PHYSICAL REVIEW A VOLUME 2, NUMBER 1 JULY 1970

Atomic Radiation in a Cavity*

P. Stehle
Sektion Physik der Universität München, Munich, Germany†
(Received 7 October 1969)

The spontaneous decay of an atom located between two parallel plane mirrors of reflectivity R is calculated using the Weisskopf-Wigner method, both when the two mirrors are very large and when they are small. When the mirrors are large and of high reflectivity, the atom radiates more rapidly than in free space and into the Fabry-Perot modes of the mirror system. When the mirrors are small, their effect on the total radiation rate is small, but radiation into the few modes defined is enhanced if the reflectivity is high. It is concluded that the effect of the presence of a laser etalon is insufficient to justify omitting the coupling of the atom to non–Fabry-Perot modes from the Hamiltonian of an atomic system in a laser etalon.

I. INTRODUCTION

The emission of electromagnetic radiation by an excited atom in free space – the process of spontaneous emission – is well described by quantum electrodynamics, by the use of either the Weisskopf-Wigner method[1] or the Furry-Low method.[2] The emission of radiation by a system of many excited atoms in free space has been investigated by Ernst and Stehle,[3] using the Weisskopf-Wigner method. They found that the photons emitted in this process are strongly correlated with each other both in frequency and direction, constituting a ray whose spread in direction and in frequency depends on the macroscopic distribution of the atoms.

The operation of conventional masers[4] and lasers[5] depends on the emission of radiation by a system of atoms within a resonant cavity of some sort. This interaction is usually described theoretically by assuming only one mode of the cavity (or at most a few) to have an appreciable interaction with the atomic system, and therefore any special characteristics of the emitted radiation are ascribed to the assumed dominance of this mode. It is of considerable interest to investigate to what extent the properties of the radiation of masers and lasers arise from the correlations shown by the photons radiated by a many-atom system in free space, and to what extent they arise from the dominance of a small number of modes of the cavity.

As a start on the study of this problem, the emission of a photon by a single excited atom is investigated when this atom is located between two parallel plane mirrors, using the Weisskopf-Wigner method. If the mirrors are large and have high reflectivity, they form a cavity with comparatively well-defined discrete modes. If the mirrors are within about a wavelength of each other they simulate a maser cavity, while if they are small and separated by many wavelengths, they constitute a laser etalon. Thus, this system is general enough to include both the maser cavity and the laser etalon. The results found here are qualitatively what would be expected on intuitive grounds. When the mirrors are large and of high reflectivity, the atom radiates predominantly into the Fabry-Perot or Fox-Li[6] modes of the mirror system, so an atom in a microwave cavity tuned to the frequency of the atomic transition radiates into the resonant mode. An atom placed between two small or poorly reflecting mirrors radiates predominantly into the free-space modes and only very slightly into any one mode of the etalon, but the radiation into etalon modes does exceed what would be expected on the basis of solid angle considerations alone. Mirrors 2 cm in radius separated by 20 cm are small in the sense needed above.

These conclusions lend support to a hypothesis of Ernst and Stehle,[3] which has been restated more explicitly by Ernst,[7] to the effect that laser action depends in an essential way on the emission of a ray of radiation by many atoms, and that it is this ray which is influenced both in its correlations and its time development by the presence of the etalon. The manner by which this comes about is the subject of another paper.

II. MODE STRUCTURE

In free space, the interaction of an atom with the electromagnetic field can be decomposed into its interaction with plane waves. In this section, we derive an expression for the field which replaces the plane wave, when the atom is between two infinite parallel plane mirrors.

Let the mirrors be perpendicular to the x axis and intersect it at $x=\pm\frac{1}{2}L$, as in Fig. 1. Let $\vec{X}$ be a point on the x axis between the mirrors. We want to construct the field $F_{\vec{k}}(\vec{X})$ with which an atom at $\vec{X}$ interacts, this field comprising the plane

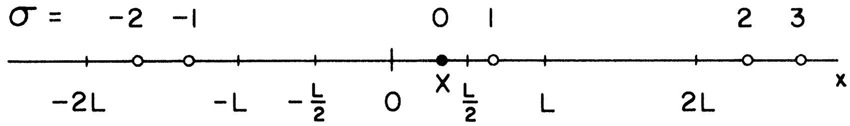

FIG. 1. Atom at X between mirros at $\pm\frac{1}{2}L$ has images shown by the circles at $X_\sigma = \sigma L + (-)^\sigma X$.

wave $\exp(i\vec{k}\cdot\vec{X})$ and all its reflections. The phases of the reflected waves must be considered carefully. The images of the point $\vec{X}$ in the mirrors occur at points $\vec{X}_\sigma$ given by

$$\vec{X}_\sigma = \sigma\vec{L} + (-)^\sigma \vec{X}, \quad \sigma = -\infty, \ldots, -1, 0, 1, \ldots, +\infty, \quad (1)$$

where $\vec{L}$ is a vector, parallel to the x axis, connecting the mirrors. The image of the wave vector $\vec{k}$ after $|\sigma|$ reflections is

$$\vec{k}_\sigma = \vec{k}_{\|} + (-)^\sigma \vec{k}_\perp . \quad (2)$$

With the image point $\vec{X}_\sigma$ we associate a plane wave $f_{\vec{k}}{}^\sigma(\vec{x})$ given by

$$f_{\vec{k}}{}^\sigma(\vec{x}) = R^{|\sigma|} \exp[i\vec{k}_\sigma\cdot(\vec{x}-\vec{X}_\sigma)] \exp(i\vec{k}\cdot\vec{X}) . \quad (3)$$

The first exponential factor contains the phase difference of the "reflected" wave between the points $\vec{x}$ and $\vec{X}_\sigma$. The second exponential factor specifies the phase of this wave at the image point $\vec{x} = \vec{X}_\sigma$ to be that of the plane wave corresponding to $\sigma = 0$, the unreflected wave, at the point $\vec{X}$. Thus, each plane wave $f_{\vec{k}}{}^\sigma(\vec{x})$ has the same phase at its associated image point $\vec{X}_\sigma$.

The image point $\vec{X}_\sigma$ is obtained by $|\sigma|$ reflections. The two signs of σ correspond to reflection in one or the other mirror first. The plane wave $f_{\vec{k}}{}^\sigma(\vec{x})$ has the amplitude appropriate to one having been reflected $|\sigma|$ times from mirrors of reflectivity R, $0 < R < 1$. We now assert that the field $F_{\vec{k}}(\vec{X})$ which an atom at $\vec{X}$ would interact with is the sum of all of the plane waves $f_{\vec{k}}{}^\sigma(\vec{X})$. This is equivalent to replacing the system of a single atom and mirrors by a line of atoms without mirrors, all atoms behaving the same way, i.e., interacting with plane waves of the same phase but with suitably attenuated amplitude. This represents merely an extension of the classical method of images used to satisfy boundary conditions. The existence of a phase change on reflection can easily be included, but is omitted for simplicity. The geometric series involved are easily summed. The result is

$$F_{\vec{k}}(\vec{X}) = \frac{(1-R^4)\, e^{i\vec{k}\cdot\vec{x}} + 2R(1-R^2)\cos(\vec{k}\cdot\vec{L})\, e^{-i\vec{k}\cdot\vec{x}}}{(1-R^2)^2 + 4R^2\sin^2(\vec{k}\cdot\vec{L})} . \quad (4)$$

For $R \approx 1$, this has a very sharp maximum where $\vec{k}\cdot\vec{L} = M\pi$, and the size of this maximum is like the one obtained by summing $2(1-R^2)^{-1}$ terms with no attenuation on reflection. Using this fact we estimate that a time T given by

$$T = \frac{L/c}{1-R^2} \quad (5)$$

is needed to establish the field $F_{\vec{k}}(\vec{X})$ by radiation between the mirrors.

For any $|k| > \pi/L$, there is a set of directions in $\vec{k}$ space for which $\sin\vec{k}\cdot\vec{L} = 0$, $\vec{k}\cdot\vec{L} = m\pi$. These are the Fabry-Perot modes of the mirrors for this k. If $R^2 \approx 1$, these modes are associated with the fields

$$F_{\vec{k}}(\vec{X}) = \frac{4\cos\vec{k}\cdot\vec{X}}{1-R^2}, \qquad m \text{ even}$$
$$F_{\vec{k}}(\vec{X}) = \frac{4i\sin\vec{k}\cdot\vec{X}}{1-R^2}, \qquad m \text{ odd} . \quad (6)$$

These mode fields do not vanish for $\vec{X} = +\frac{1}{2}\vec{L}$ because no phase reversal on reflection was introduced. If $\vec{X}$ is not on the x axis, the $\vec{k}$ in Eq. (6) must be replaced by $\vec{k}_\perp$, and the field given in Eq. (4) must be multiplied by $\exp[i\vec{k}_{\|}\cdot\vec{X}]$. For the one-atom system this is not important.

III. RADIATION BETWEEN INFINITE MIRRORS

We now imagine an atom in its excited state to be at the point $\vec{X}$ between two infinite mirrors at time $t = 0$ with no photons present. The time development of the system can be studied by use of the Weisskopf-Wigner method. If the lifetime of the atom is τ, we require only that

$$\tau > T = \frac{L/c}{1-R^2} \quad (7)$$

in order to replace the plane waves occurring in the free-space theory by $F_{\vec{k}}(\vec{X})$, as given by Eq. (4). The interaction of the atom with the electromagnetic field in free space has matrix elements of the form

$$\langle g; \vec{k}| H_{\text{int}} | e; 0\rangle = C^*_{\vec{k}}\, e^{-i(\Delta-k)t}(2Vk)^{-1/2},$$
$$C^*_{\vec{k}}(\vec{X}) = -ie\int d^3x\, \bar{u}_g(\vec{x}-\vec{X})\gamma_\mu e_\mu u_e(\vec{x}-\vec{X})\, e^{-i\vec{k}\cdot\vec{x}}, \quad (8)$$

in the notation of Ref. 3 (except for the interchange of $C_{\vec{k}}$ and $C^*_{\vec{k}}$). Δ is the Bohr frequency of the atom, and V is the field quantization volume, eventually to approach infinity. In the presence of the mirrors, $C^*_{\vec{k}}(\vec{X})$ is replaced by $\tilde{C}^*_{\vec{k}}(\vec{X})$, defined by

$$\tilde{C}^*_{\vec{k}}(\vec{X}) = -ie\int d^3x\, \bar{u}_g(\vec{x}-\vec{X})\gamma_\mu e_\mu u_e(\vec{x}-\vec{X})\, F_{-\vec{k}}(\vec{x})$$

$$= F_{-\vec{k}}(\vec{X}) D \ , \tag{9}$$

where the usual assumption that the wavelength of the emitted radiation is long in comparison with the atomic radius has been made so that the field evaluated at the center of the atom factors out. The matrix element D contains an angular factor because the vector $\vec{e}$ must be perpendicular to $\vec{k}$. This gives just the sine dependence of dipole radiation and will be ignored here.

The Weisskopf-Wigner equations may be written down as in Ref. 3. They are

$$i\frac{d}{dt}\alpha_0(t) = \sum_{\vec{k}} \tilde{C}_{\vec{k}}(\vec{X}) \frac{e^{i(\Delta-k)t}}{(2Vk)^{1/2}} \alpha_{\vec{k}}(t) \ ,$$

$$i\frac{d}{dt}\alpha_{\vec{k}}(t) = \tilde{C}_{\vec{k}}^{\,*}(\vec{X}) \frac{e^{-i(\Delta-k)t}}{(2Vk)^{1/2}} \alpha_0(t) \ . \tag{10}$$

The ansatz

$$\alpha_0(t) = \beta_0(t) \ ,$$

$$\alpha_{\vec{k}}(t) = \tilde{C}_{\vec{k}}^{\,*} \frac{e^{-i(\Delta-k)t}}{(2Vk)^{1/2}} \beta_k(t) \ , \tag{11}$$

in which $\beta_k(t)$ depends on $\vec{k}$ only through $k = |\vec{k}|$, reduce the equations to

$$i\frac{d}{dt}\beta_0(t) = \frac{1}{V}\sum_{\vec{k}} \frac{|\tilde{C}_{\vec{k}}|^2}{2k} \beta_k(t) \ ,$$

$$(\Delta - k + i\frac{d}{dt})\beta_k(t) = \beta_0(t) \ . \tag{12}$$

In the first of these equations, the sum over all $\vec{k}$ with fixed k can be carried out. Replacing the sum by integration, we define the function $\mathcal{C}_k(\vec{X})$ by

$$\mathcal{C}_k(\vec{X}) = \frac{k}{2(2\pi)^3} \int d\Omega_{\vec{k}} |\tilde{C}_{\vec{k}}(\vec{X})|^2 \ . \tag{13}$$

Then Eqs. (12) may be written

$$i\frac{d}{dt}\beta_0(t) = \int dk\, \mathcal{C}_k(\vec{X}) \beta_k(t) \ ,$$

$$\left(\Delta - k + i\frac{d}{dt}\right)\beta_k(t) = \beta_0(t) \ . \tag{14}$$

The function $\beta_k(t)$ is expected to have nonzero values only in a narrow region around $k = \Delta$, and if $\mathcal{C}_k$ is a slowly varying function of k, $\mathcal{C}_\Delta(\vec{X})$ can be factored out of the integral in the first of Eqs. (14). The equations then have the solutions, with initial conditions $\beta_0(0) = 1$, $\beta_k(0) = 0$,

$$\beta_0(t) = e^{-\Gamma_\infty t} \ ,$$

$$\beta_k(t) = \frac{e^{-\Gamma_\infty t} - e^{i(\Delta-k)t}}{\Delta - k - i\Gamma_\infty} \ , \tag{15}$$

with $\Gamma_\infty = \pi \mathcal{C}_\Delta(\vec{X})$. (16)

$2\Gamma_\infty$ is the reciprocal mean lifetime of the atom. Under the present circumstances it can in principle depend on $\vec{X}$ because the mirrors have destroyed translational invariance. It remains to show that $\mathcal{C}_k(\vec{X})$ is a slowly varying function of k.

$\tilde{C}_{\vec{k}}(\vec{X})$ is a rapidly varying function of $\vec{k}$ due to the presence of Fabry-Perot modes, but integration over the angular positions of the Fabry-Perot modes eliminates this strong dependence. We write

$$\tilde{C}_{\vec{k}}(\vec{X}) = (A_{\vec{k}} e^{i\vec{k}\cdot\vec{X}} + B_{\vec{k}} e^{-i\vec{k}\cdot\vec{X}}) D \ , \tag{17}$$

with
$$A_{\vec{k}} = \frac{(1-R^4)}{(1-R^2)^2 + 4R^2 \sin^2 \vec{k}\cdot\vec{L}} \ ,$$

$$B_{\vec{k}} = \frac{2R(1-R^2)\cos \vec{k}\cdot\vec{L}}{(1-R^2)^2 + 4R^2 \sin^2 \vec{k}\cdot\vec{L}} \ . \tag{18}$$

Then,
$$\mathcal{C}_k(\vec{X}) = \frac{kD^2}{2(2\pi)^3} \int d\Omega_{\vec{k}} \times [A_{\vec{k}}^2 + B_{\vec{k}}^2 + 2A_{\vec{k}} B_{\vec{k}} \cos(2\vec{k}\cdot\vec{X})] \ . \tag{19}$$

Contributions to the integral come from narrow regions of $\cos\theta$, θ being the angle between $\vec{k}$ and $\vec{L}$, corresponding to Fabry-Perot modes and from wider regions between these modes. We estimate these two kinds of contributions separately when $R^2 \approx 1$, the most interesting case. A mode labeled by the integer m occurs when

$$-(1-R^2) \leq 2(kL\cos\theta - m\pi) \leq 1 - R^2 \ . \tag{20}$$

In this range, $\sin^2 k\cdot L \approx (kL\cos\theta - m\pi)^2$ and $\cos(kL\cos\theta) \approx (-)^m$, so the contribution to the integral in (19) from this mode is

$$2\pi \int_{[m\pi-(1-R^2)]/kL}^{[m\pi+(1-R^2)]/kL} d(\cos\theta) \times \frac{8(1-R^2)^2(1+(-)^m \cos 2\pi m X/L)}{[(1-R^2)^2 + 4k^2L^2\cos^2\theta]^2}$$

$$\approx 32\pi \int_{-\infty}^{\infty} d\mu \frac{(1-R^2)^2}{[(1-R^2)^2 + 4k^2L^2\mu^2]^2}\, U(X) \ ,$$

where $U(\vec{X}) = \cos^2(m\pi X/L)$, even m

$$U(\vec{X}) = \sin^2(m\pi X/L), \qquad \text{odd } m$$

gives the relative energy density in mode m. Exact zeros of $U(\vec{X})$ appear only when $R=1$ and one has unit standing wave ratio.

We obtain a contribution $\mathcal{C}_k{}^m(\vec{X})$ which is proportional to the relative energy density of the mode at $\vec{X}$,

$$\mathcal{C}_k{}^m(\vec{X}) = \frac{D^2}{2\pi L(1-R^2)}\, U(\vec{X}) \quad . \tag{21}$$

For infinite mirrors, the number of modes with $k=\Delta$ is

$$M_\infty = \frac{2L\Delta}{\pi} \quad . \tag{22}$$

The total contribution of the modes is M_∞ times the average $\langle \mathcal{C}_k{}^m(\vec{X})\rangle$ over the modes. From the form of $U(\vec{X})$, this averaging introduces about a factor of $\frac{1}{2}$, which is not important for our purpose here and which will be replaced by unity.

The nonmode contribution comes from regions of $\cos\theta$, where the integrand is a rapidly varying function of $\cos\theta$ with a smooth envelope, so it can be estimated by inserting appropriate averages over $\cos\theta$ in the integrand. Thus,

$$A_{\vec{k}} \rightarrow (1-R^2) \quad ,$$

$$B_{\vec{k}} \rightarrow 0, \; B_{\vec{k}}{}^2 \rightarrow \tfrac{1}{2}(1-R^2)^2 \quad ,$$

and we obtain

$$\mathcal{C}_k{}^{\text{non}}(\vec{X}) = \frac{3kD^2}{8\pi^2}(1-R^2)^2 \quad , \tag{23}$$

which is independent of $\vec{X}$. This establishes that $\mathcal{C}_k$ is a slowly varying function of k and can be factored out of the integral in Eq. (14).

The total radiation rate Γ_∞ is the sum of the rates into the modes and between the modes:

$$\Gamma_\infty = \pi\mathcal{C}_\Delta{}^{\text{non}} + 2L\Delta\mathcal{C}_\Delta{}^m$$
$$\approx \frac{3D^2\Delta}{8\pi}(1-R^2)^2 + \frac{D^2\Delta}{\pi(1-R^2)} \quad . \tag{24}$$

The nonmode radiation is suppressed by the factor $(1-R^2)^2$ so we conclude that the radiation is overwhelmingly into the Fabry-Perot modes, and that it is emitted faster than it would be by an atom in free space. The result becomes invalid for R^2 too close to unity, because then the time T needed to establish the field exceeds the mean lifetime of the atom. The condition on the validity of Eq. (24) is

$$\tau = \frac{1}{2\Gamma_\infty} = \frac{\pi(1-R^2)}{2\Delta D^2} > \frac{L/c}{1-R^2} \quad ,$$
$$\text{or} \quad (1-R^2)^2 > \frac{2L\Delta D^2}{\pi c} \quad . \tag{25}$$

If this criterion is not met, the rate of radiation into modes is obtained approximately by replacing $(1-R^2)$ by $[2L\Delta D^2/(\pi c)]^{1/2}$ in Eq. (24).

IV. RADIATION BETWEEN FINITE MIRRORS

We must now see how the considerations of Sec. III must be modified when the mirrors involved are finite. We take them to be circular and to subtend the angle Θ at the origin. There are two extreme situations.

(a) $\vec{k}$ is so nearly parallel to the x axis that, after $(1-R^2)^{-1}$ reflections, the ray with direction $\vec{k}$ starting at the origin has not walked off the mirror system. For such modes, the results of Sec. III will be nearly exact; diffraction losses can be accounted for by decreasing R if necessary. When Θ is small, the condition for this situation is that

$$(1-R^2)^{-1}\theta \leqslant \Theta \quad . \tag{26}$$

The number of modes within this range $M(\Theta, R)$ is

$$M(\Theta,R) = \frac{2kL}{\pi}[1-\cos(1-R^2)\Theta]$$
$$= \frac{kL}{\pi}(1-R^2)^2\Theta^2 \quad , \tag{27}$$

which counts $\vec{k}$'s both nearly parallel and nearly antiparallel to $\vec{L}$. This number decreases with increasing R, because the condition for a fully developed mode becomes more stringent with increasing R. Here again, the limit $R=1$ makes no sense, because there is not sufficient time to develop a mode this completely, and the substitution given at the end of the previous section may be used. Then, for R not too large

$$\Gamma^{\text{mode}}(\Theta,R) = \frac{kD^2}{2\pi}(1-R^2)\Theta^2 \quad . \tag{28}$$

(b) $\vec{k}$ makes an angle θ with the x axis so large that no multiple reflections take place. If

$$\theta > \Theta \quad ,$$

then the radiation in this direction is at the free-space rate Γ_0. The solid angle Ω available for this radiation is

$$\Omega = 4\pi(1-\tfrac{1}{2}\Theta^2) \quad , \tag{29}$$

the cones about $\Theta=0$ and $\Theta=\pi$ being excluded. Thus,

$$\Gamma^{\text{non}} = \Gamma_0(1 - \tfrac{1}{2}\Theta)^2 \quad . \tag{30}$$

Between these two extremes lie the not fully developed modes. These could be treated with the help of the Fox-Li modes, but this is unnecessary for our purpose here. It was mentioned in connection with Eq. (27) that $M(\Theta, R)$ becomes smaller with increasing R because more reflections are needed to establish the mode. We can overestimate the contribution of modes by omitting the factor $(1 - R^2)$ in Eq. (27), and thereby count as fully developed any mode associated with a k which yields any reflections at all. Thus, an upper limit to the rate of radiation is

$$\begin{aligned} \Gamma &= \Gamma^{\text{non}} + \Gamma^{\text{mode}} \quad , \\ \Gamma &= \Gamma_0(1 - \tfrac{1}{2}\Theta^2) + \Gamma_0(2\Theta^2) \quad , \\ \Gamma &= \Gamma_0(1 + \tfrac{3}{2}\Theta^2) \quad , \end{aligned} \tag{31}$$

with $\Gamma_0 = \Delta D^2/(4\pi)$, the free-atom decay constant. This is very nearly the free-atom rate. For the case mentioned in the beginning of this paper, 2-cm mirrors 20 cm apart, $\Theta = 0.2$ and a 6% effect is involved, at most.

There is an enhancement of the radiation into the fully developed modes. This enhancement factor is just the ratio of $\mathcal{C}_\Delta{}^m$ as given in Eq. (21) to C_k multiplied by the fractional solid angle preempted by one mode,

$$\begin{aligned} \frac{\mathcal{C}_k{}^m}{C_k \delta\Omega} &= \frac{D^2/2\pi L(1 - R^2)}{D^2/(2L)} \, , \\ \frac{\mathcal{C}_k{}^m}{C_k \delta\Omega} &= \frac{1}{\pi(1 - R^2)} \quad . \end{aligned} \tag{32}$$

This factor can be large, but the solid angle involved is small so that the over-all effect is also small, because in using this one must also use the number of modes $M(\Theta, R)$ as given in Eq. (27) including the factor $(1 - R^2)^2$.

V. DISCUSSION OF RESULTS

It has been shown how the effect of the presence of mirrors on the emission of radiation by an excited atom can be included in the Weisskopf-Wigner method, yielding results which are in accord with intuitive expectations. When a cavity is present which affects many of the modes into which the atom can decay, the radiation is primarily into those modes. When only a few modes are affected by the cavity, the decay is only slightly affected, but the radiation into a mode is greater than the solid angle of the mode requires. Even allowing for this enhancement, however, an atom in a laser etalon will decay predominantly into free-space modes. The action of a laser in concentrating radiation into modes of the laser etalon is not due merely to the presence of the etalon, and the suppression of interaction of the atom with any modes other than etalon modes in the Hamiltonian of the system is an approximation which must be justified by the behavior of a many-atom system between the mirrors. That theories based on this suppression are valid in a large range of circumstances is beyond question, but the means by which it can be established are themselves of interest and will be the subject of a later paper.

It is a pleasure to thank V. Ernst and B. Neumcke for very helpful conversations. Professor F. Bopp's hospitality in Munich is gratefully acknowledged.

---

*Supported in part by the U.S. Army Research Office, Durham.

†Permanent address: Physics Department, University of Pittsburgh, Pittsburgh, Pa.

[1]V. Weisskopf and E. Wigner, Z. Physik 63, 54 (1930).

[2]W. H. Furry, Phys. Rev. 81, 115 (1951); F. E. Low, *ibid.* 88, 53 (1952); S. S. Schweber, Relativistic Quantum Field Theory (Harper and Row, New York, 1961), Sec. 15g.

[3]V. Ernst and P. Stehle, Phys. Rev. 176, 1456 (1968).

[4]J. P. Gordon, H. Zeiger, and C. H. Townes, Phys. Rev. 99, 1264 (1955); K. Shimoda, T. Wang, and C. H. Townes, *ibid.* 102, 1308 (1956).

[5]W. E. Lamb, Phys. Rev. 134, A1429 (1964); M. Scully and W. E. Lamb, *ibid.* 159, 208 (1968).

[6]A. G. Fox and T. Li, Bell Syst. Tech. J. 40, 453 (1961).

[7]V. Ernst (unpublished).