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PHYSICAL REVIEW A VOLUME 2, NUMBER 5 NOVEMBER 1970

Polarization of Lyman-α Radiation Produced in Charge-Transfer Collisions between Protons and the Inert Gases*

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(Received 8 June 1970)

The polarization of Lyman-α radiation arising from charge-transfer collisions between protons and helium, argon, and neon has been measured for incident proton energies from 0.6 to 24 keV. A lithium fluoride crystal set at the Brewster's angle was used to measure the polarization of the radiation. Structure has been observed in the energy dependence of the polarization fraction for H^+-Ne and H^+-Ar collisions which reflects the structure in the total cross sections for electron capture into the 2*P* state of atomic hydrogen for these collisions. The present results are in poor agreement with those of Gaily *et al.*

INTRODUCTION

Over the past decade considerable attention has been devoted to the measurement of cross sections for charge transfer into the 2*P* state of atomic hydrogen for collisions of the type

$$H^+ + R \rightarrow H(2P) + R^+, \tag{1}$$

where *R* is an inert-gas atom.[1-6] It is now firmly established that the cross sections for H^+-Ne and H^+-Ar collisions show two maxima. A theory of excitation via an intermediate state has been proposed[7] which helps to explain qualitatively the structure in these cross sections. The measurement of the polarization fraction of the emitted radiation will assist in both the understanding and the analysis of the collision process.

In general, the cross sections for charge transfer into the 2*P* state of the hydrogen atom have been computed from observations of the resonance radiation which arises when the excited atom decays to the ground state where the photons were detected at 90° with respect to the proton beam direction. If Q_{90} is the apparent cross section, assuming that the radiation is emitted isotropically, and Q_T is the total cross section, then it can be shown that

$$Q_T = Q_{90}(1 - \tfrac{1}{3}P), \tag{2}$$

where the polarization fraction *P* is defined as

$$P = (I_\parallel - I_\perp)(I_\parallel + I_\perp). \tag{3}$$

$I_\parallel$ and $I_\perp$ are the intensities of the radiation viewed at 90° to the incident beam direction with their electric vectors oscillating parallel and perpendicular to the direction of the proton beam.

Until quite recently the measurement of the polarization of Lyman-α radiation has been made by measuring the angular distribution of the radiation. For electric dipole radiation, the intensity at an angle θ to the incident beam direction is related to the total intensity by

$$I(\theta) = \frac{3I_T}{4\pi}\,\frac{1 - P\cos^2\theta}{3 - P}. \tag{4}$$

The measurement of intensity as a function of angle has been used to obtain polarizations in electron–hydrogen-atom collisions[8] and in heavy-particle collisions.[9,10]

The development of a polarization analyzer[11] which consists of a reflecting lithium fluoride crystal set at the Brewster's angle has enabled us to measure the polarization directly from the definition. The

present results are compared with a previous measurement[9] and are found to disagree, yet the present results appear to be consistent with the theory of Poluektov and Presnyakov[7] which takes into account excitation via an intermediate state.

EXPERIMENTAL APPROACH

The basic experimental apparatus has been described in the preceding paper.[12] In most of the present measurements a dc technique was used where the inert gas under study was admitted to the collision chamber to attain pressures on the order of 1×10^{-5} Torr. The background pressures in the chamber were typically 2×10^{-7} Torr. The interaction region viewed by the polarization analyzer was defined by collimating apertures between the analyzer and the proton beam.

The polarization analyzer was basically the same as that described in the previous paper with the major difference being that in these measurements the emitted radiation was observed at an angle of 90° to the proton beam direction (See Fig. 1). Therefore, the $I_{\parallel}$ amd $I_{\perp}$ components of radiation, as defined in Eq. (3), were measured directly. The analyzer assembly had one other modification which enabled it to be rotated into the four positions 0°, 90°, 180°, and 270° with respect to the proton beam direction. In the 0° and 180° positions the $I_{\perp}$ component of radiation was detected, while the $I_{\parallel}$ component was detected in the 90° and 270° positions.

The experimental procedure was to make several measurements in each of the four analyzer positions with the inert gas in the chamber. Then, measurements were also made with no inert gas admitted to the chamber in order to determine the dc signal arising from the protons interacting with the normal background gas in the collision chamber. This small dc background signal was then subtracted from the signal obtained with the inert gas present to obtain the signal resulting from proton–inert-gas collisions for each component of the observed radiation. The criterion for acceptability of the data was that the proton–inert-gas signals should be equal for the 0° and 180° positions, and for the 90° and 270° positions. The expression which was used to compute the polarization fraction was

$$P=\frac{1}{E}\frac{I_{\parallel}-I_{\perp}}{I_{\parallel}+I_{\perp}},$$

where $I_{\parallel}$ and $I_{\perp}$ are the proton–inert-gas signals detected for the two components of the radiation and E is the efficiency of the polarization analyzer. The efficiency E enters because the analyzer subtends a finite solid angle to the source, and radiation arrives at the polarization detector over a small range of angles near the Brewster's angle. Since radiation arriving at other than the Brewster's angle will not be completely separated according to its polarization, the true polarization of the radiation will be slightly higher than that indicated neglecting the finite aperture effect. The value of E was taken to be 0.94, a value determined experimentally by Ott *et al.*[11] for the polarization analyzer used in the present experiment.

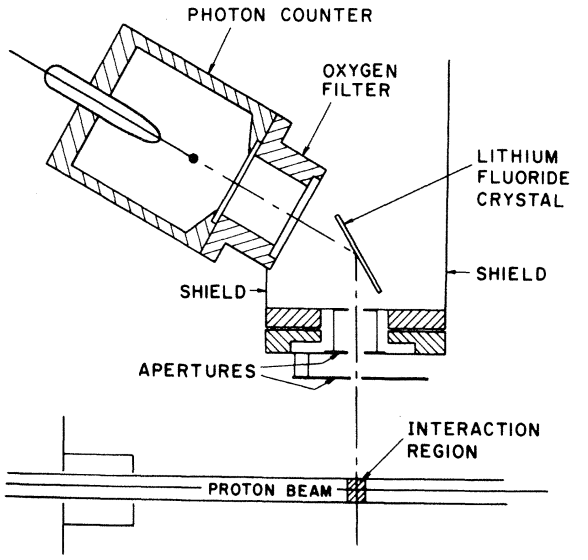

FIG. 1. Schematic of the experiment.

In these measurements the recorded count rates were kept below 20 counts/sec so that the number of counts lost due to the measured recovery time (500 μsec) of the photon counter was at most 1%.

Several of the precautions that were taken to ensure that only the desired Lyman-α radiation was being detected have been mentioned in the previous paper. With the relatively high pressure of inert gas in the collision chamber in these dc measurements, it was also necessary to investigate whether collision quenching of H(2S) atoms was occurring in view of the analyzer. This investigation was made by using a modulated beam of argon atoms for some polarization measurements. Since the background pressure in the collision region was two orders of magnitude less under these conditions than when using the dc technique, the effect of collision quenching, if it were occurring, would have been readily apparent in a comparison of the data. The results of these measurements are discussed in the Results and Discussion section.

RESULTS AND DISCUSSION

The results of the polarization measurements for proton collisions with argon, neon, and helium are shown in Figs. 2–4. Several measurements for deuteron-argon collisions are also shown in Fig. 2

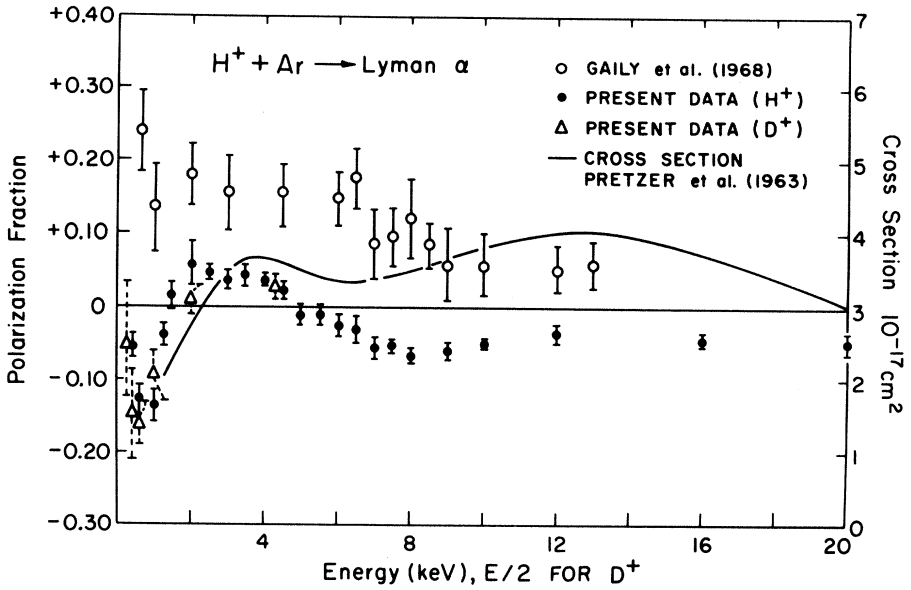

FIG. 2. Experimental values for the polarization of Lyman-α radiation produced in H^+-Ar and D^+-Ar charge-transfer collisions compared with the polarization measurements of Gaily *et al.* and the cross-section measurements of Pretzer *et al.*

and have been plotted at one-half their energy so that they may be compared with the proton data. The error bars represent plus and minus one standard deviation of the data.

The polarization results of Gaily *et al.*[9] are shown in Figs. 2–4 for comparison with the present results. For protons incident on argon and neon at energies greater than 2 keV good agreement is found in the shapes of the polarization energy dependence. However, there is a serious discrepancy of about 0.15 in the absolute values of the polarization fraction. In addition, the minima at low energies are not as pronounced in the data of Gaily *et al.* A further discrepancy has been found in comparing the H^+-He results, in that the shapes of the polarization energy dependence are not similar. The comparison between the two sets of results seems to indicate the presence of some systematic error in either or both of the experiments.

In an attempt to resolve the discrepancy between the two sets of measurements, further investigations were undertaken in the present experiment. Since Gaily *et al.* had not used an oxygen filter with their iodine-vapor-filled uv photon counter, the first investigation made was to evacuate the oxygen filter used in the present experiment and make sev-

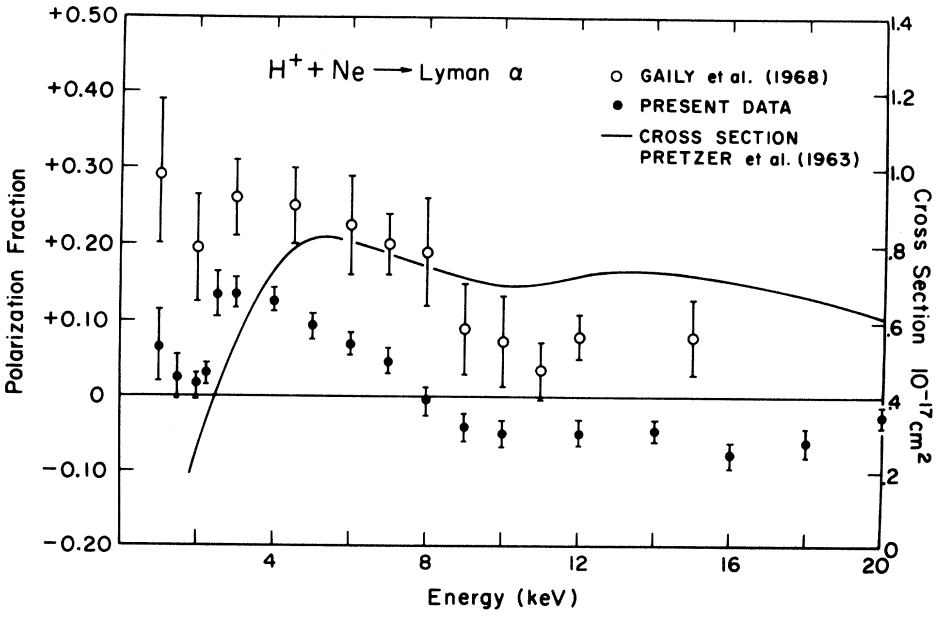

FIG. 3. Experimental values for the polarization of Lyman-α radiation produced in H^+-Ne charge-transfer collisions compared with the polarization measurements of Gaily *et al.* and the cross-section measurements of Pretzer *et al.*

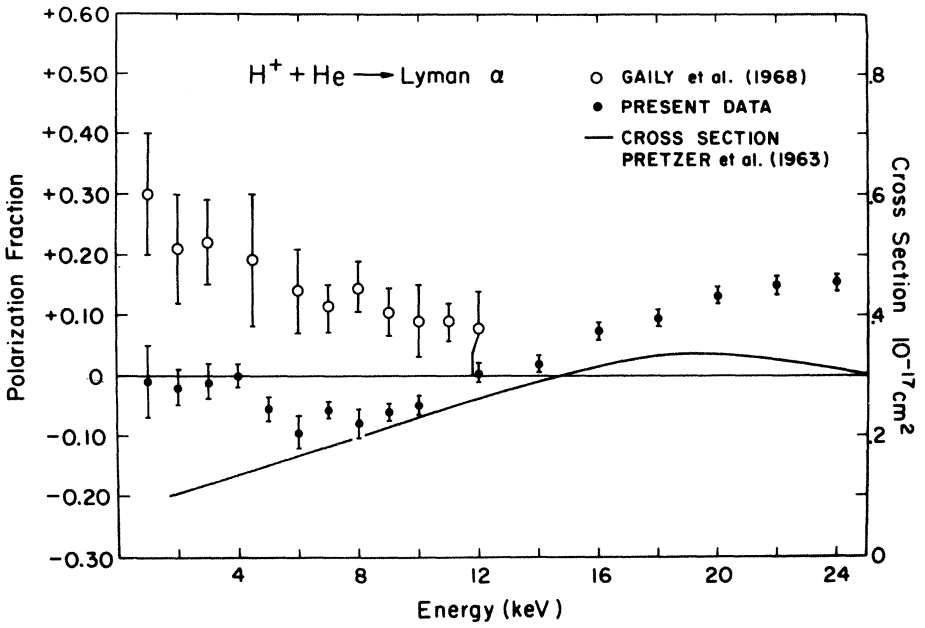

FIG. 4. Experimental values for the polarization of Lyman-α radiation produced in H^+-He charge-transfer collisions compared with the polarization measurements of Gaily *et al.* and the cross-section measurements of Pretzer *et al.*

eral remeasurements of the polarization for proton-argon collisions. Agreement was found with our measurements made using oxygen in the filter.

Another test performed was to analyze the radiation resulting from collisions of protons with a modulated argon beam. The polarization fractions obtained were $P = -0.057 \pm 0.073$ at 10 keV and $P = 0.022 \pm 0.077$ at 3 keV, which overlap the more precisely determined values obtained with the dc technique in our experiments and disagree with the results of Gaily *et al.*

In another investigation the position of the analyzer assembly was changed so that it could analyze the radiation emitted at 54.7° to the proton beam direction. In this case Eq. (6) from the previous paper[12] was used to calculate the polarization. The results obtained were in good agreement with those taken with the analyzer in the position to observe radiation at 90° to the proton beam. In view of the consistencies in our measurements we believe that the experiments of Gaily *et al.* were subject to unidentified systematic errors.

The H^+-Ar and H^+-Ne measurements in the present experiments show pronounced structure, both exhibiting a minimum at low proton energies after which the polarization fraction rises very rapidly to a maximum value. The polarization fraction then decreases and remains negative up to the highest energies used.

The similarity in the polarization curves for the argon and neon collisions is reflected in the shapes of the total cross-section curves for electron capture into the $2P$ state of atomic hydrogen. The total cross sections measured by Pretzer *et al.*[2] are shown as solid lines in Figs. 2–4. The largest polarizations were observed in the immediate vicinity of the low-energy maximum in the cross section. Poluektov and Presnyakov[7] have attributed the low-energy maximum in proton-neon collisions to charge transfer via the intermediate $1S$ state in atomic hydrogen. The maximum at higher energies is attributed to a direct charge-transfer process. Our results indicate that, if this interpretation is correct, these two processes lead to different populations of the magnetic substates of the $2P$ state of hydrogen atom.

The polarization of Lyman-α radiation arising from H^+-He collisions is near zero at the low energies. A shallow minimum is observed between 6 and 8 keV, after which the polarization increases and appears to reach a maximum at about 22 keV. This is consistent with the prediction of Poluektov and Presnyakov that the dominant contribution to the cross section in our energy range is made by charge transfer through the intermediate state.

*Research supported in part by the National Science Foundation.

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PHYSICAL REVIEW A VOLUME 2, NUMBER 5 NOVEMBER 1970

Fredholm Method. II. A Numerical Procedure for Inelastic Scattering

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A convenient and accurate numerical method is given whereby inelastic scattering information can be obtained by construction of the Fredholm determinant $\det[1-\underline{G}(E+i\epsilon)\underline{V}]$ for the coupled Lippmann-Schwinger equations. The method is noniterative and is easily applied when the potential matrix is nonlocal or energy dependent. It is shown that the determinant $\det[1-\underline{G}(E+i\epsilon)\underline{V}]$ may be factored as $\det[1-\mathcal{P}\underline{G}(E)\underline{V}]\det(1-i\underline{R})$ when $\mathcal{P}\underline{G}(E)$ is the principal-value Green's function and R is the usual R matrix of principal-value Lippmann-Schwinger theory; the R matrix may be obtained from $\det[1-\underline{G}(E+i\epsilon)\underline{V}]$ by a single partial triangularization. As a simple example of the extraction of the R matrix from the Fredholm determinant, the problem of electron scattering from hydrogen atoms is considered in the $1s$, $1s$-$2s$, $1s$-$2s$-$3s$, and $1s$-$2s$-$3s$-$4s$ close-coupling approximations. The use of optical potentials in the Fredholm theory is discussed: The two-channel problem originally suggested by Huck is solved numerically by construction of an optical potential.

I. INTRODUCTION

The fact that the S-matrix elements for many-channel potential scattering can be extracted from a single function of the channel momenta has been pointed out by LeCouteur[1]; Newton[2] has noted that this function is the Fredholm determinant for coupled many-channel Lippmann-Schwinger equations. Blankenbecler[3] has rederived these results using functional techniques; the necessary functional derivatives are evaluated using a prescription for the analytic continuation of the determinant as a function of a single complex energy. More recently, Newton[4] has given a formal extension of the method to the case of continuous channels and discussed the relation of Blankenbecler's analytic-continuation methods to the original derivation[2] based on the generalized Jost function.

Although these formal results are of great intrinsic interest, they have not been used in practical applications, except in very low orders of approximation.[3] These low-order results display the analytic properties and other qualitative features expected of the exact solutions, but certainly do not provide a computational method suitable for generating results which might be compared with experiment. It is the purpose of this paper to provide such a computational method.

In an earlier paper on one-channel scattering,[5] it was shown how a modification of Fredholm's original formal derivation of the method which bears his name provides a practical and accurate numerical scheme for the approximation of the determinant of an infinite-dimensional operator. In Sec. II, these earlier results are reviewed and the extension to many-channel scattering is presented via Newton's "substitution" rules.[2,4] It is noted that the Fredholm determinant $D(E+i\epsilon)=\det[\underline{1}-\underline{G}(E+i\epsilon)\underline{V}]$ may be factored as $\det[1-\mathcal{P}\underline{G}(E)\underline{V}]\det[\underline{1}-i\underline{R}(E)]$, where $\mathcal{P}G(E)$ denotes the principal-value Green's function, and R is the usual R matrix of principal-value Lippmann-Schwinger theory.[6] It is then shown that this factorization allows the R matrix to be extracted from $D(E+i\epsilon)$ by a single partial triangularization. Section III contains a simple example of the method in the case where the potential matrix is nonlocal and energy dependent; the problem of inelastic electron–hydrogen-atom scattering is treated in the $1s$-$2s$, $1s$-$2s$-$3s$, and $1s$-$2s$-$3s$-$4s$ close-coupling approximations. In Sec. IV the use of optical potentials is briefly discussed and a simple numerical example presented. Section V contains a summary and a conclusion.