Scattering of a charged particle by a model atom in the presence of a low-frequency laser

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The problem of the scattering of a charged particle by a two-state atom in the presence of a low-frequency laser is considered. The atom is allowed to distort adiabatically due to the laser, and the scattering is obtained as a power series in ω, the laser frequency. When the distortion of the atom is neglected the multichannel generalization of the Kroll-Watson result is obtained but the distortion introduces new terms in first order of ω which are not simply understandable. The relevance of the model to the real problem is discussed.

I. INTRODUCTION

When an electron scatters from an atom in the presence of a laser field the laser photons may play the role of a third body thereby allowing off-shell electron-atom scattering and other exotic effects to be observed. The laser is a particularly useful "third body" in that its coupling to the other two is simple and its "density" may be high. The full problem is too difficult to have been solved in full but it can be handled in a variety of special cases.[1] One useful one is the limit in which the laser photon energy ω ($\hbar = 1$) is low compared to the other relevant energies of the problem. Kroll and Watson[2] have treated this problem in the approximation in which the internal degrees of freedom of the atom could be neglected. In that case the problem is one of potential scattering of a charged particle in a laser field. They showed that the scattering could be expressed in terms of the on-shell T matrix for the scattering in the absence of the laser in the lowest two orders in ω, ω^0, and ω^1. Another method[3] confirmed their results and showed that off-shell effects entered in order ω^2.

A recent experiment[4] in which 11-eV electrons were scattered off argon in the presence of a CO_2 laser beam ($\omega \sim 1.2\times 10^{-3}$ eV) gave qualitative agreement with the theory and thus raised the question of the effect of the internal degrees of freedom of the target on the scattering.

In this paper the second method[3] is used to describe the scattering of a charged particle on a model atom in a laser field. The atom is described by a two-state model, but it is clear that the results generalize simply to a multistate atom as long as only a *finite* number of states are allowed and exchange is neglected. The result, Eq. (42), shows that the effect of the laser in lowest order ω^0 can be described by the appropriate average of a T matrix, but that the next higher order ω^1 introduces off-shell effects due to the internal degrees of freedom of the atom.

In Sec. III some conjectures are made concerning the real problem in which both the effect of the Pauli principle and the atomic continuum are included.

II. FORMAL DEVIATION

In describing the scattering of an electron by a two-state model atom our starting point is the atomic wave functions in the presence of the laser. These can be obtained analytically[5] in the adiabatic approximation in which the laser photon energy is low compared to the energy difference between the atomic states,

$$\omega \ll W . \tag{1}$$

(We shall set the energy zero such that the two states lie at $\pm\frac{1}{2}W$.) The details of the states will not be needed but they are included for completeness:

$$\Phi_+ = \frac{e^{-i\bar{\epsilon}t/2 - \frac{1}{2}i\int_0^t dt'[\epsilon(\omega t') - \bar{\epsilon}]}}{[2\epsilon(\omega t)(\epsilon(\omega t) - w)]^{1/2}} \times \{[\epsilon(\omega t) - W]u_0 + 2\Lambda^* \cos\omega t u_1\} , \tag{2}$$

$$\Phi_- = \frac{e^{i\bar{\epsilon}t/2 + \frac{1}{2}i\int_0^t dt'(\epsilon(\omega t') - \bar{\epsilon})}}{[2\epsilon(\omega t)(\epsilon(\omega t) + w)]^{1/2}} \times \{-[\epsilon(\omega t) + W]u_0 + 2\Lambda \cos\omega t u_1\} , \tag{3}$$

where u_0 and u_1 are the bare atomic states, Λ is the dipole matrix element of the laser-atom interaction connecting the two, $\epsilon(\omega t)$ is a generalization of the Rabi frequency,

$$\epsilon(\omega t) = (W^2 + 4|\Lambda|^2 \cos^2\omega t)^{1/2} , \tag{4}$$

and $\bar{\epsilon}$ is the average value of $\epsilon(\omega t)$. The total wave function of atom plus projectile is then written

$$\psi = F_+(\vec{r}, t)e^{i\bar{\epsilon}t/2}\Phi_+ + F_-(\vec{r}, t)e^{-i\bar{\epsilon}t/2}\Phi_- , \tag{5}$$

where $\vec{r}$ is the projectile coordinate. This can be substituted into the Schrödinger equation and with the use of the orthonormality of $\Phi_\pm$, equations for F are obtained of the form

$$\left[i\frac{\partial}{\partial t} - \left(\frac{\bar{\epsilon}}{2}\sigma_z + \frac{\vec{p}^2}{2m} + \frac{e}{m}\vec{p}\cdot\vec{A}(t)\right) - V(\vec{r},\omega t)\right]F(\vec{r},t) = 0\,, \tag{6}$$

where a matrix notation has been introduced:

$$F(\vec{r},t) = \begin{pmatrix} F_+(\vec{r},t) \\ F_-(\vec{r},t) \end{pmatrix} \tag{7}$$

and

$$V(\vec{r},\omega t) = \begin{pmatrix} V_{++} & V_{+-} \\ V_{-+} & V_{--} \end{pmatrix}. \tag{8}$$

The choice (5) assures that V can be expanded in a Fourier series

$$V(\vec{r},\omega t) = \sum_{n=-\infty}^{\infty} V_n(\vec{r})e^{-in\omega t} \tag{9}$$

since the $e^{\pm i\bar{\epsilon}t}$ factors have been removed. The initial and final states can be taken as the states of (6) in the absence of the interaction, $V = 0$. These can be written

$$x = \exp[i\vec{k}\cdot\vec{r} - i\vec{k}\cdot\vec{\alpha}(t) - i\epsilon_k t - \tfrac{1}{2}i\bar{\epsilon}t\sigma_z]u\,, \tag{10}$$

where

$$\vec{\alpha}(t) = \frac{e}{mc}\int_0^t dt'\,\vec{A}(t') = \vec{\alpha}_0 \sin\omega t\,, \quad \epsilon_k = \frac{k^2}{2m}\,, \tag{11}$$

and u is the atomic state which in this notation is $u = \binom{0}{1}$ for the ground state or $u = \binom{1}{0}$ for the excited state.

The exact S matrix for scattering from the initial to final state is

$$S_{fi} = -i\langle f|V + V\tilde{G}V|i\rangle\,, \tag{12}$$

where $\tilde{G}$ is the Green's function for the full problem. It can be expanded in powers of V yielding

$$S_{fi} = -i\sum_{n=0}^{\infty}\langle f|V(GV)^n|i\rangle = \sum_{n=0}^{\infty} S_{fi}^{(n+1)}\,, \tag{13}$$

where G is the Green's function for the operator in Eq. (6) with $V = 0$. It can be written

$$G(\vec{r}t;\vec{r}'t') = -i\theta(t-t') \times\int \frac{d^3k}{(2\pi)^3}\exp i\{\vec{k}\cdot(\vec{r}-\vec{r}') - \vec{k}\cdot[\vec{\alpha}(t) - \vec{\alpha}(t')] - (\epsilon_k + \tfrac{1}{2}\bar{\epsilon}\sigma_z)(t-t')\}\,. \tag{14}$$

Let us consider one of the terms, $S_{fi}^{(n+1)}$, in (13). If we make explicit use of (14), it can be written

$$S_{fi}^{(n+1)} = (-i)^{n+1}\int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{t_1} dt_2 \cdots \int_{-\infty}^{t_n} dt_{n+1} \int \frac{d^3k_1 \cdots d^3k_n}{(2\pi)^{3n}} \sum_{\substack{l_1 \cdots l_{n+1} \\ s_1 \cdots s_{n+1}}} J_{l_1}(\vec{K}_1\cdot\vec{\alpha}_0)\cdots J_{l_{n+1}}(\vec{K}_{n+1}\cdot\vec{\alpha}_0)(-1)^{\Sigma_j l_j} \times (u_f e^{iA_1t_1}\tilde{V}_{s_1}(\vec{K}_1,t_1)e^{iA_2t_2}\tilde{V}_{s_2}(\vec{K}_2,t_2)\cdots \tilde{V}_{s_{n+1}}(\vec{K}_{n+1},t_{n+1})u_i)\,, \tag{15}$$

where we have used

$$e^{i\vec{K}_j\cdot\vec{\alpha}(t_j)} = \sum_{l_j=-\infty}^{\infty} J_{l_j}(\vec{K}_j\cdot\vec{\alpha}_0)e^{-il_j\omega t_j}(-1)^{l_j} \tag{16}$$

and

$$\tilde{V}_{s_j}(\vec{K}_j,t) = e^{i\bar{\epsilon}t\sigma_z/2}\tilde{V}_{s_j}(\vec{K}_j)e^{-i\bar{\epsilon}t\sigma_z/2} = \int d^3r\, e^{-i\vec{K}_j\cdot\vec{r}}e^{i\bar{\epsilon}t\sigma_z/2}V_{s_j}(r)e^{-i\bar{\epsilon}t\sigma_z/2}\,. \tag{17}$$

We have also defined

$$\vec{K}_1 = \vec{p}_f - \vec{k}_1,\ \vec{K}_2 = \vec{k}_1 - \vec{k}_2 \cdots \vec{K}_{n+1} = \vec{k}_n - \vec{p}_i \tag{18}$$

and

$$A_j = \epsilon_{k_{j-1}} - \epsilon_{k_j} - (l_j + s_j)\omega - i\eta \tag{19}$$

with

$$\epsilon_{k_0} = \epsilon_{p_f}\,, \quad \epsilon_{k_{n+1}} = \epsilon_{p_i}\,.$$

The first objective is the extraction of the energy-conservation δ function. This can be accomplished by noting that the time dependence of the potential matrix (17) has a simple form which can be written as

an expansion in Pauli matrices:

$$\tilde{V}_s(\vec{K},t)=\tilde{V}_s^{(0)}(\vec{K})+\tilde{V}_s^{(z)}(\vec{K})\sigma_z+\tilde{V}_s^{(+)}(\vec{K})e^{i\bar{\epsilon}t}\sigma_+ +\tilde{V}_s^{(-)}(\vec{K})e^{-i\bar{\epsilon}t}\sigma_- . \tag{20}$$

The algebra of these matrices limits the combinations of σ_+ and σ_- that can occur in (15). For instance, if u_i is the ground state and u_f the excited state than one more power of σ_+ than of σ_- must occur in (15), and from (20) this says that one more power of $e^{i\bar{\epsilon}t}$ than of $e^{-i\bar{\epsilon}t}$ must occur. Now with the substitutions

$$\begin{aligned} t_2&=t_1-\tau_1,\\ t_3&=t_1-\tau_1-\tau_2,\\ &\cdot\\ &\cdot\\ &\cdot\\ t_{n+1}&=t_1-\tau_1-\tau_2\cdots-\tau_n, \end{aligned} \tag{21}$$

(15) becomes

$$\begin{aligned} S_{fi}^{(n+1)}=(-i)^{n+1}\int_{-\infty}^{\infty}dt_1\int_0^{\infty}d\tau_1\cdots\int_0^{\infty}d\tau_n\int\left(\frac{d^3k}{(2\pi)^3}\right)^n\sum_{\substack{l_1\cdots l_{n+1}\\ s_1\cdots s_{n+1}}}J_{l_1}(\vec{K}_1\cdot\vec{\alpha}_0)\cdots J_{l_{n+1}}(\vec{K}_{n+1}\cdot\vec{\alpha}_0)(-1)^{\Sigma_i l_i}\\ \times(u_f e^{iA_1t_1}\tilde{V}_{s_1}(K_1,t_1)e^{iA_2(t_1-\tau_1)}\tilde{V}_{s_2}(\vec{K}_2,t_1-\tau_1)\cdots\\ e^{iA_{n+1}(t_1-\tau_1-\tau_2\cdots-\tau_n)}\tilde{V}_{s_{n+1}}(\vec{K}_{n+1},t_1-\tau_1\cdots-\tau_n)u_i). \end{aligned} \tag{22}$$

Since t_1 enters in every potential and since the number of $e^{\pm i\bar{\epsilon}t}$ factors from the potential product is fixed by the initial and final states then the total t_1 behavior from the product of potentials is also fixed by them. The t_1 integral can then be done yielding the energy δ function of the form

$$\delta\left(\epsilon_{p_f}-\epsilon_{p_i}-(\delta_i-\delta_f)\frac{\bar{\epsilon}}{2}-\omega\sum_{i=1}^{n+1}(s_i+l_i)\right),$$

where

$$\sigma_z u_{i,f}=\delta_{i,f}u_{i,f}. \tag{23}$$

The summation variables in (22) can be shifted by writing

$$l_1=l-\sum_{i=1}^{n+1}s_i-\sum_{i=2}^{n+1}l_i, \tag{24}$$

yielding

$$S_{fi}^{(n+1)}=-2\pi i\sum_{l=-\infty}^{\infty}\delta\left(\epsilon_{p_f}+\delta_f\frac{\bar{\epsilon}}{2}-\epsilon_{p_i}-\delta_i\frac{\bar{\epsilon}}{2}-\omega l\right)T_{fi}^{(n+1)}(l), \tag{25}$$

thereby defining the transition matrix for scattering with transfer of l photons

$$\begin{aligned} T_{fi}^{(n+1)}(l)=(-i)^n\int_0^{\infty}d\tau_1\cdots\int_0^{\infty}d\tau_n\int\left(\frac{d^3k}{(2\pi)^3}\right)^n\sum_{\substack{s_1\cdots s_{n+1}\\ l_2\cdots l_{n+1}}}J_{l_1}(\vec{K}_1\cdot\vec{\alpha}_0)\cdots J_{l_{n+1}}(\vec{K}_2\cdot\vec{\alpha}_0)(-1)^{l-\Sigma_i s_i}\\ \times(u_f\tilde{V}_{s_1}(\vec{K}_1,0)e^{-iA_2\tau_1}\tilde{V}_{s_2}(\vec{K}_2,-\tau_1)\cdots\\ e^{-iA_{n+1}(\tau_1+\cdots+\tau_n)}\tilde{V}_{s_{n+1}}(\vec{K}_{n+1},-\tau_1\cdots-\tau_n)u_i). \end{aligned} \tag{26}$$

The τ dependence of the potentials can be extracted with the use of the first part of (17) and then all the τ integrals can be performed with the result

$$\begin{aligned} T_{fi}^{(n+1)}(l)=(-1)^l\int\left(\frac{d^3k}{(2\pi)^3}\right)^n\sum_{\substack{L_1\cdots L_n\\ S_0\cdots S_n}}(-1)^{S_0}J_{l-S_0-L_1}(\vec{K}_1\cdot\vec{\alpha}_0)J_{L_1-L_2}(\vec{K}_2\cdot\vec{\alpha}_0)\cdots J_{L_n}(\vec{K}_{n+1}\cdot\vec{\alpha}_0)\\ \times\left(u_f\tilde{V}_{S_0-S_1}(\vec{K}_1)\frac{1}{D_1}\tilde{V}_{S_1-S_2}(\vec{K}_2)\frac{1}{D_2}\cdots\frac{1}{D_n}\tilde{V}_{S_n}(\vec{K}_{n+1})u_L\right), \end{aligned} \tag{27}$$

where we have shifted variables by

$$S_i = \sum_{i=j+1}^{n+1} s_i, \quad L_j = \sum_{i=j+1}^{n+1} l_i \tag{28}$$

and defined

$$D_j = \epsilon^+_{p_i} + \tfrac{1}{2}\bar{\epsilon}\,\delta_i - (\epsilon_{k_j} + \tfrac{1}{2}\bar{\epsilon}\,\sigma_z) + \omega(S_j + L_j)\,. \tag{29}$$

Equation (27) is our starting point for the expansion in powers of ω. If only terms up to ω^l are retained this becomes

$$T^{(n+1)}_{fi}(l) = (-1)^l \int \left(\frac{d^3k}{(2\pi)^3}\right)^n \sum_{\substack{L_1\cdots L_n \\ S_0\cdots S_n}} (-1)^{S_0} J_{l-S_0-L_1}(\vec{K}_1\cdot\vec{\alpha}_0) J_{L_1-L_2}(\vec{K}_2\cdot\vec{\alpha}_0)\cdots J_{L_n}(\vec{K}_{n+1}\cdot\vec{\alpha}_0)$$
$$\times\left(1 - \sum_{j=1}^{n} \omega(S_j+L_j)\frac{\partial}{\partial\epsilon_{k_j}}\right)\left(u_f\,\tilde{V}_{S_0-S_1}(\vec{K}_1)\,\frac{1}{\Delta_1}\,\tilde{V}_{S_1-S_2}(\vec{K}_2)\cdots\frac{1}{\Delta_n}\,\tilde{V}_{S_n}(\vec{K}_{n+1})u_i\right), \tag{30}$$

where

$$\Delta_j = D_j(\omega = 0)\,.$$

The inverse of (9) can be Fourier transformed to give

$$\tilde{V}_S(\vec{K}) = \int_0^{2\pi} \frac{d\theta}{2\pi}\, e^{iS\theta}\,\tilde{V}(\vec{K},\theta)\,. \tag{31}$$

If each of the n factors of $\tilde{V}$ in (30) replaced in this way then the S sums can be performed with the aid of the identities

$$\sum_{r=-\infty}^{\infty} e^{-ir\phi} J_r(z) = e^{-iz\sin\phi} \tag{32}$$

and

$$\sum_{r=-\infty}^{\infty} e^{ir(\theta-\theta')} = 2\pi\,\delta(\theta-\theta')\,, \tag{33}$$

which then allows the performance of all but one of the θ integrals. The L sums can then be done in a manner identical with that used in Ref. 3, with the result

$$T^{(n+1)}_{fi}(l) = \int \left(\frac{d^3k}{(2\pi)^3}\right)^n \int_0^{2\pi} \frac{d\theta_1}{2\pi}\, e^{il\theta_1 + i\vec{q}\cdot\vec{\alpha}_0\sin\theta_1} \left\{1 + \omega\sum_{j=1}^{n}(\vec{k}_n - \vec{p}_i)\cdot\vec{\alpha}_0\cos\theta_1\frac{\partial}{\partial\epsilon_{k_j}}\right.$$
$$\left. + \omega\sum_{j=1}^{n}\sum_{r=j+1}^{n+1}\frac{1}{i}\,\frac{\partial}{\partial\theta_r}\,\frac{\partial}{\partial\epsilon_{k_j}}\right\}$$
$$\times\left(u_f\,\tilde{V}(\vec{K},\theta_1)\,\frac{1}{\Delta_1}\,\tilde{V}(K_2\theta_2)\cdots\frac{1}{\Delta_n}\,\tilde{V}(\vec{K}_{n+1},\theta_{n+1})u_i\right)_{\theta_j=\theta_1}, \tag{34}$$

where $\vec{q} = \vec{p}_f - \vec{p}_i$. The factor $\cos\theta_1$ in the second term of the curly brackets can be replaced by

$$e^{i\vec{q}\cdot\vec{\alpha}_0\sin\theta_1}\cos\theta_1 = \frac{1}{i\vec{q}\cdot\vec{\alpha}_0}\,\frac{\partial}{\partial\theta_1}\,e^{i\vec{q}\cdot\vec{\alpha}_0\sin\theta_1}, \tag{35}$$

and then an integration by parts on θ_1 results in a modified form of the curly brackets which is

$$\{\ \} = \left\{1 - l\omega\sum_{j=1}^{n}\frac{(\vec{k}_j - \vec{p}_i)\cdot\vec{\alpha}_0}{\vec{q}\cdot\vec{\alpha}_0}\,\frac{\partial}{\partial\epsilon_{k_j}} - \omega\sum_{j=1}^{n}\frac{(\vec{k}_j - \vec{p}_i)\cdot\vec{\alpha}_0}{\vec{q}\cdot\vec{\alpha}_0}\,\frac{1}{i}\,\frac{\partial}{\partial\theta_1}\,\frac{\partial}{\partial\epsilon_{k_j}} + \omega\sum_{j=1}^{n}\sum_{r=j+1}^{n+1}\frac{1}{i}\,\frac{\partial}{\partial\theta_r}\,\frac{\partial}{\partial\epsilon_{k_j}}\right\}. \tag{36}$$

The first two terms of this expression are identical with those occurring in the potential scattering problem and have been dealt with previously.[2,3] They can be absorbed by a shift in momenta in the zero-order term

$$\vec{P}_i = \vec{p}_i - \vec{\lambda}, \quad \vec{P}_f = \vec{p}_f - \vec{\lambda}, \quad \vec{k}'_j = \vec{k}_j - \vec{\lambda}, \quad \lambda = m\omega l\vec{\alpha}_0/(\vec{q}\cdot\vec{\alpha}_0)\,. \tag{37}$$

Then summing over all n, we obtain

$$T_{fi}(l)=\int_0^{\pi}\frac{d\theta_1}{2\pi'}\,e^{il\theta_1+i\vec{q}\cdot\vec{\alpha}_0\sin\theta_1}\sum_{n=0}^{\infty}\int\left(\frac{d^3k'}{(2\pi)^3}\right)^n\left\{1-\omega\sum_{j=1}^{n}\frac{(k_j'-\vec{P}_i)\cdot\alpha_0}{\vec{q}\cdot\vec{\alpha}_0}\frac{1}{i}\frac{\partial}{\partial\theta_1}\frac{\partial}{\partial\epsilon_{kj}}+\omega\sum_{j=1}^{n}\sum_{r=j+1}^{n+1}\frac{1}{i}\frac{\partial}{\partial\theta_r}\frac{\partial}{\partial\epsilon_{kj}}\right\}$$
$$\times\left(u_f\tilde{V}(\vec{K}_1\theta_1)\frac{1}{\Delta_1'}\tilde{V}(\vec{K}_2,\theta_2)\cdots\tilde{V}(\vec{K}_{n+1},\theta_{n+1})u_1\right)_{\theta_j=\theta_1}, \tag{38}$$

where the primes indicate the shift to the new variables of (37). The n sum in the first term is simply performed by use of

$$\sum_{n=0}^{\infty}\left(u_f\tilde{V}_1\frac{1}{\Delta_1}\tilde{V}_2\cdots\tilde{V}_{n+1}u_i\right)=(\vec{P}_f,f|T(\theta)|\vec{P}_i,i), \tag{39}$$

where

$$T(\theta)=V(\theta)+V(\theta)[1/(E_i^+-H)]T(\theta), \tag{40}$$

where $V(\theta)$ is the potential of (9) with $\omega t=\theta$ and

$$E_i^+=\epsilon_{Pi}+\tfrac{1}{2}\delta_i\bar{\epsilon}+i\eta,\quad H=P^2/2m+\tfrac{1}{2}\bar{\epsilon}\sigma_z. \tag{41}$$

The last two terms may also be expressed in terms of this T operator with the result

$$T_{fi}(l)=\int_0^{2\pi}\frac{d\theta}{2\pi}\,e^{il\theta+i\vec{q}\cdot\vec{\alpha}_0\sin\theta}(\vec{P}_f,f|T(\theta)+\frac{i\partial}{\partial\theta}\left(\frac{T(\theta)\omega\vec{\alpha}_0\cdot(\vec{P}-\vec{P}_i)T(\theta)}{\vec{\alpha}_0\cdot\vec{q}(E_i^+-H)^2}\right)-iT(\theta)\frac{\omega}{(E_i^+-H)^2}\frac{\partial}{\partial\theta}T(\theta)|\vec{P}_i,i), \tag{42}$$

which can be written in a more symmetric form as

$$T_{fi}(l)=\int_0^{2\pi}\frac{d\theta}{2\pi}\,e^{il\theta+i\vec{q}\vec{\alpha}_0\sin\theta}(\vec{P}_f,f|T(\theta)+\frac{i\omega}{\vec{\alpha}_0\cdot\vec{q}}\frac{\partial}{\partial\theta}\left(T(\theta)\frac{\vec{\alpha}_0\cdot\vec{P}}{(E_i^+-H)^2}T(\theta)\right)+\frac{i\omega}{2}\left(\frac{\partial T(\theta)}{\partial\theta}\frac{1}{(E_i^+-H)^2}T(\theta)\right)$$
$$-\frac{i\omega}{2}\left(T(\theta)\frac{1}{(E_i^+-H)^2}\frac{\partial T(\theta)}{\partial\theta}\right)|\vec{P}_i,i). \tag{43}$$

The interpretation of the first term is interesting: $V(\omega t)$ is the electron-atom interaction matrix as modified by the distortion of the atom by the laser. $T(\theta)$ is the exact scattering T operator in the approximation that the distorted atom is frozen during the scattering (a low-frequency approximation), and the θ integral is a coherent average over the cycle of the laser. The last two terms are difficult to understand but if one makes the approximation that the atom is undistorted by the laser, then T is independent of θ and equal to the T operator in the absence of the laser. Thus, the last two terms vanish, and the first term is just the multichannel generalization of the Kroll-Watson result.

This T matrix can then be used to form a cross section in the conventional way,

$$\frac{d\sigma_{fi}}{d\Omega}(l)\sim[p_f(l)/p_i]|T_{fi}(l)|^2, \tag{44}$$

which in the no-distortion limit can be written

$$\frac{d\sigma_{fi}}{d\Omega}(\vec{p}_f(l);\vec{p}_i;(l))=J_l^2[\vec{q}(l)\cdot\vec{\alpha}_0]\frac{p_f(l)}{p_i(0)}\frac{P_i(l)}{P_f(l)}$$
$$\times\frac{d\sigma_{fi}}{d\Omega}(\vec{P}_f(l),\vec{P}_i(l)). \tag{45}$$

Here the magnitude of the final momentum $p_f(l)$ is obtained from the energy-conserving δ function in (25). It, and the momentum transfer $\vec{q}$ both depend upon l as do the shifted momenta $\vec{P}_f(l)$ and $\vec{P}_i(l)$. If the product $l\omega$ is small then these l dependencies are all weak and the multichannel generalization of the Kroll and Watson sum rule may be obtained from (45):

$$\sum_l\frac{d\sigma_{fi}}{d\Omega}(\vec{p}_f,\vec{p}_i;l)=\frac{d\sigma_{fi}}{d\Omega}(\vec{P}_f,\vec{P}_i)\Big|_{\alpha_0=0}. \tag{46}$$

It should be noted that the same type of sum rule applies even when the distortions of the target due to the laser are retained in (43). Again neglecting the l dependence in $\vec{p}_f$ and $\vec{q}$, we may perform the l sum

$$\sum_l|T_{fi}(l)|^2=\int_0^{2\pi}\frac{d\theta}{2\pi}\,|(\vec{p}_f,f|T(\theta)|\vec{p}_i,i)|^2, \tag{47}$$

where terms of order ω have been neglected. This can be converted to a cross section with the result

$$\sum_l\frac{d\sigma_{fi}}{d\Omega}(\vec{p}_f,\vec{p}_i;l)=\int_0^{2\pi}\frac{d\theta}{2\pi}\frac{d\sigma}{d\Omega}(p_f,p_i;\theta) \tag{48}$$

where the cross section on the right-hand side can be understood as follows: The target states are distorted by the laser and so depend upon the phase ($\theta = \omega t$) of the laser. The scattering cross section is calculated with the phase fixed and then an *incoherent* average over this phase is performed.

The calculation has assumed a single-mode description of the laser which is unrealistic. A more realistic description can be obtained by allowing the laser amplitude to be slowly varying (on the time scale of ω^{-1}). This modification can be included by performing an ensemble average of the cross section over the various possible values of the laser amplitude.[6]

III. DISCUSSION

The development above made no explicit use of the form of the atomic wave functions other than the periodicity of V [Eq. (9)]. It is obvious that there is no difficulty in generalizing to a multistate atomic model. Again, only the periodicity is necessary but this is a general result for a real atom.[7] However inclusion of the atomic continuum presents some real problems since these states are dense and it is questionable whether the adiabatic approximation for the wave functions are ever justified.

The inclusion of exchange between the projectile and one of the bound electrons can be effected through the device of the optical potential. The principal modification of that inclusion is to make the interaction-potential matrix nonlocal. If one returns to the procedure of Sec. II it is evident that the only use of the locality of V was made at the introduction of the translations (37), where we used

$$(\vec{k}|V|\vec{k}') = V(\vec{k} - \vec{k}') .$$

This is not true for a nonlocal potential so the Kroll-Watson treatment of the first-order terms will not work. This means that the effect of the Pauli principle (exchange scattering) will introduce a correction of order ω in addition to the ones already contained in (43). The zero-order term is still justified so barring our misgivings about the adiabatic treatment of the continuum atomic states we may expect that the first term of (43) can be applied in the real world.

The experiment of Weingartshofer *et al.*[4] gave only qualitative agreement with the details of Kroll-Watson result but gave quantitative agreement with the sum rule

$$\sum_l \frac{d\sigma_{fi}(l)}{d\Omega} = \left(\frac{d\sigma_{fi}}{d\Omega}\right)_{\alpha_0 = 0} . \tag{49}$$

If one assumes that the argon target was not distorted at all by their laser, then Eq. (43) reduces to the Kroll-Watson result and this sum rule is justified. We see from the two-state model that the condition

$$|\Lambda| \ll W$$

would justify that approximation. The excitation energy of argon is a few eV and their laser intensity was of the order of[8] 10^7 W/cm^2, which yields a value of Λ of the order of 5×10^{-6} eV which justifies the approximation. However, the analysis given above shows that the sum rule would be satisfied for much higher laser intensities.

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[1]See, for example, M. H. Mittleman, Phys. Rev. A 16, 1549 (1977).
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