PHYSICAL REVIEW A VOLUME 2, NUMBER 6 DECEMBER 1970

Observation of Intrinsic Quantum Fluctuations in Semiconductor Lasers

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(Received 23 June 1970)

In this paper we report the first experimental observation of the excitation of the spiking resonance in a semiconductor laser by the quantum fluctuations intrinsic to the lasing process. Such excitation and the resulting anomalous peaks in the microwave noise spectrum of the laser intensity were initially predicted by McCumber and later calculated in detail for semiconductor lasers by Haug. In the present observations made with continuously operating GaAs junction lasers, this anomalous peak has been observed at currents I from less than 1% to nearly 100% above threshold I_{th}. The frequency of the peak varied with current as $(I/I_{\text{th}}-1)^{1/2}$, in agreement with previous analytical results. However, the resonant frequency was observed to be independent of temperature between 80 and 150 °K, in apparent contradiction with the recent calculations of Haug.

I. INTRODUCTION

A fundamental source of intensity fluctuations in the output of continuously operating lasers is the quantum shot noise intrinsic to the laser. Physically, this shot noise originates in the discrete and random nature of the transitions which produce the spontaneous or stimulated emission. Such quantum fluctuations were first described theoretically by McCumber[1] for lasers in general. The calculations predicted that the frequency spectra of the fluctuating intensity from solid-state and semiconductor lasers should show anomalous peaks due to the shot-noise excitation of the natural laser resonance. As shown previously,[2] this "spiking resonance" originates in the dynamic feedback between the electromagnetic field in the laser cavity and the population inversion. More recently, Haug[3] has presented detailed calculations made explicitly for semiconductor lasers in order to indicate the dependence of the intensity noise on pump level and temperature.

Experimentally, the existence of an anomalous peak in the intensity noise spectrum has been verified for the output of the yttrium aluminum garnet Nd oscillator.[4] In this case, the resonance frequency was in the range of 1–200 kHz. For semiconductor lasers, the resonance frequency is expected[1,3] to be somewhere above 100 MHz. As a result, previous low-frequency studies[5] of the fluctuating intensity from junction lasers did not reveal contributions from the increased noise at the resonance frequency. This early work did, however, provide a measurement of the low-frequency portion of the intensity noise spectrum which agreed quantitatively with theory.[1,3] In addition, the resonance frequency of junction lasers has also been studied[6] indirectly by modulating the injection current at microwave frequencies.

In this paper, we report the first experimental observation of the predicted anomalous peak in the intensity noise spectrum of a semiconductor laser. In Sec. II, the details of the experimental system and the lasers used in this work are briefly described. In Sec. III, the experimental results are presented and compared with the previously published calculations of Haug.[3] Conclusions of the present work are presented in Sec. IV.

II. EXPERIMENTAL DETAILS

The microwave frequency spectrum of the fluctuating light intensity from GaAs injection lasers was observed with the single-detector system shown schematically in Fig. 1. Because of the low level of the photocurrent generated by the intensity fluctuations, phase sensitive detection was necessary. Consequently, the laser light was collected and chopped before being focused into a coaxially mounted Si PIN photodiode.[7] An optical isolator was inserted between the laser and the photodiode to avoid interaction of the laser with reflections from the detection system. The photodiode was biased at constant voltage through a capacitive coupling and the dc bias current was monitored. The rf output of the photodiode was coupled to a microwave spectrum analyzer[8] through a dc short and a microwave filter to eliminate possible ambiguities in the spectral display. The frequency response of the microwave coupling circuit was constant within 2 dB from 60 MHz to 5 GHz. Variations in the incident light intensity produced by changes in the focusing or by the insertion of neutral density filters had no effect on the observed spectra, indicating that the microwave photoresponse of the detection circuit was independent of the laser intensity. In addition, the spectral observations were independent of the particular photodiode used (either a Ge p-n avalanche type or the nonavalanching Si PIN diode) and its biasing level. The vertical output of the spectrum analyzer was directed to the lock-in amplifier whose

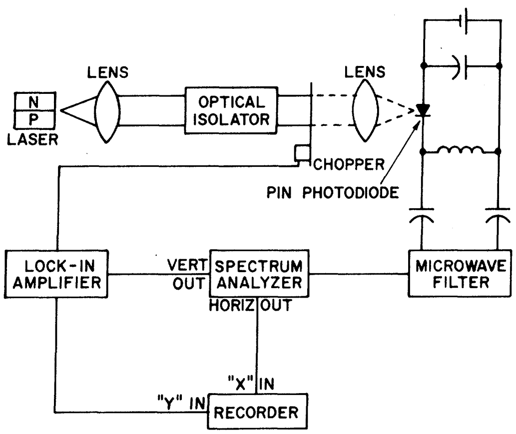

FIG. 1. Schematic diagram of the system used to observe the microwave spectrum of the fluctuating intensity from a GaAs injection laser.

output was displayed as the vertical scale on an x-y recorder. The x scale was derived from the horizontal sweep of the analyzer. The time constant of the lock-in was usually 30 msec while the total sweep time was 30 sec. The sensitivity of the system not including the PIN photodiode was estimated with a calibrated signal generator as – 110 dBm. This level is determined primarily by noise intrinsic to the spectrum analyzer.

The output at frequency $\omega = 2\pi\nu$ from the system described above is proportional to[1]

$$\int d\tau\, e^{i\omega\tau} \langle J_{\text{ph}}(t) J_{\text{ph}}(t+\tau)\rangle_t = [\eta\langle I\rangle_t + \eta^2 P(\omega)]\, |b(\omega)|^2, \tag{1}$$

where $J_{\text{ph}}(t)$ and $I(t)$ are, respectively, the photocurrent in electrons/sec and laser intensity in photons/sec, $\langle\rangle_t$ denotes a time average, $b(\omega)$ and η are, respectively, the frequency response function and the quantum efficiency of the detector, and $P(\omega)$ is the power spectrum of the intensity fluctuations. The first term on the right-hand side of Eq. (1) arises from the shot-noise fluctuations of the signal and tends to obscure $P(\omega)$, the signal of interest. However, the calculations of McCumber[1] and Haug[3] indicate that the peak of $P(\omega)$ is more than 500 times the shot-noise level at 2% above laser threshold and should therefore be observable if within the absolute sensitivity of the system. Direct calculation shows that for the conditions of our experiments the shot-noise level generated with the laser operating near threshold is approximately 20 dB below the system sensitivity, thus indicating that the peak of $P(\omega)$ is detectable.

The observations reported below were made on GaAs junction lasers with stripe metallic contacts[9,10] of dimensions either 25×380 μm or 50×380 μm.[11] They were bonded on copper or diamond[10] heatsinks and operated continuously on a cold finger placed within an inverted glass Dewar. The heatsink temperature was maintained constant within ± 0.1 °K.[12] A stable-current source was used to maintain the laser injection current within ± 0.1 mA. Digital control of the current supply allowed accurate repeatable changes as small as 1 mA. During the course of this work, the temperature of each laser was maintained in the vicinity of 77 °K from day to day to avoid any change in its characteristics due to temperature cycling.

The lasing threshold for each laser and temperature was determined graphically from a plot of light output power versus injection current over the threshold region. The exact value in each case was taken as the intersection of extrapolations of the spontaneous and lasing portions of this curve.

III. EXPERIMENTAL RESULTS

Typical intensity noise spectra obtained as described in Sec. II are shown in Fig. 2 as a function of laser pump level at a constant heatsink temperature of 80 °K. Each point of the traces represents the rms of the absolute intensity noise in a frequency interval of 1 MHz centered about the frequency indicated on the horizontal scale. Although not shown in the figure, the noise peak was detectable at current levels much less than 1% above threshold. With increasing current near threshold, the noise spectral density was typically observed to shift rapidly to higher frequencies and to increase in magnitude. At currents well above threshold, the absolute noise tended to reach a maximum, as indicated in Fig. 2, before decreasing somewhat

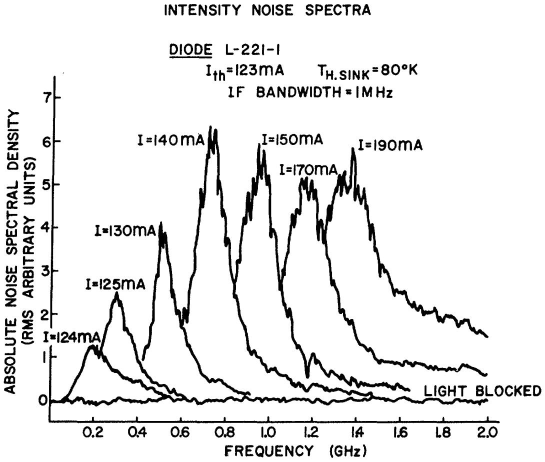

FIG. 2. Intensity noise spectra obtained from a continuously operating GaAs injection laser. The superposition of traces shows the rapid increase of the absolute noise at low currents and the frequency shift of the peak with increasing current level.

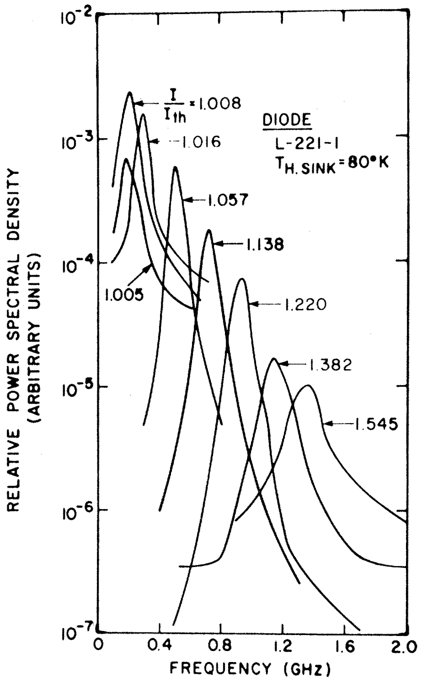

FIG. 3. Relative intensity noise spectra obtained for the same conditions as Fig. 2. The superposition of traces shows the initial increase of the relative fluctuations near threshold and then the sharp decrease of the relative fluctuations with further increases in current.

with further increases in current. At the same time, the frequency of the peak continued to increase although at a somewhat slower rate. In addition, the background noise level began to increase at the higher injection levels and eventually obscured the peak. This behavior agrees qualitatively with the theoretical results of Haug.[3] It should be emphasized that the frequency spectra shown in Fig. 2 were below the intrinsic noise level of the spectrum analyzer without phase-sensitive detection and therefore were not observed on the visual display of the analyzer itself.

Although the absolute intensity noise was increasing at low currents as shown in Fig. 2, the rms fluctuations relative to the intensity were decreasing rapidly over most of this current range, as expected in an amplitude-stabilized oscillator. This behavior is demonstrated in Fig. 3 where the log of the *relative* fluctuations is plotted as a function of frequency. Estimates of the relative intensity noise were made by normalizing the absolute spectra of Fig. 2 by the square of the average photocurrent. This normalization provides only an estimate of the relative noise since it neglects the frequency dependence of $|b(\omega)|^2$ in Eq. (1). However, the frequency response of the photodiode is not falling off rapidly over the frequency range of interest,[7] so the estimate is reasonable. More accurate but involved calibration can be achieved as suggested by McCumber.[1]

Although decreasing over most of the current range studied, the relative intensity fluctuations were observed to increase slightly with currents up to approximately 1% above threshold. This behavior is demonstrated in Fig. 3 by spectra obtained at 1.005 and 1.008 times threshold. Unfortunately, the relative intensity fluctuations of semiconductor lasers have not been studied theoretically at pump levels less than 1% above threshold, where this increase in the relative noise was observed. However, with solid-state lasers, such an increase at very low pump levels is expected.[13] The stability with which junction lasers operate near threshold makes the observation of this effect much less difficult than with other types of lasers.

The current dependence of the frequency at the peak of the fluctuations is presented in Fig. 4. In this figure, the resonance frequency $\nu_{\max}$ is plotted on log-log scale as a function of pump level above threshold for the same conditions as Figs. 2 and 3. In this case, the slope of the linear approximation to the experimental points was approximately 0.45. For all lasers tested, the slope was found to be between 0.55 and 0.45, indicating that $\nu_{\max}$ varies very nearly as $(I/I_{\text{th}}-1)^{1/2}$ as expected from the theoretical calculations.[1,3] For comparison, Haug's[3] extrapolation of his high-temperature calculation to low temperatures is also shown in Fig. 4. Some of the difference in absolute values between the theoretical and experimental curves is presumably due to the lack of agreement between the nominal parameter values used in the theory and those which are characteristic of our lasers.

The analysis of Haug[3] has also shown that at a constant pump level above threshold a shift in $\nu_{\max}$

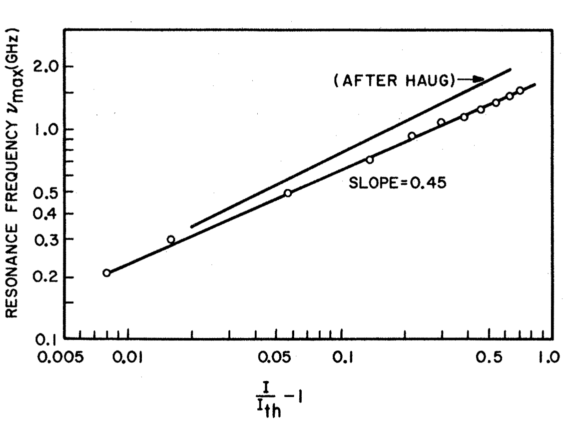

FIG. 4. The resonance frequency as a function of current level above threshold for the same conditions as Figs. 2 and 3. The theoretical curve is taken from the analysis of Ref. 3.

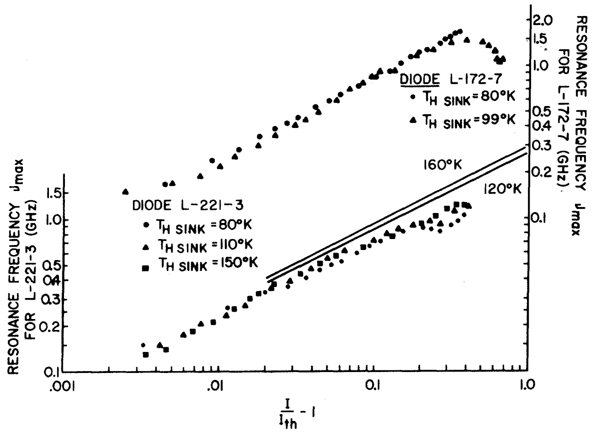

FIG. 5. Resonance frequency as a function of current for two different lasers at several heatsink temperatures. The proximity of the experimental points for various temperatures indicates that no temperature variation was observed. The solid lines are taken from the analysis of Ref. 3 to indicate the predicted temperature dependence. No calculations (Ref. 3) were made for pump levels below 1.02 times threshold, where the approximations of the theory were presumed to break down. Additional large-signal effects are occurring at pump levels where the experimental points depart from an approximately straight line.

is expected due to the temperature dependence of both the laser intensity and the quasi-Fermi level which describes the population inversion. In the present work, experiments were also conducted to determine the dependence of the resonance frequency on temperature. Results of this study, which are typical of all diodes tested, are shown in Fig. 5 where $\log\nu_{max}$ is plotted as a function of $\log(I/I_{th} - 1)$ for several heatsink temperatures up to 150 °K. For comparison, theoretical curves from Ref. 3 are shown for two temperatures as solid lines. The indicated frequency shift is certainly within the resolution of this experiment. However, it is clear that within our experimental accuracy the resonance frequency showed essentially no variation with heatsink temperature between 80 and 150 °K.

In some diodes, departures from the small-signal-noise behavior described above occurred at levels more than 10% above threshold over certain ranges of heatsink temperature. At such currents and temperatures in these lasers, the resonance becomes excited through the nonlinear interaction of the longitudinal laser modes as reported previously.[14] This excitation leads to several large-signal effects such as a reduction of the spectral width of the resonance, a strong increase of the amplitude of the microwave signal, and a reduction in the resonance frequency. The latter behavior is in fact indicated in Fig. 5, especially for diode L-172-7 at 99 °K. This depression of ν_{max} and other related large-signal effects which occur when the laser intensity exhibits self-pulsing[15] are beyond the scope of this paper and will be published elsewhere. However, it should be emphasized that the resonantlike peak described in Sec. III was observed in the intensity noise spectrum of all our lasers, whether or not they exhibited self-induced intensity pulsations over some region of operation.

IV. CONCLUSIONS

In this paper, we have reported the experimental observation of the excitation of the natural resonance of a GaAs injection laser by the quantum fluctuations intrinsic to the laser. Qualitative agreement with theoretical calculations[1,3] has been found for the pump dependence of the resonance frequency and of the absolute and relative magnitudes of the intensity noise. However, the resonance frequency as a function of I/I_{th} was observed to be independent of heatsink temperatures between 80 and 150 °K. This lack of temperature dependence is in apparent contradiction to the theoretical calculations[3] and requires further study.

ACKNOWLEDGMENTS

The authors thank J. H. Rowen, J. C. Dyment, L. A. D'Asaro, E. A. Ulmer, and E. I. Gordon for their helpful comments on the manuscript and L. H. Drewes for his competent technical assistance.

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PHYSICAL REVIEW A VOLUME 2, NUMBER 6 DECEMBER 1970

Crystalline Order in Restricted Geometries

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(Received 11 May 1970)

It is proven that a system of nuclei and electrons, confined by walls in the shape of an infinite slab or a rectangular rod of infinite length, cannot have *maximum* long-range crystalline order. The proof is based on the Bogoliubov inequality.

I. INTRODUCTION

It was shown by Peierls[1] that for an infinite two-dimensional system of particles with nearest neighbors connected by linear springs, one obtains

$$\langle u_i^2 \rangle = \infty \quad , \tag{1}$$

where u_i is the displacement of the ith particle from its average position. This result shows an instability in the crystal structure. Landau,[2] using phenomenological arguments, obtained the same result for two-dimensional systems. He also showed that there could be no periodicity in the particle density, contrary to what one would normally require of a crystalline structure. More recently, Mermin,[3] from first principles, has obtained the same result (lack of periodicity in the density) for an infinite two-dimensional system of particles interacting through the pair potential $\phi_{(r)}$, satisfying the conditions

$$\phi_{(r)} \to 1/r^{2+\epsilon} \qquad \text{as } r \to \infty \quad ,$$

$$\phi_{(r)} \to |A|/r^{2+\epsilon} \qquad \text{as } r \to \infty \quad .$$

The proof of this result was extended by the author[4] to cover the infinite two (one)-dimensional system of nuclei and electrons interacting through the two (one)-dimensional Coulomb potential.

It is pertinent at this point to elaborate a little on the meaning of instability [Eq. (1)] and lack of periodicity in the density. By lack of periodicity in the density, we mean

$$\langle \Psi_{\vec{G}} \rangle = 0 \quad \text{for any } \vec{G} \neq 0,$$

where

$$\langle \Psi_{\vec{G}} \rangle = \lim_{N \to \infty} (\langle \rho_{\vec{G}} \rangle / N) \; . \tag{2}$$

N is the total number of particles in the system;

$$\rho_{\vec{G}} = \int \rho(\vec{r}) \exp[-i\vec{G}\cdot\vec{r}]\, d\vec{r} \quad ,$$

where $\rho(\vec{r})$ is the number-density operator at point $\vec{r}$, and the limit is taken with N/Ω constant (Ω is the volume of the system). In the case of nuclei and electrons (Sec. II), we will be dealing with electrically neutral systems. In that case, N will refer to the number of nuclei.

It is worth noting that Eq. (2) probably implies Eq. (1), but the converse is not generally true (see Appendix A in I).

We will refer to Eq. (1) as "lack of stability" (or "instability"), and Eq. (2) as "lack of maximum crystalline order," or "lack of maximum long-range order" in the system. The reason for the word "maximum" is that there are systems with some kind of crystalline order (weak long-range order), in which

$$\langle \rho(\vec{R})\rho(\vec{R}+\vec{r}) \rangle \underset{r\to\infty}{\to} (N/\Omega)^2 + C r^{-T/T_c} \cos(\vec{G}\cdot\vec{r}) \quad ,$$

where T_c is a fixed temperature and T is the temperature of the system. This behavior of the density-density correlation function is known to exist in systems such as the one-dimensional electron gas (see Dyson[5]), and the two-dimensional "harmonic solid" (see Jancovici[6]). However, in all of these cases, $\langle \Psi_{\vec{G}} \rangle = 0$, in contrast with the order (maximum crystalline order) that one normally associates with crystals, where $\langle \rho(\vec{R})\rho(\vec{R}+\vec{r}) \rangle$, as $|\vec{r}| \to \infty$, goes into a periodic function with constant amplitude, implying that $\langle \Psi_{\vec{G}} \rangle \neq 0$.

In Sec. II, we show that maximum order cannot exist in partially finite geometries, viz., and infinite slab or an infinite rod of finite rectangular cross section, for a system of nuclei or electrons. The proof is based on the Bogoliubov inequality[7] and the following assumptions: (a) The first derivatives