

$$1 = \frac{1}{2} \Gamma \frac{\partial \delta_i}{\partial E} \sum_{i=1}^n y_i^2 \sec^2[\delta_{0i} - \delta_j(E)], \quad (10)$$

which shows that δ_j always increases with energy. Near $E = E_r$ the eigenphases change rapidly with energy. The sharpness of the bend in the curve for δ_i near $E \lesssim E_r$ is a measure of the magnitude of y_i , i. e., a measure of the coupling of the resonance to the i th eigenchannel. This can be stated quantitatively by considering the point at which a tangent to the δ_j curve meets the energy axis at 45° . Then $\partial \delta_j / \partial E$ just equals the scale factor s , which gives the ratio of the radian scale to the energy scale. At such an energy near $E \sim E_r$, $\delta_j - \delta_{0j}$ is small, and we can neglect all terms on the right-hand side of Eq. (10) with $i \neq j$. Approximating the sin by its argument, we find

$$(\frac{1}{2}\Gamma)^{1/2} y_j \approx |\delta_{0j} - \delta_j| / \sqrt{s}, \quad (11)$$

for an energy such that $\partial \delta_j / \partial E = s$ and $E \lesssim E_r$. From Eq. (11), it follows that the width for the decay of the resonance into the j th eigenchannel of S_0 is approximately one-half of the perpendicular distance d between the j th and $(j+1)$ th curves divided by \sqrt{s} :

$$\sqrt{\Gamma} y_j \approx \frac{1}{2} d / \sqrt{s}. \quad (12)$$

This relation is illustrated in Fig. 1. We emphasize that this construction gives only a rough determination of y_j . It is presented here to give precise meaning to the statement that the degree of curve repulsion measures the partial decay width for the resonance to decay into the j th eigenchannel.

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Interpretation of Transport Measurements in Electronically Conducting Liquids.

III. Metals in Liquid Ammonia

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This note presents data on liquid metal-ammonia solutions that were omitted from an earlier tabulation and discussion of the transport characteristics of electronically conducting liquids. Similarities and differences between the metal-ammonia solutions and the liquids included in the earlier publication are noted and briefly discussed.

The transport properties of over 100 electronically conducting liquids were tabulated and discussed in two earlier publications¹ hereafter identified as ECL. The data were arranged according to the magnitude of the electrical conductivity σ . Three ranges of σ , labeled A, B, and C, were identified, covering the values $10^5 - 5 \times 10^3$, $5 \times 10^3 - 10^2$, and $10^2 - 10^{-12} \Omega^{-1} \text{cm}^{-1}$, respectively.

As discussed in ECL, liquids in range A exhibit more or less conventional metallic characteristics. These can be understood in terms of Ziman's nearly free-electron theory of liquids.² It seems clear that liquids in range B should also be classified as metallic, despite the semiconductorlike

behavior of their conductivity and thermoelectric power.¹

Solutions of various metals in liquid ammonia show electronic conduction when the metallic concentration is high enough, and they should have been included in ECL. The soluble metals are Li, Na, K, Rb, Cs, Ca, Sr, Ba, Yb, and Eu.^{3,4}

In traversing the composition range from pure Cs to pure NH_3 , for example, electronic conductivity is observed in σ ranges A and B,⁵ while a transition to ionic conductivity occurs in range C when the conductivity drops below $10^{-1} \Omega^{-1} \text{cm}^{-1}$ or so.⁴

Some transport properties of solutions of Cs

TABLE I. Transport properties of metal-ammonia solutions at 233 K.

Liquid	Composition (MPM)	σ^a ($\Omega^{-1}\text{cm}^{-1}$)	Sign of $d\sigma/dT$	α^b ($\mu\text{V}/\text{K}$)	Sign of $d \alpha /dT$	R^c (cm^3/C)	$R/ R_0 $	Sign of $d R /dT$	μ_H ($\text{cm}^2/\text{V sec}$)
Cs ^d	100	27800	-	+6.4	+				
Cs-NH ₃	55	11400	-						
Cs-NH ₃	29	6300	0						
Na-NH ₃	15	4800 ^e	+	+2	+	-1.9×10^{-3}	-1.0	0	9.1
Na-NH ₃	13	3250	+	0	+	-2.2	-1.0	0	7.1
Na-NH ₃	10	1650	+	-3	+	-2.9	-1.0	0	4.9
Na-NH ₃	8	950	+	-5	0	-3.7	-1.05	0	3.5
Na-NH ₃	6	370	+	-7	-	-5.2	-1.08	0	1.9
Na-NH ₃	4	90	+	-13	-	-10.	-1.24	0	0.9

^aReference 5; C. A. Kraus, J. Am. Chem. Soc. **43**, 749 (1921); C. A. Kraus and W. W. Lucasse, *ibid.* **43**, 2420 (1921); **44**, 1941 (1922).

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^dECL (Ref. 1).

^eSolutions of Na and of other metals saturate near 16 MPM, but saturation is not observed in the Cs-NH₃ system.

and Na in NH₃ (the latter is the most widely studied system) are presented in Table I. The metal concentration range is 100-4 MPM;⁶ this leads to values of σ which cover most of ranges A and B. Russakoff and Ashcroft⁷ have had remarkable success in calculating σ from a Faber-Ziman-type alloy theory for metal concentrations above 8 MPM.

The sign of $d\sigma/dT$ in Table I changes from minus to plus as σ drops below 6000 $\Omega^{-1}\text{cm}^{-1}$. This behavior is very similar to that which was revealed by the tabulation in ECL, though the changeover occurs at different values of σ with different solutes.

There is considerable scatter in the behavior of the thermoelectric power α and its temperature coefficient in the data presented in ECL. However, the over-all trends do not correspond to what is shown in Table I. Moreover, the sign change in α does not appear in K-NH₃ solutions;⁸ the difference in the results for the K and Na solutions is not understood. Presumably such effects result from the influence of the changing NH₃ frac-

tion on the scattering. Scattering from the ion clearly dominates at high concentrations and scattering by NH₃ molecules may take over at lower concentrations.^{4,7}

The Hall measurements in Table I were obtained on range-B liquids. The behavior is metallic, as it is for almost all of the range-B data from ECL. It should be noted, however, that in Table I, R_0 specifies the free-electron Hall coefficient for the valence electrons from the metallic component only, while in ECL, it refers to the total valence electron density for all constituents of the liquids. The quantities are equivalent since the NH₃ is inert (ionization energies exceed 11 eV). The Hall mobilities listed in Table I decrease with decreasing σ as they generally do in the ECL data. The values shown in Table I, however, are about one order of magnitude larger than in the earlier tabulation.

Below 4 MPM, ionic conductivity begins to contribute significantly to σ . Hence a comparison with range-C liquids from ECL is not feasible.

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⁶Concentrations are expressed in mole percent metal (MPM) = 100 × moles metal / (moles metal + moles NH₃).

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