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Influence of a Strong Magnetic Field on Plasma-Broadened $2P$-$4Q$ ($Q=P,D,F$) He I Lines*

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The profiles of He I 4471- and 4921-Å lines and their forbidden components have been calculated for $N_e = 6\times10^{15}\ e\,\mathrm{cm}^{-3}$ and $T_e = 2\times10^4\,°\mathrm{K}$ in the presence of a 70-kG magnetic field, with the aid of an extended formulation of the impact theory. In both cases, the lines exhibit important modifications, which are not shown by isolated or completely degenerate lines in similar situations.

Recently, it has been shown[1] that the static Stark patterns of the hydrogenic lines $2P$-$4Q$($Q = P, D, F$) located at 4471 and 4921 Å in the spectrum of neutral helium are deeply modified in the presence of a static magnetic field. These lines, of great interest in plasma diagnostics, have been the object of several recent investigations [2,3] for the case when there is no magnetic field. The purpose of this article is to present the given profiles for the case that a strong magnetic intensity confines the emitting discharge. The formalism used for that study is based on the generalized impact theory developed by Griem *et al.*[4] conveniently extended[1,3] in order to take into account the full structure of the static patterns in presence of combined Stark and Zeeman effects.

Then the light intensity polarized along a unit vector $\hat{e}$ may be written

$$I(\omega,\hat{e}) = \pi^{-1}\,\mathrm{Re}\int W(\vec{\mathrm{F}})\,d\vec{\mathrm{F}} \sum_{i,j,k,l} \langle n_i|\hat{e}\cdot\vec{\mathrm{R}}|n_j\rangle \times\langle n'k|\hat{e}\cdot\vec{\mathrm{R}}|nl\rangle\langle n_i|\langle n_j| \times\{i[\omega-\hbar^{-1}(H_n-H_{n'})]-\phi_{nn'}\}^{-1}|nl\rangle\;|n'k\rangle\,, \quad (1)$$

with $\vec{\mathrm{R}}$ the optical electron position vector. H_n [$H_{n'}$] is the atomic Hamiltonian[5] taking into account the full static electromagnetic perturbation operating on the sublevels $|n_i\rangle$ and $|n_l\rangle$ of the upper state (n) [$|n_j\rangle$ and $|n_k\rangle$ of the lower state (n')] of the line. $\phi_{nn'}$ denotes the electron collision (or relaxation) operator.

As in most line-broadening theories, the ions are regarded as infinitely massive classical particles over the time of interest (static ion approximation). Moreover, it may be shown that the low-frequency microfield distribution[6] $W(\vec{\mathrm{F}})$ is rigorously unaffected[7] in presence of a magnetic field of any strength when Doppler broadening is negligible in a thermal plasma. Therefore, it remains to evaluate the $\phi_{nn'}$ matrix elements. We restrict our attention to a sufficiently high electron density, such that the Larmor radius remains greater than the corresponding Debye length, i.e.,

$$r_G/\lambda_D = 4.544\times10^{-3}N_e^{1/2}/B \geq 1\,, \quad (2)$$

where N_e is in cm^{-3} and B in gauss.

The electron-atom interaction may then be evaluated with the usual monopole-dipole approximation and a straight-line trajectory for the perturbing electron travelling in the Debye sphere surrounding the emitter.

Leaving apart the cumbersome, but trivial, algebraic quantities involving the atomic matrix elements,[5] and restricting ourselves for the purpose of discussion to the upper state (n), the electron-impact contribution of a second-order transition $i \to l' \to l$ will be given by the average of the generalized width function $A(z, z')$ and of the generalized shift function $B(z, z')$ (z and z' are the dimensionless adiabaticity parameters $z = \omega_{il'} \times \rho / V$ and $z' = \omega_{ll'} \times \rho / V$) taken over the impact parameter ρ and the velocity V, weighted with a Maxwellian distribution. It is important to realize that the off-diagonal dynamic quantities with $z \neq z'$ keep a physical meaning in the impact theory[4] in the presence of a sufficiently high electron density fulfilling inequality (2). More precisely, the relation

$$\omega_{il} \ll \omega_p \tag{3}$$

is satisfied for all the $2P$-$4Q$ components with $N_e \gtrsim 10^5 \text{ cm}^{-3}$ and $B \lesssim 10^5$ G. Hence, the neglect of the very slow (as referenced to the plasma frequency) time dependence of the statically perturbed $(4, l, m)$ states[5] appears well justified for the considered off-diagonal S-matrix elements.[1,3] Another way to appreciate this fact in the Baranger scattering formulation[8] of the impact theory is to remark that these off-shell matrix elements remain always close to the energy shell in the no-back reaction hypothesis. Therefore, their absolute value is relatively small compared to the diagonal terms (less than $\frac{1}{10}$ of the latter), but they are numerous and they make the real and imaginary parts of the impact profile noncommutative, a feature that increases their numerical importance.

The inequality (3) allows us to consider $A(z, z')$ and $B(z, z')$ with

$$|z - z'| \lesssim 0.15 , \tag{4}$$

for the conditions given above, which are more than sufficient in practice. A too restrictive interpretation[9] of the impact theory forbidding the use of off-diagonal $\phi_{nn'}$ matrix elements would lead us to consider only the two unphysical limits of complete degenerate lines without static splitting and superposed isolated lines without mutual interaction. The latter approximation is clearly ruled out for the $2P$-$4Q$ lines, even in the absence of magnetic field, as evidenced by precise experimental measurements.[10] The latter show that the singlet line 2^1P-4^1Q keeps a greater half-width than the triplet one does, although the unperturbed-levels system 4^1Q is spread over a narrower interval[11] than the 4^3Q one. Actually, this fact can only be understood with a more intense interaction between the split sublevels than that produced by a simple isolated lines superposition.

The dynamical parts of the collision-operator matrix elements may then be written explicitly for the upper state (n):

$$\mathrm{Re}\,\langle n_i|\phi_n|nl\rangle \sim \int_0^\infty dy\, e^{-y} \left[A\!\left(\frac{c}{\sqrt{y}}, \frac{ac}{\sqrt{y}}\right) + \mathcal{A}\!\left(\frac{c}{\sqrt{y}}\;\frac{ac}{\sqrt{y}}\right) - \mathcal{A}\!\left(\frac{b}{\sqrt{y}}, \frac{ab}{\sqrt{y}}\right)\right], \tag{5a}$$

$$\mathrm{Im}\langle n_i|\phi_n|nl\rangle \sim \int_0^\infty \frac{dy\, e^{-y}}{2y} \left[B\!\left(\frac{c}{\sqrt{y}}, \frac{ac}{\sqrt{y}}\right) - B\!\left(\frac{b}{\sqrt{y}}, \frac{ab}{\sqrt{y}}\right)\right], \tag{5b}$$

where $y = m_e v^2 / 2kT_e$, $a = \omega_{il'} / \omega_{ll'}$,

$b = \omega_{il'} \times \rho_{\max} / \overline{V}$,

$c = b \rho_{\min} / \rho_{\max}$, $\overline{V} = (2kT_e / m_e)^{1/2}$,

$\mathcal{A}(z, az) = \int_z^\infty A(z', az') dz'/z'$.

$\omega_{il'}$ and $\omega_{ll'}$ are the statically perturbed angular frequencies. The impact cutoffs $\rho_{\min}$ and $\rho_{\max}$ are those discussed by Baranger,[8] conveniently generalized for the off-diagonal ϕ matrix elements. In Eq. (5a), the A function accounts for the strong collision contribution.

The complete electron contribution also includes a very weak term arising from the lower level $n' = 2$ and a small real cross term which are both exactly handled as those made explicit in Eqs. (5a) and (5b).

An accurate evaluation of the foregoing dynamic quantities [especially (5b)] requires cumbersome analytical manipulations. Fortunately for the $2P$-$4Q$ lines, with $N_e \gtrsim 10^{15}\, e\, \text{cm}^{-3}$ and $B \lesssim 10^5$ G, it is sufficient to replace the complete expressions for A, $\mathcal{A}$, and B with their asymptotic expansions[12] which are much easier to handle.

The resulting electron profile has to be factorized with the quasistatic matrix elements[5] and the low-frequency microfield distribution.[6] In the general case, this operation demands an angular average of the electric field $\vec{F}$ taken with respect to $\vec{B}$, which can be safely neglected in presence of a strong magnetic intensity. It is worthwhile to emphasize that the degenerate perturbation method used in Ref. 5 allows us to evaluate simultaneously and accurately the static electromagnetic perturbation of the $4Q(Q = P, D, F)$ sublevels, thus avoiding complicated manipulations of questionable accuracy with the quadratic Stark effect of separate sublevels.

III. RESULTS

The complete profiles are shown in Figs. 1 and 2, respectively, for the 4921-Å singlet line and the 4471-Å triplet line with the intensities ob-

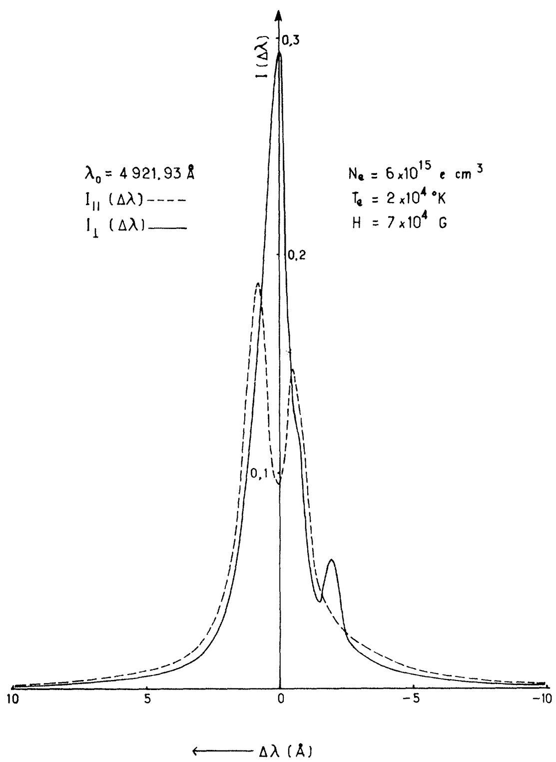

FIG. 1. Profiles of the singlet 2^1P-4^1Q ($Q=P, D, F$) line centered at 4921.93 Å.

served parallel and perpendicular to $\vec{H}$ and expressed in terms of the polarized intensities by the relations

$$I_{\parallel}(\Delta\lambda)=\tfrac{1}{2}[I(\Delta\lambda,\hat{x})+I(\Delta\lambda,\hat{y})] \ , \tag{6a}$$

$$I_{\perp}(\Delta\lambda)=\tfrac{1}{2}[I_{\parallel}(\Delta\lambda)+I(\Delta\lambda,\hat{z})] \ . \tag{6b}$$

At first sight, the given profiles show a structure which looks very different from the isolated-line Lorentz triplet[13] and from the hydrogen H_β line broadened in the presence of a strong magnetic field[14] with the same (N_e, T_e) values.

More precisely, the parallel intensities $I_{\parallel}(\Delta\lambda)$ keep their central peaks $2P$-$4D$ and $2P$-$4F$. The perpendicular intensities $I_{\perp}(\Delta\lambda)$ again exhibit this structure, but with a strong asymmetry. The second peak of $I_{\parallel}(\Delta\lambda)$ may be surely attributed to the $2P$-$4F$ maximum, as would be shown in a plot of $I(\Delta\lambda, \hat{z})$ (polarized along the magnetic field and free from the σ components[13]) with the same two-peaked structure.

Another striking result is the absence of σ components on the $2P$-$4D$ side and the appearance of a weak σ component on the 2^3P-4^3F side only, located at (see Fig. 2)

$$\Delta\lambda_z=\lambda^2\times 4.688\times 10^{-13}B \ , \tag{7}$$

where λ_z and λ are in angstroms, and B in gauss (see Fig. 2). This feature is easily explained by the interpenetration of the $2P$-$4D$ and $2P$-$4F$ static patterns in the presence of a strong Zeeman effect[5] and also by the dominating electron-impact effect in the line center, at the vicinity of the $2P$-$4D$ maximum. This behavior is more pronounced for the 4^1Q sublevels, which have a stronger mutual interaction, than for the 4^3Q ones. In fact, the line-center broadening of these partially degenerate lines is the result of a competition between Zeeman, static Stark, and electron-impact broadening.

As a by-product it clearly appears that even a strong Zeeman effect cannot be studied with a linear superposition of Lorentz triplets to each component of the static Stark pattern.

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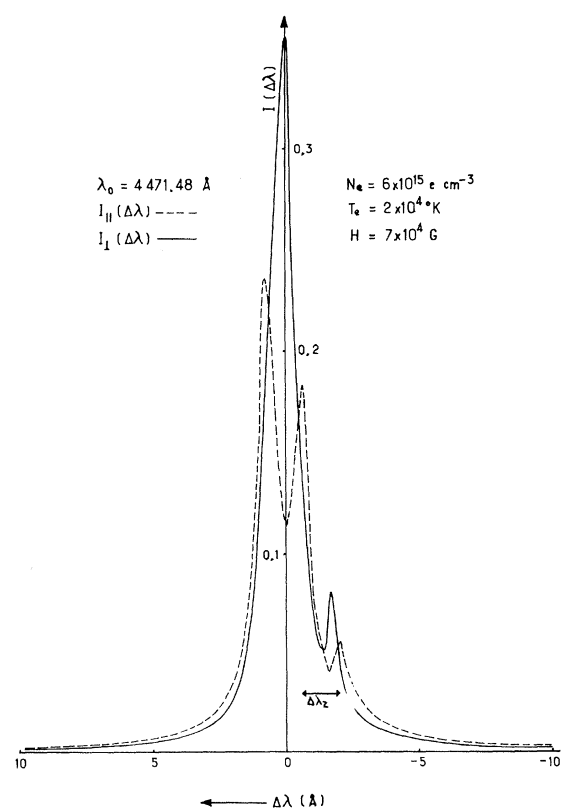

FIG. 2. Profiles of the triplet 2^3P-4^3Q ($Q=P, D, F$) line centered at 4471.48 Å.

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Hyperfine Contact Interaction in the Iron Atom Calculated by Many-Body Theory*

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The contribution of the hyperfine contact interaction in the iron atom has been calculated by many-body theory. Terms involving one and two Coulomb interactions have been included, and our result for the contact constant is −4.55 MHz as compared with the experimental result of −5.1 MHz obtained by Childs and Goodman. Inclusion of an approximate relativistic correction changes the calculated value to −4.87 MHz. An estimate of higher-order terms gives a theoretical result −5.35 MHz or −5.72 MHz when the relativistic correction factor is included.

I. INTRODUCTION

The many-body perturbation theory of Brueckner[1] and Goldstone[2] is used to calculate the hyperfine contact interaction in the iron atom. The methods used to evaluate the diagrams of perturbation theory are taken from our previous work.[3–5] Our methods for applying the Brueckner-Goldstone theory to atoms have also been used in hyperfine calculations by Dutta *et al.*[6]

The effect of the contact interaction in the iron atom has been analyzed previously by Watson and Freeman[7] who carried out an unrestricted Hartree-Fock (UHF) calculation in an analytic expansion and obtained the value −3.4 MHz for the contact hyperfine-interaction constant C. A more accurate UHF calculation using an analytic expansion was later carried out by Bagus and Liu[8] who obtained the value −4.4 MHz for C. An experimental value for C equal to −5.1 MHz has been obtained by Childs and Goodman from their measurements of the magnetic dipole hyperfine-interaction constants for the $^5D_{4,3,2,1}$ states of the ground term of Fe^{57} in an atomic-beam magnetic-resonance experiment.[9]

The contact contribution to the hyperfine splitting is written[8–10]

$$E_C = C\vec{I}\cdot\vec{J}\,, \tag{1}$$

where $\vec{I}$ is the nuclear spin and $\vec{J}$ is the electronic angular momentum. The contact constant C may be written[8,9]

$$C = \tfrac{8}{3}\pi (g_J - 1) g_e \mu_e g_I (\mu_N/S) \times\langle L S, M_L, M_S = S \,|\textstyle\sum_i \delta(\vec{r}_i) s_{zi}\,| LS, M_L = L, M_S = S \rangle\,, \tag{2}$$

where

$$g_J = 1 + [J(J+1) + S(S+1) - L(L+1)]/[2J(J+1)]\,. \tag{3}$$

The usual g factors of the electron and of the nucleus are represented by g_e and g_I, respectively.[11] The symbols μ_e and μ_n represent the Bohr magneton and nuclear magneton.[11]

We may also express C in terms of

$$\chi = \frac{4\pi}{S}\left\langle LS; M_L, M_S = S \left| \sum_{i=1}^{N} \delta(\vec{r}_i)\, s_{zi} \right| LS; M_L, M_S = S \right\rangle. \tag{4}$$

In order to calculate Eq. (4), we require the state