Differential cross sections for the production of H atoms in collisional electron detachment of H^- ions on He, H, and H_2 targets

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The differential cross sections for collisional electron detachment of H^- ions by He, H, and H_2 targets in the 100-keV–10-MeV energy region is investigated. The summation over all the final states which contribute to electron detachment is carried out in the closure approximation using the concept of the average energy loss at a fixed scattering angle. In the Born approximation, the differential cross section for electron-detachment collisions is strongly peaked in the forward direction. We have also studied nondetachment collisions in the Born approximation. One of the interesting features associated with nondetachment collisions is the zero in the differential cross section near the forward direction. Results for the total electron-detachment cross section are also presented and compared with experiment where measurements are available.

I. INTRODUCTION

The recent interest in neutral beams for heating of thermonuclear plasma requires a better understanding of atomic processes associated with the production of the beam. High-current beams of hydrogen or deuterium atoms with energies in the keV to MeV energy range are required. Presently, two approaches to the problems of generating such neutral beams are being investigated.[1] In the first approach a neutral beam is produced using the charge-transfer mechanism by passing a positive ion beam of hydrogen or deuterium through a suitable conversion cell. In the alternative approach, the neutral beam is generated using the electron-deteachment mechanism by passing a negative ion beam of hydrogen or deuterium through a suitable conversion cell. In the present paper we are concerned with the angular distribution of the neutral beam produced by collisional electron detachment of a negative ion beam.

The electron-detachment cross sections for negative hydrogen ions on hydrogen and helium atoms have been calculated by McDowell and Peach,[2] and by Sida[3] in the Born approximation using relatively simple wave functions for H^- ions and target atoms. A free-collision approximation developed by Dimitriev and Nikolaev[4] and the quasiclassical impulse approximation given by Bates and Walker[5] have also been applied to the electron-detachment problems. Recently, Gillespie[6] has calculated the electron-detachment cross section in the Born approximation using the atomic form factors and the incoherent scattering functions evaluated by Inokuti and Kim.[7,8] The total cross sections,[6] obtained by summing over all the allowed excited states of H^- ions and target states, are in good agreement with experiment at high energies ($E \geqslant 1$ MeV). The Born approximation will also be adopted in the present work for the calculation of electron-detachment amplitudes. The problems with target distortions are considered in a subsequent paper.[9]

The use of the Born approximation for the electron-detachment amplitude is given in Sec. II. In Sec. III, we develop a method for calculating the electron-detachment differential cross section. Since most excited states of H^- ions lead to electron detachment, all the allowed final states associated with these excited H^- states must be summed over to give the total electron-detachment differential cross section. To our knowledge, this kind of differential cross section has not been previously calculated. The summations in the evaluation of the electron-detachment differential cross section are carried out in the closure approximation with the help of an averaged momentum transfer defined by using the concept of average energy loss. The average energy loss for small angle scatterings is estimated in the Born approximation using the sum rules[10] for the generalized oscillator strength. This permits us to obtain the differential cross section for electron detachment at small scattering angles. We have applied our method to calculate the electron-detachment differential cross section for H^- collisions on He, H, and H_2 targets.

The differential cross sections for elastic and nondetachment inelastic collisions are presented in Sec. IV. One of the interesting features for the elastic and nondetachment inelastic cross sections is the zero in the differential cross section near the forward direction. This is a consequence of the Born approximation since, for any negative ion, the elastic atomic form factor has a zero[8] at a certain momentum transfer q_0. In Sec. V, the total cross sections for electron-detachment are evaluated using the concept of the averaged momentum transfer at the forward direction. Results are then compared with the experimental and theoretical cross section obtained in the Bethe-Born approximation.

II. BORN APPROXIMATION

In the Born approximation the differential cross section for the excitation of the collisional systems, a and b, from their initial states α_i and β_i to the final states α_f and β_f, is given in terms of their corresponding atomic form factors. We have

$$\frac{d\sigma_{\alpha_f\beta_f}(\theta)}{d\Omega}=\left(\frac{\mu}{2\pi}\right)^2\frac{k_f}{k_i}\left(\frac{4\pi}{q^2}\right)^2|F^{(a)}_{\alpha_f\alpha_i}(q)|^2|F^{(b)}_{\beta_f\beta_i}(q)|^2, \tag{2.1}$$

where μ is the reduced mass of the scattering system. q is the magnitude of momentum transfer, related to the initial and final magnitude of momenta, $\vec{k}_i$ and $\vec{k}_f$, and the scattering angle θ, as

$$q=(k_i^2+k_f^2-2k_ik_f\cos\theta)^{1/2}. \tag{2.2}$$

The atomic form factor $F^{(a)}_{\alpha_f\alpha_i}(q)$ for system a is defined by

$$F^{(a)}_{\alpha_f\alpha_i}(q)=\langle\phi^{(a)}_{\alpha_f}|\left(Z^{(a)}_{N\ i}-\sum_l^{Z_e^{(a)}}e^{i\vec{q}\cdot\vec{r}_l}\right)|\phi^{(a)}_{\alpha_i}\rangle, \tag{2.3}$$

where $Z_e^{(a)}$ is the number of electrons, $Z_N^{(a)}$ is the charge of the nucleus, $\phi^{(a)}_{\alpha_f}$ are the eigenstates of the system a, and $\vec{r}_l$ is the coordinate of the lth electron relative to the center of mass of system a. The atomic form factor for system b is similarly defined.

For the application of the Born approximation to the electron-detachment problem, we let atomic systems a and b represent the incident negative ion and target, respectively.

At high energies, we can use the approximation $k_f\cong k_i$ which is valid as long as the energy of the incident negative ion is large compared with the loss of energy due to electron detachment and excitation.

III. ELECTRON-DETACHMENT DIFFERENTIAL CROSS SECTION

In this section we examine the electron-detachment differential cross section of negative ions. The negative ion in the $(-1)^L$ parity sub-Hilbert space has only one ^1S bound state. Although there are ^3P bound states in the $(-1)^{L+1}$ parity sub-Hilbert space, these states being of $(-1)^{L+1}$ parity are not easily populated during collisions. Therefore, we can assume that all the excited states of negative ion are contributing to electron detachment. Consequently, the total electron-detachment differential cross section can be obtained simply by summing over all the allowed excited states of negative ion and target states.

$$\frac{d\sigma(\theta)}{d\Omega}=\left(\frac{\mu}{2\pi}\right)^2\sum_{\alpha_f\neq\alpha_i}\sum_{\beta_f}\left(\frac{4\pi}{q^2}\right)^2|F^{(a)}_{\alpha_f\alpha_i}(q)|^2\times|F^{(b)}_{\beta_f\beta_i}(q)|^2, \tag{3.1}$$

where $F^{(a)}_{\alpha_f\alpha_i}(q)$ and $F^{(b)}_{\beta_f\beta_i}(q)$ are the atomic form factors of the negative ion and target, respectively.

In the calculation of electron-detachment differential cross section, the summations in Eq. (3.1) sum over the atomic form factors which are evaluated at different values of momentum transfer q. To simplify the calculation, the summations are carried out in the closure approximation with the help of an averaged momentum transfer $\bar{q}(\theta)$ defined in terms of an average energy loss $\Delta E(\theta)$ as

$$\bar{q}(\theta)=k_i[2-\Delta E(\theta)/E_i-2(1-\Delta E(\theta)/E_i)^{1/2}\cos\theta]^{1/2} \tag{3.2}$$

with $\Delta E(\theta)$ defined by

$$\Delta E(\theta)=\sum_{\alpha_f\neq\alpha_i}\sum_{\beta_f}(E^{(a)}_{\alpha_f}+E^{(b)}_{\beta_f})\frac{d\sigma_{\alpha_f\beta_f}(\theta)}{d\Omega}\times\left(\sum_{\alpha_f\neq\alpha_i}\sum_{\beta_f}\frac{d\sigma_{\alpha_f\beta_f}(\theta)}{d\Omega}\right)^{-1/2} \tag{3.3}$$

where $E^{(a)}_{\alpha f}$ and $E^{(b)}_{\beta f}$ are the eigenvalues of the Hamiltonians of the negative ion and target, respectively. In the closure approximation states which do not satisfy the energy and momentum conservations are included. The effect of this on angular dependence needs careful study. The closure approximation also includes the contribution from double ionization of H^- ion. However, the error introduced by the inclusion of double ionization is expected to be small since, experimentally, it has been found[11] that the cross section for double ionization is only 4% of the electron-detachment cross section.

$\Delta E(\theta)$ can be estimated for small scattering angles in the Born approximation using the following sum rules[10] for the generalized oscillator strength:

$$\sum_{\alpha_f\neq\alpha_i}f^{(a)}_{\alpha_f\alpha_i}(q)=Z_e^{(a)} \tag{3.4}$$

and

$$\sum_{\alpha_f \neq \alpha_i} f^{(a)}_{\alpha_f \alpha_i}(q) \frac{1}{E^{(a)}_{\alpha_f}} = \frac{Z_e^{(a)} S^{(a)}_{\rm inc}(q)}{q^2}, \qquad (3.5)$$

where $Z_e^{(a)} S^{(a)}_{\rm inc}(q)$ is the incoherent scattering function of the negative ion defined by

$$Z_e^{(a)} S^{(a)}_{\rm inc}(q) = \langle \phi^{(a)}_{\alpha_i} | \left| Z_N^{(a)} - \sum_{l=1}^{Z_e^{(a)}} e^{i\vec{q}\cdot\vec{r}_l} \right|^2 | \phi^{(a)}_{\alpha_i} \rangle - |F^{(a)}_{\alpha_i \alpha_i}(q)|^2 . \qquad (3.6)$$

The atomic form factor is related to the generalized oscillator strength by

$$f^{(a)}_{\alpha_f \alpha_i}(q) = [\,|F^{(a)}_{\alpha_f \alpha_i}(q)|^2/q^2] E^{(a)}_{\alpha_f} . \qquad (3.7)$$

Then $\Delta E(\theta)$ takes the following simple form

$$\Delta E(\theta) = \frac{\bar{q}^2(\theta)}{S^{(a)}_{\rm inc}[\bar{q}(\theta)]} + \frac{Z_e^{(b)} \bar{q}^2(\theta)}{Z_e^{(b)} S^{(b)}_{\rm inc}[\bar{q}(\theta)] + |F^{(b)}_{\beta_i\beta_i}[\bar{q}(\theta)]|^2} . \qquad (3.8)$$

The unit of energy is in rydberg.

$\bar{q}(\theta)$, the averaged momentum transfer as a function of scattering angles, can be obtained by solving Eqs. (3.2) and (3.8) for a given incident energy. With the help of $\bar{q}(\theta)$ the electron-detachment differential cross section is simplified to the form

$$\frac{d\sigma(\theta)}{d\Omega} = \frac{4\mu^2}{[\bar{q}(\theta)]^4} Z_e^{(a)} S^{(a)}_{\rm inc}[\bar{q}(\theta)] \times \{ |F^{(b)}_{\beta_i\beta_i}[\bar{q}(\theta)]|^2 + Z_e^{(b)} S^{(b)}_{\rm inc}[\bar{q}(\theta)] \}, \qquad (3.9)$$

where $Z_e^{(a)} S^{(a)}_{\rm inc}$ and $Z_e^{(b)} S^{(b)}_{\rm inc}$ are the incoherent scattering functions of the negative ion and the target. For the H^- ion, the incoherent scattering function $Z_e^{(a)} S^{(a)}_{\rm inc}(q)$ has been calculated by Inokuti and Kim[7] using a 20-term Hylleraas wave function[12] and by Kim[8] using a 39-term Weiss wave function.[13] Both calculations agree well with each other (1% or better) and their results will be used in our calculation for the electron-detachment differential cross section.

Equations (3.2) and (3.8) give the solution for $\bar{q}$, the averaged momentum transfer, only over the angular range 0 to θ_m. At high energies the contribution to the total cross section from collisions with scattering angles greater than θ_m is negligible. Since the electron-detachment differential cross section is strongly peaked in the forward direction at these energies, integrating the differential cross sections obtained in Eq. (3.9) over the angular range 0 to θ_m gives results which agree well with the cross sections recently evaluated by Gillespie[6] for energies $E \geq 1$ MeV. As the energy decreases the angular range shrinks and eventually Eqs. (3.2) and (3.8) fail to have a solution for $\bar{q}$ at the forward direction (scattering angle $\theta = 0$). For specific cases we have studied, this occurs at energies below the region of validity of the Born approximation. The breakdown of Eqs. (3.2) and (3.8) is due to the evaluation of $\Delta E(\theta)$ in the Born approximation and closure approximation and the assumption that $\bar{q}$ can be defined in terms of $\Delta E(\theta)$ by Eq. (3.2). However, the averaged momentum transfer concept is still very useful in calculating differential cross sections near the forward direction at the energy region where the Born approximation is applicable.

The application of Eq. (3.9) to the problems of electron detachment of H^- ions by the He atom, H atom, and H_2 molecule are given below.

A. Electron detachment of H^- ions by He atoms

For helium atom, both the incoherent scattering function and ground-state atomic form factor have been calculated by Kim and Inokuti[14] for several different wave functions available in the literature. Utilizing the most accurate results given, for a 20-term Hylleraas wave function,[12] the electron-detachment differential cross section for H^- collisions on a He target is calculated using Eq. (3.9). The results are presented in Fig. 1.

We have also calculated the electron-detachment differential cross section using the following Hartree-Fock wave function[15,16] for the He ground state.

$$\Psi_0^{\rm HF}(\vec{r}_1, \vec{r}_2) = \chi(\vec{r}_1)\chi(\vec{r}_2) \qquad (3.10)$$

and

$$\chi(\vec{r}) = (1/4\pi)^{1/2}(a_1 e^{-a_2 r} + b_1 e^{-b_2 r})$$

with

$a_1 = 2.60505$, $b_1 = 2.08144$;

$a_2 = 1.41$, $b_2 = 2.61$.

Using this wave function, both the ground-state atomic form factor and the incoherent scattering function can be evaluated analytically; the results are

$$F_0^{\rm He}(q) = 2\left(1 - \frac{4a_2 a_1^2}{(4a_2^2 + q^2)^2} - \frac{4b_2 b_1^2}{(4b_2^2 + q^2)^2} - \frac{4a_1 b_1 (a_2 + b_2)}{[(a_2 + b_2)^2 + q^2]^2}\right), \qquad (3.11)$$

$$2S^{\rm He}_{\rm inc}(q) = 2\left[1 - \left(\frac{4a_2 a_1^2}{(4a_2^2 + q^2)^2} + \frac{4b_2 b_1^2}{(4b_2^2 + q^2)^2} + \frac{4a_1 b_1 (a_2 + b_2)}{[(a_2 + b_2)^2 + q^2]^2}\right)^2\right]. \qquad (3.12)$$

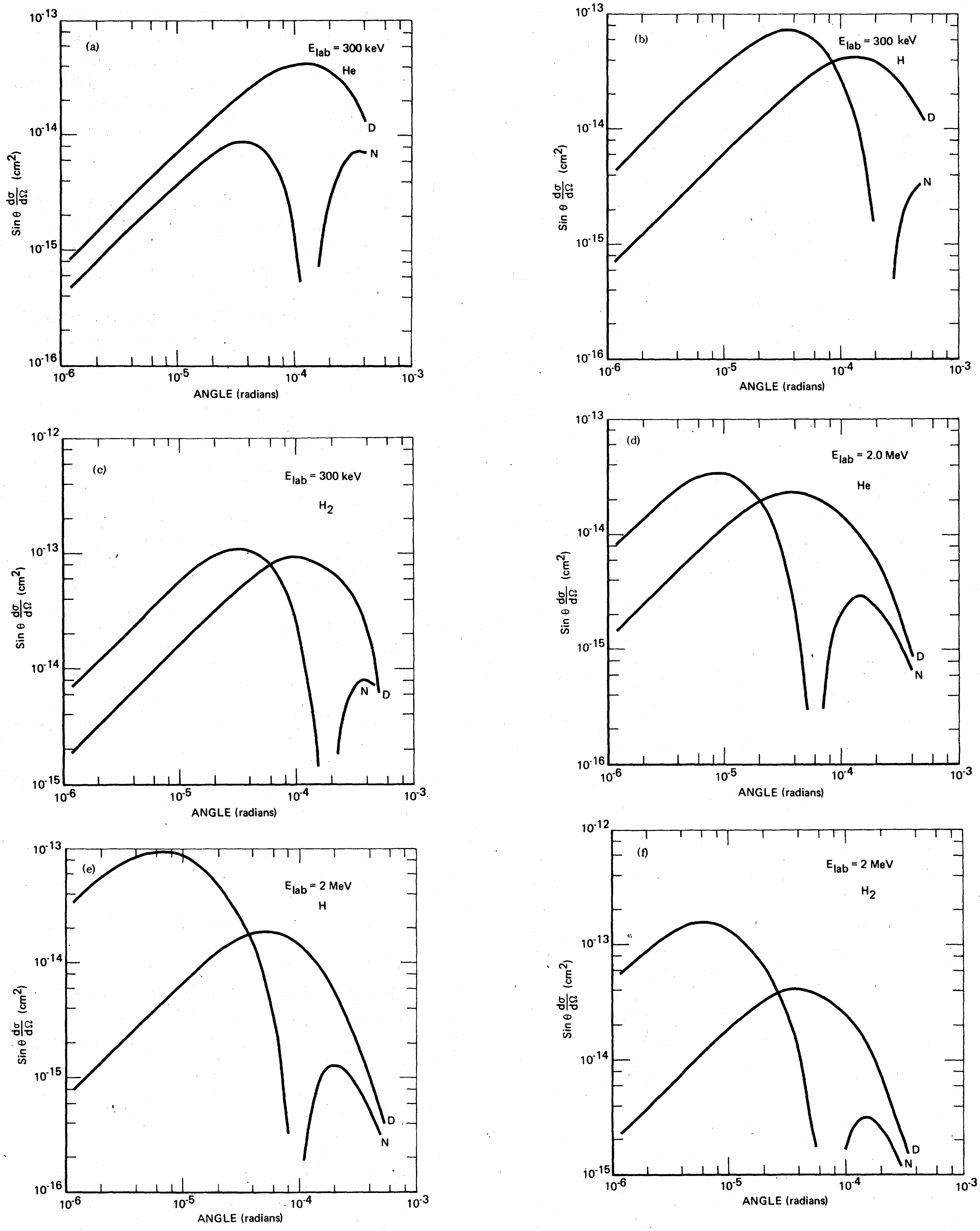

FIG. 1. Differential cross sections for H^- ion collisions on He, H, and H_2 targets with laboratory energies $E = 300$ keV (a)–(c) and 2 MeV (d)–(f). Curve D is the total electron-detachment differential cross section and curve N is the total nondetachment differential cross section. Note that the total nondetachment differential cross section has a zero at certain a angle.

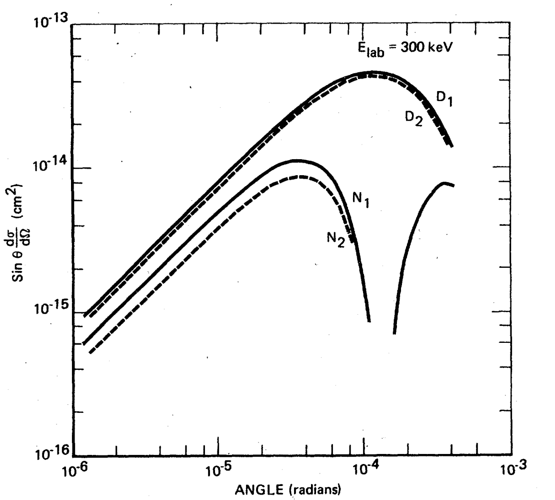

FIG. 2. Comparison of differential cross sections for (H^-, He) collisions obtained from using different He ground-state wave functions at laboratory energy, $E = 300$ keV. Curves D_1 and D_2, respectively, are the electron-detachment differential cross sections obtained using the Hartree-Fock wave function and the 20-term Hylleraas wave function for He. Curves N_1 and N_2, respectively, are the nondetachment differential cross sections obtained using the Hartree-Fock wave function and the 20-term Hylleraas wave function for He.

The electron-detachment differential cross sections obtained from using Eqs. (3.11) and (3.12) are presented in Fig. 2. The results agree well with the calculations using a more accurate 20-term Hylleraas wave function for He.

B. Electron detachment of H^- ions by H atoms

For the H atom, both the ground-state atomic form factor and incoherent scattering function are known analytically. The results for the electron-detachment differential cross section are presented in Fig. 1.

C. Electron detachment of H^- ions by H_2 molecules

For the H_2 molecule, the ground-state atomic form factor has been calculated by Liu and Smith[17] using the Davidson-Jones wave function,[18] and the incoherent scattering function has also been calculated by Liu[19] using a 5-term configuration-interaction wave function.[20] Using their results, we have calculated the electron-detachment differential cross sections which are presented in Fig. 1.

IV. NONDETACHMENT DIFFERENTIAL CROSS SECTION

The nondetachment differential cross section for H^- ion collisions on He, H, and H_2 targets in the Born approximation may be calculated easily since the H^- ion ground-state atomic form factor is available. Using the H^- ion ground-state atomic form factor given by Inokuti and Kim[7] and the He, H, and H_2 incoherent scattering functions discussed before, the nondetachment differential cross sections are calculated and presented in Fig. 1.

There are many interesting features associated with nondetachment collisions. In the Born approximation, Gillespie[6] found that the nondetachment cross section exceeds the total electron-detachment cross section at high energies. The total nondetachment cross section decreases as $(1/E_i)\ln(E_i)$ in comparison with the total electron-detachment cross section which decreases as $1/E_i$. It is well known that in electron-atom scattering the inelastic cross section decreases as $(1/E_i)\ln(E_i)$ in the Bethe-Born approximation. The reason for the same energy dependence in both cases is that the incident particles are charged particles. However, there is a difference between the nondetachment total cross section and the total inelastic cross section due to electron impact because of the structure of the incident particles. It has been suggested[6] that the breakdown of the Bethe theory for the nondetachment process is due to the structure of the negative ion. We have found[9] that the distortion of the target affects the electron-detachment differential cross section at the forward direction. Taking into account the distortion effect neglected in the Born approximation may modify the non detachment cross section because our present calculations show the contribution to the total nondetachment cross section mainly comes from small-angle scatterings.

Another interesting feature associated with nondetachment collisions is the zero in the differential cross section at the forward direction within the Born approximation. For negative ions, the elastic atomic form factor has a node at certain momentum transfer q_0, which is equal to 0.78 a.u.[8] for H^- ion. Utilizing the concept of energy loss the corresponding scattering angles θ_0 (i.e., the angles where the nondetachment differential cross section is zero) can be expressed in terms of q_0 by

$$\theta_0 \cong \{q_0^2/k_i^2 - [\Delta E(q_0)/2E_i]^2\}^{1/2} \tag{4.1}$$

with

$$\Delta E(q_0) \cong \frac{Z_e^{(b)} q_0^2}{Z_e^{(b)} S_{\text{inc}}^{(b)}(q_0) + |F_{\beta_i\beta_i}^{(b)}(q_0)|^2}\,. \tag{4.2}$$

For high-energy scatterings, Eq. (4.1) further reduces to

$$\theta_0 \cong q_0/k_i\,. \tag{4.3}$$

The zero in both the elastic and nondetachment inelastic differential cross sections is unique to the Born approximation. The inclusion of the contribution from the second Born approximation will remove the zero in the differential cross section at the angle θ_0. But the nondetachment differential cross section will still exhibit a dip near the forward direction which may be accessible to experimental tests in the future.

V. TOTAL ELECTRON-DETACHMENT CROSS SECTION

Applying the results obtained in Sec. II, the total cross section for the excitation of the collisional systems from their initial states α_i and β_i, to the final states α_f and β_f, can be written as an integral over momentum transfer,

$$\sigma_{\alpha_f\beta_f} = \frac{8\pi\mu^2}{k_i^2}\int_{q_{\min}}^{q_{\max}} |F^{(a)}_{\alpha_f\alpha_i}(q)|^2 |F^{(b)}_{\beta_f\beta_i}(q)|^2 \frac{dq}{q^3}\,. \tag{5.1}$$

The minimum and maximum values of q appearing in Eq. (5.1) correspond to scattering angles of 0 and π, respectively, and are given by

$$q_{\min} = k_i - k_f\,, \tag{5.2a}$$

$$q_{\max} = k_i + k_f\,. \tag{5.2b}$$

Since k_f depends on the excitation energies, the integration limits are state dependent.

The total electron-detachment cross section can be obtained simply by summing over all the allowed excited states of the negative ion and target.

$$\sigma = \sum_{\alpha_f\neq\alpha_i}\sum_{\beta_f}\sigma_{\alpha_f\beta_f} \tag{5.3a}$$

$$= \frac{8\pi\mu^2}{k_i^2}\sum_{\alpha_f\neq\alpha_i}\sum_{\beta_f}\int_{q_{\max}}^{q_{\max}} |F^{(a)}_{\alpha_f\alpha_i}(q)|^2 \times |F^{(b)}_{\beta_f\beta_i}(q)|^2 \frac{dq}{q^3}\,. \tag{5.3b}$$

Writing the integrals in Eq. (5.3b) as the sum of three terms,

$$I_{\alpha_f\beta_f} = \int_0^{\infty} |F^{(a)}_{\alpha_f\alpha_i}(q)|^2 |F^{(b)}_{\beta_f\beta_f}(q)|^2 \frac{dq}{q^3}\,, \tag{5.4a}$$

$$J_{\alpha_f\beta_f} = \int_0^{q_{\min}} |F^{(a)}_{\alpha_f\alpha_i}(q)|^2 |F^{(b)}_{\beta_f\beta_i}(q)|^2 \frac{dq}{q^3}\,, \tag{5.4b}$$

$$K_{\alpha_f\beta_f} = \int_{q_{\max}}^{\infty} |F^{(a)}_{\alpha_f\alpha_i}(q)|^2 |F^{(b)}_{\beta_f\beta_i}(q)|^2 \frac{dq}{q^3}\,, \tag{5.4c}$$

then the total electron-detachment cross section becomes

$$\sigma = \frac{8\pi\mu^2}{k_i^2}\sum_{\alpha_f\neq\alpha_i}\sum_{\beta_f}(I_{\alpha_f\beta_f} - J_{\alpha_f\beta_f} - K_{\alpha_f\beta_f})\,. \tag{5.5}$$

The integrals defined in Eqs. (5.4a), (5.4b), and (5.4c) are all well behaved at both small- and large-q values provided that the final states of the H^- ion is not the ground state. Since the integral $I_{\alpha_f\beta_f}$ is independent of the incident energy of the H^- ion, it provides the leading contribution to the total electron-detachment cross section. The integrals $J_{\alpha_f\beta_f}$ and $K_{\alpha_f\beta_f}$, which are dependent on the incident energy, give the correction to the asymptotic cross section.

Since the limits of integration for the evaluation of $I_{\alpha_f\beta_f}$ are state independent, the summations over the final states appearing in Eq. (5.5) can be carried out using the closure approximation. The final result is

$$A = \sum_{\alpha_f\neq\alpha_i}\sum_{\beta_f} I_{\alpha_f\beta_f} = \int_0^{\infty} Z_e^{(a)} S^{(a)}_{\rm inc}(q)[Z_e^{(b)} S^{(b)}_{\rm inc}(q) + |F^{(b)}_{\beta_i\beta_i}(q)|^2]\frac{dq}{q^3}\,. \tag{5.6}$$

Then the asymptotic cross section for electron detachment takes the form

$$\sigma \to 8\pi a_0^2(\alpha^2/\beta^2)A\,, \tag{5.7}$$

where α is the fine-structure constant and $\beta = v/c$.

The contribution to the asymptotic cross section arising from $K_{\alpha_f\beta_f}$ can be estimated in the closure approximation. In the first-order approximation, $q_{\max}$ is simply equal to $2k_i$, independent of the final states α_f and β_f. Evaluating the summations in Eq. (5.5) in the closure approximation gives

$$C = \sum_{\alpha_f\neq\alpha_i}\sum_{\beta_f} K_{\alpha_f\beta_f} \cong \int_{2k_i}^{\infty} Z_e^{(a)} S^{(a)}_{\rm inc}(q)[Z_e^{(b)} S^{(b)}_{\rm inc}(q) + |F^{(b)}_{\beta_i\beta_i}(q)|^2]\frac{dq}{q^3}\,. \tag{5.8}$$

This integral can be easily estimated using the following asymptotic forms for the incoherent scattering functions and atomic form factors:

$$\lim_{q\to\infty} Z_e^{(a)} S^{(a)}_{\rm inc}(q) \to Z_e^{(a)} \tag{5.9a}$$

and

$$\lim_{q\to\infty} F^{(b)}_{\beta_i\beta_i}(q) \to Z_N^{(b)}\,. \tag{5.9b}$$

The result is

$$C \cong Z_e^{(a)}[Z_e^{(b)} + (Z_N^{(b)})^2]/8k_i^2\,. \tag{5.10}$$

The value of C is smaller than the value of A by a factor of $1/k_i^2$; consequently, the contribution of the $K_{\alpha_f \beta_f}$ terms to the asymptotic cross section in the energy region, keV to MeV, of our interest will be negligible.

The main contribution to the asymptotic cross section will come from the $J_{\alpha_f \beta_f}$ terms. To apply the closure approximation, the $q_{\min}$ appearing as the limit of integration in Eq. (5.5) is replaced by the average $\bar{q}_{\min}$ for all the possible final states. This approximation is good at high energies. $\bar{q}_{\min}$, the averaged momentum transfer corresponding to the scattering angle $\theta = 0$, is given by

$$\bar{q}_{\min} = \lim_{\theta \to 0} \bar{q}(\theta) \tag{5.11}$$

with $\bar{q}(\theta)$ defined in Eq. (3.2). Within the Born approximation the calculation of the averaged momentum transfer as the function of scattering angles θ has been discussed in Sec. III.

With the help of the $\bar{q}_{\min}$ the summations of the $J_{\alpha_f \beta_f}$ terms reduce to the form

$$B = \sum_{\alpha_f \neq \alpha_i} \sum_{\beta_f} J_{\alpha_f \beta_f} = \int_0^{\bar{q}_{\min}} Z_e^{(a)} S_{\text{inc}}^{(a)}(q) \left[Z_e^{(b)} S_{\text{inc}}^{(b)}(q) + |F_{\beta_i \beta_i}^{(b)}(q)|^2 \right] \frac{dq}{q^3} . \tag{5.12}$$

Then the total electron-detachment cross section becomes

$$\sigma = (8\pi \mu^2 / k_i^2)(A - B) . \tag{5.13}$$

B depends on the incident energy through $\bar{q}_{\min}$.

In an expansion in β^{-2}, the Born cross section may be written as

$$\sigma = 8\pi a_0^2 \left(A' \frac{\alpha^2}{\beta^2} - B' \frac{\alpha^4}{\beta^4} + \cdots \right) . \tag{5.14}$$

The first term in Eq. (5.14) gives the asymptotic cross section at high energies. The second-order term,[6] arising from the $J_{\alpha_f \beta_f}$ terms, is obtained by expanding $q_{\min}$ to the first order in β^{-2}. Both A' and B' may be readily calculated from ground-state properties of the incident negative ion and target atom. For electron-detachment problems, the series expansion appears to diverge well before $\beta \approx \alpha$ in several specific cases for which the first two terms have been evaluated. The second-order term is found to be the same order as the leading term at the energy region where the Born approximation is applicable. Using the concept of the averaged momentum transfer, the β^{-2} expansion is avoided. In the present formulation the second term in Eq. (5.13) contains contribution from all orders in the β^{-2} expansion.

Equation (5.13) has been applied to calculate

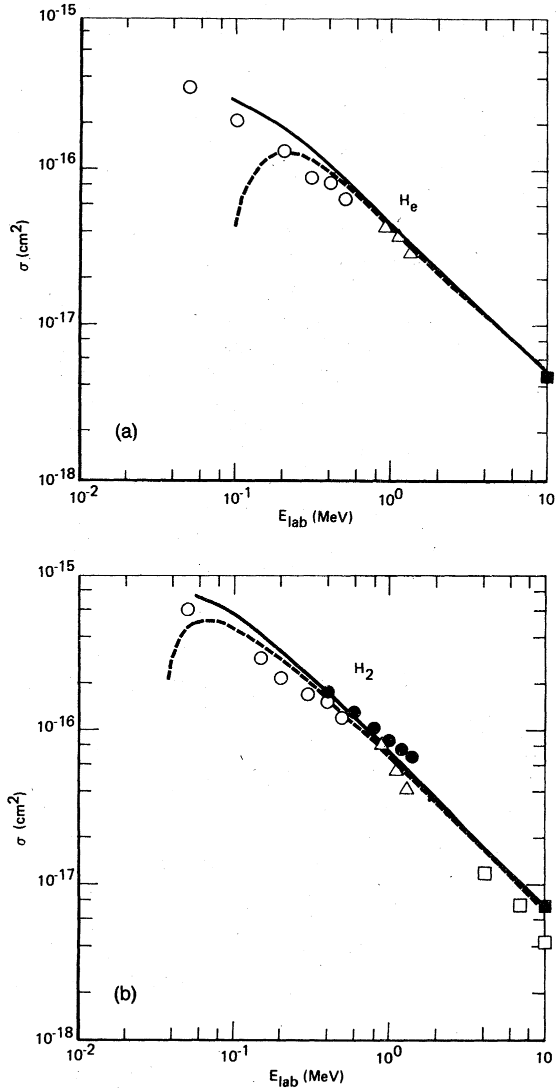

FIG. 3. Total electron-detachment cross section (electron-detachment and double ionization) for H^- ion collisions on (a) He and (b) H_2 target gases. Solid curves give the results of the present calculation; broken curves are the theoretical cross sections obtained from the first two terms in β^{-2} ($\beta = v/c$) expansion. Experimental data include that of Berkner (□, Ref. 21); Smythe and Toevs (■, Ref. 22); Dimov and Dudnikov (△, Ref. 23, data on H_2 at 1.3 MeV includes electron detachment only); Rose *et al.* (●), Ref. 24, electron detachment only); and Heinemeier *et al.*, (○, Ref. 11, data on He for 200–500 keV include electron detachment only).

the electron-detachment cross section for H^- ions on H_2 and He. Calculated cross sections are shown in Fig. 3, together with the available experimental data[11,21-24] and the theoretical cross sections[6] obtained from the first two terms in the β^{-2} expansion.

VI. CONCLUSIONS

In this paper, we apply the concept of average momentum transfer to study the problem of electron detachment of negative ions. The total electron-detachment differential cross section near the forward direction is evaluated using the average momentum transfer as a function of scattering angle. At high energies, $E \geqslant 1$ MeV, the total cross section obtained by integrating the differential cross section gives results which agree well with the cross section[6] obtained by extending the Bethe theory to treat explicitly the structure of the negative ion. The total electron-detachment cross section can also be evaluated directly using the concept of average momentum transfer. In this approach the total cross section contains contribution from all orders in the β^{-2} expansion of the cross section in comparison with the usual Bethe theory which includes only the first two terms in the expansion. In several specific cases for which the first two terms in the β^{-2} expansion of the electron-detachment cross section has been evaluated, the second-order term is found to be the same order as the leading term at the energy region where the Born approximation should still be applicable (see Fig. 3). For the problem of electron detachment of negative ions the Born approximation is expected to be valid at energies $E \geqslant 100$ keV. Our method avoids the β^{-2} expansion and can be used to give an estimate for the cross section at lower energies.

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