.0-Hz sinusoidal current. The temperature was 1.70 K.