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# Stark-profile calculations for Lyman-series lines of one-electron ions in dense plasmas

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The frequency distributions of the first six Lyman lines of hydrogenlike carbon, oxygen, neon, magnesium, aluminum, and silicon ions broadened by the local fields of both ions and electrons are calculated for dense plasmas. The electron collisions are treated by an impact theory allowing (approximately) for level splittings caused by the ion fields, finite duration of the collisions, and screening of the electron fields. These calculations are fully quantum mechanical and include the full Coulomb interaction. Ion effects are calculated in the quasistatic, linear-Stark-effect approximation, using distribution functions of Hooper and Tighe which include correlation and shielding effects. Theoretical uncertainties from the various approximations are estimated, and the scaling of the profiles with density, temperature, and nuclear charge is discussed. A correction for the effects caused by low-frequency field fluctuations is suggested.

## I. INTRODUCTION

Interest in Stark-broadened spectral lines emitted or absorbed by dense high-temperature plasmas arises mainly from three classes of problems. In conjunction with a reliable theory of the broadening, measured profiles can first be used to determine one of the most important parameters of the plasma, namely, its density. The theory applied in the present work actually yields profiles of absorption and emission coefficients (or cross sections) for plasmas in which Stark broadening due to thermal fluctuations of the electric microfield dominates all other line-broadening mechanisms, including Stark effects arising from plasma waves and microinstabilities. The corresponding restrictions must be kept in mind when applying this method of density diagnostics, e.g., to plasmas used in laser or particle-beam fusion research (imploding pellets).

A second major class is the calculation of radiative transfer in stellar interiors and, again, imploding pellet plasmas. For such applications, one is mainly interested in the behavior of absorption coefficients at large frequency separations, compared to the halfwidth, from line center. Also, since Stark profiles follow approximately an inverse 2.0–2.5 power law as function of this separation, Stark broadening may be dominant in this context even in circumstances where the corresponding halfwidth is smaller than the Doppler width. This can happen because thermal Doppler profiles are Gaussian, i.e., decay exponentially on the wings of the lines.

The third class of problems is concerned with turbulent plasmas, in which fields from strongly excited collective modes may have a substantial influence on spectral line shapes. Appropriate measurements and analysis enable one to determine electrical energy densities and dominant frequencies in such cases. We shall assume, however, that wave-produced fields are of minor importance in comparison with particle-produced fields, and merely refer the reader to an introduction[1] to this rapidly developing method of plasma diagnostics.

Particle-produced fields are, on the average, a factor ~3 larger than the Holtsmark normal field strength[1,2]

$$F_p = 2.603 p e N_p^{2/3} . \tag{1}$$

This is very nearly equal to the field produced by an ion of charge $p$ which is at a mean ion-ion distance $r_p = (4\pi N_p/3)^{-1/3}$ from the perturbed ion. Since in a neutral plasma containing, e.g., only a single ion species, the electron density is related to the ion density by $N = pN_p$, the electronic Holtsmark field

$$F_0 = 2.603 e N^{2/3} \tag{2}$$

is typically related to that produced by ions by

$$F_0 = p^{-1/3} F_p . \tag{3}$$

Except for very highly charged ions, typical fields produced by the two kinds of charged particles are therefore about the same, and we may use, say, $5F_0$ for estimating Stark effects in thermal plasmas.

To the extent that the actual time dependence of the perturbing fields can be neglected (quasistatic approximation, see Sec. II), linear Stark shifts are therefore of the order[3] (in angular frequencies)

$$\Delta\omega_S \approx 5\frac{n^2}{z}a_0\frac{e}{\hbar}F_0 \approx 13\frac{n^2\hbar}{zm}N^{2/3}, \qquad (4)$$

where $a_0 = \hbar^2/me^2$ is the Bohr radius and $n$ the principal quantum number of the upper state of the radiating ion with nuclear charge $z$. For the Stark effect to be linear rather than quadratic, $\Delta\omega_S$ must be much larger than the fine-structure spread[3] of the upper states, i.e.,

$$\Delta\omega_S \gg \Delta\omega_F \approx \frac{mc^2}{2\hbar}\frac{(\alpha z)^4}{n^4}(n-1). \qquad (5)$$

(Here $\alpha \approx 1/137$ is the fine-structure constant.) We may define a critical electron density $N_F$ by the corresponding equality. This density is

$$N_F \approx 8\times 10^{-3}(\alpha/a_0)^3(z/n)^{15/2}(1-1/n)^{3/2} \qquad (6)$$

and must be well exceeded for our calculations to be valid.

Another characteristic density $N_D$ is obtained by comparing $\Delta\omega_S$ with the thermal Doppler width, which is for typical temperatures $kT \approx \frac{1}{8}mc^2(\alpha z)^2$ and ion masses $m_z \approx 2zm_H$ estimated by

$$\Delta\omega_D \approx (mc^2/8\hbar)(m/zm_H)^{1/2}(\alpha z)^3. \qquad (7)$$

For density measurements based on our calculations, a second lower limit is therefore given by

$$N_D \approx (104\alpha)^{-3/2}\left(\frac{m}{zm_H}\right)^{3/4}\left(\frac{\alpha}{a_0}\right)^3\frac{z^6}{n^3}$$

$$\approx 5\times 10^{-3}\left(\frac{\alpha}{a_0}\right)^3\frac{z^{21/4}}{n^3}, \qquad (8)$$

again expressed in terms of $(\alpha/a_0)^3 \approx 3\times 10^{18}$ cm$^{-3}$. We also note $N_D/N_F \approx (n^2/z)^{9/4}$, which indicates that fine structure is more important than Doppler broadening for $n^2 \lesssim z$. (Natural broadening need not be considered because it is always less than the fine-structure splitting.)

High-density limits for the validity or usefulness of the present calculations might be expected from a number of sources. First of all, for the lines of different principal quantum numbers not to overlap and the linear-Stark-effect approximation to remain valid, Stark shifts should certainly be smaller than separations between the upper level in question and the next level,

$$\omega_{n+1} - \omega_n = \frac{mc^2}{2\hbar}(\alpha z)^2\left(\frac{1}{n^2} - \frac{1}{(n+1)^2}\right) \approx \frac{m}{\hbar}c^2(\alpha z)^2 n^{-3}. \qquad (9)$$

Near equality of this separation with $\Delta\omega_S$ from Eq. (4) corresponds to the Inglis-Teller limit,[4] in our case

$$N_{\mathrm{IT}} \approx 2\times 10^{-2}a_0^{-3}z^{9/2}/n^{15/2} \approx 1.5\times 10^{23}z^{9/2}/n^{15/2}\ \mathrm{cm}^{-3}. \qquad (10)$$

As can be seen from $N_{\mathrm{IT}}/N_D \approx 10^7 z^{-3/4}n^{-9/2}$ and $N_{\mathrm{IT}}/N_F \approx 10^7 z^{-3}$, there is a considerable range (by factors $10^3$ to $10^5$) between the lowest densities of interest and the Inglis-Teller limit for the lines and ions considered in this paper. However, it is not obvious that this entire range can be explored with the usual approximations of Stark-broadening theory, which will be discussed next. Limitations imposed by the various approximations are further discussed in Sec. III and in Sec. IV, where we present quantum-theoretical results for the broadening by electrons via the complete Coulomb interaction rather than the usual semiclassical calculations of dipole interactions. Stark profiles and linewidths are given and discussed in Sec. V.

## II. THEORY

As in many previous calculations of Stark broadening,[1] we use the following expression for the spectral line shape:

$$L(\Delta\omega) = -\frac{1}{\pi}\mathrm{Re}\,\mathrm{Tr}\int_0^\infty dF\,W(F)\{D[i\Delta\omega - iCF + \phi]^{-1}\}. \qquad (11)$$

Here $\Delta\omega$ is the frequency separation from the unperturbed line, Re indicates the real part, and Tr the trace over unperturbed states of the radiating ion for which the principal quantum number equals that of the upper state of the line in question. The integral is over the field strength $F$ produced by the perturbing ions, with $W(F)$ being the corresponding distribution function.[5] The dipole operator $D$ generates appropriate products of matrix elements between states of principal quantum number $n$ and the ground state, and $C$ is an operator whose matrix elements (in terms of parabolic wave functions) are the linear Stark coefficients for the components of the line. Finally, $\phi$ is an operator describing the effects of electron collisions on the line shape whose fully quantum-mechanical calculation to all contributing orders in the multipole expansion of the Coulomb interaction will be discussed in Sec. IV.

To the leading orders in both the perturbation (Dyson series) expansion for the (perturbing) electron- (radiating) ion system and the multipole expansion for the interaction Hamiltonian, this collision operator is[1,6]

$$\phi = -(4\pi/3v)N(\hbar/m)^2\vec{\mathrm{R}}\cdot\vec{\mathrm{R}}\ln(\rho_{\max}/\rho_{\min}), \qquad (12)$$

where $v$ is the velocity of the perturbing electrons, and $\vec{\mathrm{R}}$ is the (radiator) electron position operator (in atomic units) operating in the subspace of principal quantum number $n$. The quantities $\rho_{\max}$ and $\rho_{\min}$ correspond to the limits of the integral over impact parameters, which arises in the straight classical path approximation (for the perturbing electrons) used in Ref. 6. (The impact parameters $\rho$ were of course assumed to be larger than the radial coordinate of the radiating electron.)

At small impact parameters, curvature of the classical path due to the strong Coulomb interaction with the radiating ion may be important. Because the dipole-monopole interaction can now change sign, a hyperbolic classical path calculation[7] actually gives a convergent result from which an equivalent (Coulomb) cutoff follows as $\rho_c \approx (z-1)e^2/mv^2$. However, since this is smaller than the excited-state Bohr radius, $\rho_n \approx n^2 a_0/z$, by a factor $\sim(ze^2/n^2\hbar v)^2 \lesssim 1$, other than dipole terms in the multipole expansion (and penetrating orbits) will be very important for $\rho \approx \rho_c$. Higher-order terms in the perturbation expansion must be considered as well, giving rise to an equivalent (strong collision) cutoff $\rho_s \approx n^2\hbar/zmv$ in the straight classical path calculations, to be supplemented by a quantum-mechanical cutoff $\rho_q \approx \hbar/mv$ corresponding to the de Broglie wavelength of the perturbing electron. Because of $\rho_s/\rho_n = e^2/\hbar v < 1$ for our high-temperature plasmas and, even, $\rho_q/\rho_n = ze^2/n^2\hbar v \lesssim 1$ we conclude that $\rho_n$ is always the largest minimum impact parameter for typical electron velocities in plasmas emitting the lines under consideration. We therefore provisionally choose

$$\rho_{\min} = (n^2/z)a_0 . \tag{13}$$

Corrections and errors associated with this choice will be discussed in Sec. III, and we now proceed with the determination of the maximum impact parameter $\rho_{\max}$. Three physical effects[1,8] were ignored in arriving at the divergent expression for the collision operator: correlations between perturbing electrons, splitting of the levels by quasistatic fields, and the finite duration of the collisions. The equivalent cutoff accounting for electron-electron correlations is close to the (electron) Debye radius,

$$\rho_D = (kT/4\pi N e^2)^{1/2} \approx v/\omega_P , \tag{14}$$

where $\omega_P = (4\pi N e^2/m)^{1/2}$ is the electron plasma frequency. The quasistatic splitting may be allowed for by $\rho_{\max} \approx v/\Delta\omega_S$ with $\Delta\omega_S$ estimated by Eq. (4), while the finite duration of the collisions requires $\rho_{\max} \lesssim v/|\Delta\omega|$. The combined effects are accounted for by choosing

$$\rho_{\max} = v(\omega_P^2 + \Delta\omega_S^2 + \Delta\omega^2)^{-1/2} , \tag{15}$$

and the corresponding theoretical uncertainties will again be discussed in Sec. III. (Since deviations from straight paths are not important near $\rho_{\min}$, we may neglect them for $\rho_{\max}$.)

From Eqs. (12), (13), and (15) the Maxwell average of the collision operator is as in Eq. (29) of Ref. 6,

$$\phi \approx -\frac{4\pi}{3}\left(\frac{2m}{\pi kT}\right)^{1/2} N\left(\frac{\hbar}{m}\right)^2 \vec{\mathrm{R}}\cdot\vec{\mathrm{R}}\left(\frac{1}{2}\int_y^\infty e^{-x}\frac{dx}{x}\right), \tag{16}$$

with

$$y = \frac{mv_{\min}^2}{2kT} \approx \left(\frac{\hbar n^2}{2z}\right)^2 \frac{\omega_P^2 + \Delta\omega_S^2 + \Delta\omega^2}{E_{\mathrm{H}}kT} \tag{17}$$

determined from the requirement $\rho_{\max} > \rho_{\min}$. (We use $E_{\mathrm{H}} = e^2/2a_0$, i.e., the ionization energy of hydrogen, to combine the various atomic constants.) The fraction of electrons excluded by this requirement is estimated[9] by $\frac{4}{3}\pi^{-1/2}y^{3/2}$ and must be small for our calculations of the broadening by electrons to be reasonably accurate.

The most important quantity that remains to be determined is the ion field strength distribution function $W(F)$ in Eq. (11) for the line shape. We have used distribution functions calculated by Tighe and Hooper,[5] which depend on electron and ion densities and on the ion charges. It is convenient to use reduced field strengths defined by

$$\beta = F/F_0 \tag{18}$$

with $F_0$ according to Eq. (2). The $\beta$ distribution, besides on ionic charges and ion density ratios, depends on the dimensionless parameter

$$a = \frac{r_0}{\rho_D} \approx \left(\frac{e}{F_0\rho_D^2}\right)^{1/2} \approx 2.2\left(\frac{e^2N^{1/3}}{kT}\right)^{1/2} . \tag{19}$$

This parameter is primarily a measure of how strongly coupled singly charged particles are. (Note that $\frac{1}{3}a^2 = e^2/r_0kT$ is the ratio of characteristic potential and kinetic energies.) The parameter $a$ is also a measure of the relative (frequency-dependent) contributions of wave- and particle-produced fields, their ratio being estimated by[1] $0.17a^{1/2}/\bar{p}^{1/3}$, if $\bar{p}$ is the mean charge of perturbing ions. Since only static correlations are allowed for in the calculated field strength distribution functions, we must certainly restrict our calculations to $a^{1/2}/\bar{p}^{1/3} \lesssim 1$. However, since for fixed electron density the ion Debye radius scales as $(\bar{p})^{-1/2}$ and since the mean ion-ion separation is $\bar{p}^{1/3}r_0$, the generalized parameter corresponding to $a$ is a factor $\bar{p}^{5/6}$ larger than the value given by Eq. (19). For the modified cluster expansion used in the calculation of the distribution function to converge rapidly one would, *a priori*, therefore expect the more severe constraint $a\bar{p}^{5/6} < 1$.

## III. ERRORS AND RESTRICTIONS

The two parameters in our problem that should be small are the quantities $y$ for the electron broadening and, at least, $a^{1/2}/\bar{p}^{1/3}$ for the ion broadening. They are both increasing functions of density and decreasing functions of temperature. For any given ion, the temperature range is relatively small, and we shall simply use $kT=\frac{1}{4}z^2E_{\rm H}$ in this section. Also, the Inglis-Teller estimate should give a reasonable upper density limit for the lines to remain discrete, provided $0.17a^{1/2}/\bar{p}^{1/3}$ is indeed small. To verify this condition, we substitute $N_{\rm IT}$ from Eq. (10) into Eq. (19) and obtain

$$a\lesssim 3(zn^5)^{-1/4}\lesssim z^{-1/4}.$$

The omitted wave contributions to the microfield are therefore for $\bar{p}\approx z$ of order $0.2/\bar{p}^{11/24}$ or less, and we may conjecture that corresponding theoretical uncertainties in the ion-broadening calculations are $\lesssim$10% for the Lyman-$\alpha$ lines (and the highest densities) and $\lesssim$5% for the other lines. Should the wave fields exceed their appropriate thermal levels or act mostly on a small portion of the line profile, e.g., near the center of Lyman-$\alpha$ (see also Sec. V),[10,11] their omission here would cause proportionally larger errors. Equally or more serious appears, especially for the $\alpha$ lines, the possible violation of the constraint $a\bar{p}^{5/6}<1$ although comparison with Monte Carlo calculations[12] suggests that the calculated distribution functions remain accurate to better than 10% up to $a\bar{p}^{5/6}\approx 2$. The corresponding limit on the density is often more restrictive than that imposed by approximations in the electron broadening.

Other errors in the ion-broadening calculations are connected with the use of the linear Stark effect, dipole interaction approximations. While quadratic Stark effects should be small, except near the Inglis-Teller limit, quadrupole interactions are important already at lower densities, as emphasized by Demura and Sholin.[13] These interactions cause asymmetries in the line shapes of order

$$\frac{\rho_n}{r_0}=\frac{n^2a_0}{z}\left(\frac{F_0}{e}\right)^{1/2}\lesssim 0.4\left(\frac{z}{n}\right)^{1/2}$$

if the Inglis-Teller limit is used for an upper bound. Especially for the Lyman-$\alpha$ lines, one might therefore expect substantial asymmetries from this source. However, detailed calculations[13] of the ion-produced asymmetry yield additional factors $\lesssim$0.3 so that omission of the higher-order effects discussed in this paragraph should generally be less serious than other uncertainties in our calculations.

Before proceeding to the electron broadening, we wish to suggest that errors introduced by the quasistatic approximation *per se* are small as well. The corresponding ion-dynamical corrections[1] are likely to occur only near $\Delta\omega\approx\omega_{Pi}$ from line center, where $\omega_{Pi}$ is the ion plasma frequency. For the plasmas to which our calculations might apply, we have $\omega_{Pi}\approx 10^{-2}\omega_P$, and by comparing this with the average Stark shift from Eq. (4), we find that the quasistatic approximation is reasonably valid for all densities fulfilling

$$N\gg 10^{-9}\frac{z^6}{n^{12}}\left(\frac{\alpha}{a_0}\right)^3\approx 2\times 10^{-5}\frac{z^{3/4}}{n^9}N_D$$
$$\approx 10^{-7}\frac{z^{-3/2}}{n^{9/2}}\left(1-\frac{1}{n}\right)^{-3/2}N_F\,,$$

a criterion which is therefore well met for all densities exceeding the low-density limits for the utility of our calculations as estimated by Eqs. (6) and (8).

A corresponding criterion for the electrons, obtained by using the electron plasma frequency, might suggest that the quasistatic approximation is reasonably valid for them as well, in spite of an additional factor $10^6$ in the condition on the density. However, most contributions to the field are characterized by frequencies higher that $\omega_P$, and a more quantitative criterion is called for or, conversely, an examination of the high-density limit for the validity of the impact approximation used in this work. This limit must be imposed to meet the requirement $y\lesssim 1$ discussed below Eq. (17). Since we are not interested in frequencies which exceed the inherent level splittings, Eqs. (9) and (17) can be used to find that $y$ values of interest fulfill

$$y\lesssim z^2E_H/n^2kT\lesssim 1,$$

unity actually being approached only for the Lyman-$\alpha$ lines and near the Inglis-Teller limit. (Note that $\omega_P<\Delta\omega_S$ is always true near the Inglis-Teller limit so that $\Delta\omega_S$ is indeed the characteristic frequency in this regime.)

It is clear from Eq. (16) that the collision operator becomes sensitive to the actual value of $y$ as this parameter approaches unity. We therefore restricted our calculations by imposing the requirement $y\leqslant 0.1$, in which case a factor 2 uncertainty in $y$ corresponds to ~± 30% error in the approximate collision operator while the excluded fraction of electrons is entirely negligible. This restriction alone prevents our reaching the Inglis-Teller limit by factors $\lesssim 10$, a gap that could be narrowed by a (much more involved) unified theory calculation.[5,14,15] [See also the remarks following Eqs. (24) and (25).]

Comparison of Eqs. (12) and (16) allows the identification

$$\rho_{\max}/\rho_{\min} \approx y^{-1/2} \approx L_{\max}/L_{\min} \,, \tag{20}$$

using

$$\int_y^\infty e^{-x}\,dx/x \approx \ln(1/y)$$

and replacing impact parameters by (dimensionless) angular momenta, $L = m\rho v/\hbar$. According to Eq. (13), we have

$$L_{\min} \approx \frac{n^2}{z}\frac{\hbar v}{e^2} = n^2\left(\frac{mv^2}{2z^2E_{\rm H}}\right)^{1/2} \approx 0.5n^2 \,. \tag{21}$$

Given the restriction $y \lesssim 0.1$, we therefore need to consider only cases with $L_{\max} \gtrsim 1.5n^2$ and should expect the largest total error from ambiguities in $\rho_{\max}$ if $L_{\max}$ is near this limit. (Errors connected with $\rho_{\min}$ will be almost eliminated by the modification in the electron-broadening operator discussed next.)

## IV. ELECTRON BROADENING

Since the diagonal matrix elements of the collision operator equal $\frac{1}{2}$ of the total rate coefficients for electron-ion (non-Coulomb) collisions, the approximate collision operator in Sec. II corresponds to total cross sections[1]

$$\sigma_{nl} = 6\pi a_0^2\left(\frac{2E_{\rm H}}{mv^2}\right)\left(\frac{n}{z}\right)^2(n^2 - l^2 - l - 1)\ln\left(\frac{L_{\max}}{L_{\min}}\right), \tag{22}$$

if the ion is in the $n, l$ level. This semiclassical cross section for the $2p$ and $4p$ levels of O VIII and Al XIII is compared in Table I with distorted-wave (quantum-mechanical) calculations,[16] with $L_{\max} = 20$ in both cases and $L_{\min}$ according to Eq. (21) for the semiclassical cross section. The last column contains a modified semiclassical cross section, in which a strong-collision term[1,6] is added to the logarithm. [$\ln(\cdots)$ is replaced by $C_n + \ln(\cdots)$, $C_2 = 1.5$, $C_4 = 0.75$.] After this modification the distorted-wave result is represented by Eq. (22) to within ~10%, except for the high-energy values for $2p$. The distorted-wave calculations as such should be almost exact, since all partial-wave contributions to the cross section stay well below the unitarity limit and since exchange, which is neglected, is not likely to be important for total cross sections. (Corresponding calculations for $n = 3$ and 5 give $C_3 = 1.0$ and $C_5 = 0.5$.)

To account for strong (close) collisions, the Maxwell-averaged collision operator estimated by Eq. (16) can therefore be replaced by

$$\phi = -\frac{4\pi}{3}\left(\frac{2m}{\pi kT}\right)^{1/2} N\left(\frac{\hbar}{m}\right)^2 \vec{\rm R}\cdot\vec{\rm R}\left(C_n + \frac{1}{2}\int_y^\infty e^{-x}\frac{dx}{x}\right) \tag{23}$$

and at least the diagonal matrix elements for the $p$ states may, for $y < 0.1$, be expected to have an accuracy of ~20%. Comparisons with close-coupling calculations[17,18] for $2p$ states of H I and He II support this estimated accuracy for the Lyman-$\alpha$ lines, for which these states are most important. It also seems reasonable to assume that Eq. (23) can be used for all levels involved in our calculations, both for diagonal and off-diagonal matrix elements (in the parabolic quantum-number representation). While we suspect that the strong-collision terms adopted here may lead to an overestimate for these off-diagonal

TABLE I. Cross sections[a] for electron scattering on $np$ levels of oxygen VIII and aluminum XIII.

| $E/E_{\rm H}$ | $2p$<br>Elastic[b] | <br>Inelastic | <br>Total | <br>Semicl. | $4p$<br>Elastic[b] | <br>Inelastic[c] | <br>Total | <br>Semicl. |
|---|---|---|---|---|---|---|---|---|
| | | | | O VIII | | | | |
| 10 | 1.09 | 0.40[d] | 1.49 | 1.51 | 28.6 | 7.9 | 36.4 | 37.1 |
| 20 | 1.03 | 0.51 | 1.54 | 1.38 | 23.2 | 7.0 | 30.2 | 30.3 |
| 40 | 0.94 | 0.64 | 1.58 | 1.25 | 17.2 | 5.7 | 22.9 | 23.6 |
| | | | | Al XIII | | | | |
| 20 | 0.42 | 0.15[d] | 0.57 | 0.59 | 11.6 | 3.0 | 14.6 | 15.1 |
| 50 | 0.39 | 0.19 | 0.58 | 0.53 | 9.0 | 2.7 | 11.7 | 11.7 |
| 100 | 0.36 | 0.24 | 0.60 | 0.48 | 6.7 | 2.2 | 8.9 | 9.1 |

[a] In units of $\pi a_0^2$ and multiplied by the incoming electron energy $E$ in rydberg units. Only partial waves to $L = 20$ are included.

[b] For the monopole interaction contribution, the difference of $np$ and $1s$ scattering amplitudes was used.

[c] For the $\Delta n = 2$, 3, etc., $4p$ cross sections, including ionization, a correction corresponding to $6E_{\rm H}/\Delta E$ was added which is consistent with estimates of ionization cross sections. (Here $\Delta E$ is the energy gap between levels 4 and 6.) This correction is 2.7 for O VIII and 1.0 for Al XIII.

[d] Near-threshold values for the $n = 2$ to 3 transitions.

dipole-interaction matrix elements, one must remember that the corresponding error in the calculated profiles may be counteracted by contributions from off-diagonal quadrupole-interaction, etc., matrix elements which were neglected.

The reader may also wonder whether the use of a Maxwell distribution for Eqs. (16) and (23) is indeed appropriate for the very dense plasmas in which Lyman-$\alpha$ lines, e.g., of Si XIV show significant Stark broadening. To address this question, we estimate the Fermi energy for densities corresponding to the Inglis-Teller limit in Eq. (10) and obtain

$$E_F = (\hbar^2/2m)(3\pi^2 N)^{2/3} \lesssim z^3 E_{\rm H}/n^5 . \qquad (24)$$

This characteristic energy is therefore considerably smaller than typical thermal energies, $kT \approx z^2 E_{\rm H}/4$, except for the Lyman-$\alpha$ lines, for which the restriction on the parameter $y$ prevents us from calculating profiles for densities close to this limit. Extensions of our calculations based on the unified theory to these densities, should, however, account for the degeneracy of the electron gas and be based on a fully quantum-mechanical treatment of all multipole interactions. A corresponding calculation[19] for the H I Lyman-$\alpha$ line wings now exists but would have to be generalized to allow for inelastic collisions, quasistatic splitting, and degeneracy. We also note that an approximate version[20] of the unified theory suggests corrections to the impact approximation of the order $10^{-3}z^{-1}$ for $y \lesssim 0.1$. An application[21] of the unified theory to ion lines gives corrections of order $10^{-4}n^4/z(z-1)^2$, both smaller than those corresponding to our choice of the maximum impact parameter. (Note that these semiclassical calculations[20,21] are based on the dipole approximation. They would have to be supplemented by an impact-parameter cutoff near the excited-state Bohr radius and by a quantum calculation for small $L$.)

Lastly, there is the question of the accuracy of the semiclassical cross section of ions in $p$ states for partial waves $L > 20$. We therefore compare in Table II the $L = 20$ partial-wave contribution to the cross section corresponding to Eq. (22), namely,

$$\Delta\sigma_{nl} = 6\pi a_0^2 \left(\frac{2E_{\rm H}}{mv^2}\right)\left(\frac{n}{z}\right)^2 \frac{(n^2 - l^2 - l - 1)}{L} , \qquad (25)$$

with our distorted-wave quantum-mechanical calculations. Except for the $4p$ values at intermediate energies, the agreement is well within the 20% overall theoretical error expected for our calculations, which are presented in Sec. V. (For $L \approx 40$, deviations were found to be $\lesssim 5\%$.) For $n = 2$ and 3, the requirement $L_{\max} \gtrsim 20$ imposes, according to Eqs. (20) and (21), an additional restriction ($y \lesssim 0.01$ and $y \lesssim 0.05$) on the parameter $y$ to ensure $L_{\max}$ values large enough for a quasiclassical treatment of Debye shielding, finite duration of collisions, and quasistatic level splittings. For the Lyman-$\alpha$ lines we must therefore keep the density below the Inglis-Teller limit by factors $\lesssim 50$ or treat inelastic collisions in more detail. (Inelastic collisions are responsible for the rather large deviations between semiclassical partial cross sections and distorted-wave results for, say, $L = 10$.)

## V. RESULTS AND DISCUSSION

Using Eqs. (11), (17), and (23) line profiles $L(\Delta\omega)$ were calculated for electron temperatures corresponding to near maximum abundance of the ion in question and, except for aluminum, also for half and twice these temperatures. Electron densities were varied by factors of 10 over ranges determined as discussed in Secs. I, III, and IV. Instead of the angular frequency displacement from line center, we used the reduced wavelength separation,

$$\alpha = |\Delta\lambda|/F_0 = (|\Delta\omega|\lambda^2/2\pi c F_0)10^{-8} , \qquad (26)$$

with the Holtsmark normal field strength given by Eq. (2). (The factor $10^{-8}$ is connected with our use of angstrom units.) This choice ensures

TABLE II. $L = 20$ partial-wave contributions to the cross sections[a] for electron scattering on $np$ levels of O VIII and Al XIII. Semiclassical values are given in the last row.

|  | O VIII |  |  | Al XIII |  |
|---|---|---|---|---|---|
| $E/E_{\rm H}$ | $2p$ ($\times 10^2$) | $4p$ | $E/E_{\rm H}$ | $2p$ ($\times 10^3$) | $4p$ ($\times 10$) |
| 10 | 2.02 | 1.13 | 20 | 7.6 | 4.1 |
| 20 | 1.83 | 1.24 | 50 | 6.9 | 4.7 |
| 40 | 2.15 | 0.99 | 100 | 7.9 | 3.9 |
|  | 1.88 | 0.98 |  | 7.1 | 3.7 |

[a] In units of $\pi a_0^2$ and multiplied by $E/E_{\rm H}$ and the factors following the $np$ designations.

that the reduced profiles

$$S(\alpha)=L(\Delta\omega)|d\Delta\omega/d\alpha| \tag{27}$$

have a relatively weak dependence on electron density. From Eqs. (4), (26), and the Bohr (Balmer) formula for $\lambda$ follow typical $\alpha$-values

$$\alpha_S \approx 1.7\times 10^{12}(a_0^3/e)(n^2/z^5) \approx 5\times 10^{-4} n^2/z^5 \tag{28}$$

for a given line (with principal quantum number $n$ of the upper state) and radiating ion (of nuclear charge $z$).

From the calculated $S(\alpha)$ profiles, which were tabulated and graphed,[22] we determined Stark half, quarter, and eighth intensity widths defined by

$$\tfrac{1}{2}S(\alpha_{\max})=S(\alpha_{1/2}), \tag{29a}$$

$$\tfrac{1}{4}S(\alpha_{\max})=S(\alpha_{1/4}), \tag{29b}$$

$$\tfrac{1}{8}S(\alpha_{\max})=S(\alpha_{1/8}), \tag{29c}$$

where $\alpha_{\max}$ corresponds to the maximum of $S(\alpha)$. For even-$n$ values $S(\alpha)$ is like a triplet, i.e., has a sharp central peak ($\alpha_{\max}=0$) and two symmetrically placed secondary maxima or shoulders; for odd-$n$ values $S(\alpha)$ is a doublet with a central minimum and two symmetrically located maxima ($\alpha_{\max}\neq 0$). Table III contains values of $\alpha_{\max}$, $\alpha_S$, $\alpha_{1/2}$, $\alpha_{1/4}$, and $\alpha_{1/8}$. It should be sufficient for many applications of our numerical results[22] and will serve as a basis for the concluding discussion of the dependence of Stark profiles on density, temperature, and nuclear charge.

Also given in Table III is a quantity $\alpha_C$ calculated from

$$\alpha_C \approx \tfrac{1}{5}(\tfrac{1}{2}a)^{1/2}\alpha_S, \tag{30}$$

where $a$ is the parameter defined by Eq. (19) and $\alpha_S$ the estimated Stark broadening from Eq. (28). The factor $\frac{1}{5}$ corresponds to the estimated ratio between Holtsmark normal field strength and effective mean field strength used in Eq. (5) and the factor $(\frac{1}{2}a)^{1/2}$ is from Eq. (13) of Ref. 11, generalized by replacing the dipole matrix element for Lyman-$\alpha$ (3 in atomic units) with $n^2$.

According to Ref. 11, $\alpha_C$ is a measure of the broadening from field fluctuations caused by electrons in the Debye shielding clouds of ions. In the case of the hydrogen Lyman-$\alpha$ line, a substantial theory-experiment[10] discrepancy (factor ~2 in halfwidth) was removed[11] by assuming a statistically independent Gaussian broadening with $1/e$ width given by $\alpha_C$. We therefore suggest compounding $\alpha_C$ with the thermal Doppler width $\alpha_D$ before convolving our calculated Stark profiles with Gaussian profiles. The coresponding modifications will often lead to substantial modifications of central profile structures, e.g., the narrow peaks of $\alpha$ and $\gamma$ lines and the central dips of $\beta$ and $\delta$ lines. For density measurements, reliance on these central structures should therefore be avoided until the physical model proposed in Ref. 11 has been verified. We note here that alternate explanations for the H I–Lyman-$\alpha$ discrepancy[10] have been given in terms of ion-dynamical corrections[23,24] which would probably scale differently.

Our calculations were all done using microfield distributions calculated for the case of equal charges on radiating and perturbing ions, i.e., for $p=z-1$. As can be seen from new calculations by Tighe and Hooper,[25] this results for fixed electron density in profiles that are broader than those one would obtain if most of the perturbing ions were of lower charge than the radiating ions. Also, we assumed equal ion and electron temperature and again refer the reader to Ref. 25 for a discussion of distribution functions for cases where these temperatures differ.

On the other hand, although our calculations are strictly valid only for one-electron radiating ions, they will often provide a good approximation for resonance lines of helium like ions of the element with the next-higher nuclear charge. This will be true as soon as densities are high enough for the profiles to be broader than the electrostatic splittings of $np$ and $ns$, etc., levels of the unperturbed ions.

Returning to Table III, it is seen that the fractional intensity widths for lines with odd $n$ are only weak functions of density and temperature. The strong dependence on plasma conditions found for lines with even $n$ results from the tripletlike profile structure mentioned above. (Relatively large fractional widths correspond to $\alpha_{1/n}$ values beyond the secondary maxima, while smaller values lie on the sides of the central peak.) Comparing $\alpha_{1/n}$ values for analogous lines from different elements, one finds that they scale approximately as $z^{-5}$ for near maximum abundance temperatures and densities corresponding to similar fractions of the largest density for which calculations were made. This scaling is in accordance with the original Holtsmark theory[2] and therefore with Eq. (28). It suggests that reductions of the quasistatic broadening by ions due to ion-ion correlations and Debye screening by electrons are largely balanced by electron-impact broadening. That electron-impact broadening remains sufficiently important can be inferred from Eqs. (4) and (23), which with $kT\sim z^2$ and $\vec{R}\cdot\vec{R}\sim z^{-2}$ indicate that the ratio of electron and

TABLE III. Calculated fractional widths $\alpha_{1/n}$ of Stark profiles for Lyman lines from $1 \leftarrow n$ transitions in one-electron ions for various temperatures and electron densities. The fractional widths are in units of $10^{-8}$ Å per cgs field strength, as are the positions $\alpha_m$ of the off-center maxima of the $n=3$, 5, and 7 lines and the quantities $\alpha_C$, $\alpha_D$, and $\alpha_S$, which are defined in the text.

| $10^{-6}T$ (K) | $\log_{10}N_e$ | $\alpha_m$ | $\alpha_C$ | $\alpha_D$ | $\alpha_{1/2}$ | $\alpha_{1/4}$ | $\alpha_{1/8}$ |
|---|---|---|---|---|---|---|---|
| Carbon 1–2 $\alpha_S=49.0$ | | | | | | | |
| 0.5 | 20 | | 3.60 | 10.9 | 0.594 | 1.03 | 1.59 |
| 1.0 | 20 | | 3.00 | 15.4 | 0.440 | 0.76 | 1.17 |
| 1.0 | 21 | | 3.70 | 3.3 | 0.782 | 1.36 | 2.13 |
| 2.0 | 20 | | 2.50 | 21.8 | 0.325 | 0.56 | 0.86 |
| 2.0 | 21 | | 3.10 | 4.7 | 0.581 | 1.01 | 1.56 |
| 2.0 | 22 | | 3.70 | 1.0 | 0.992 | 1.75 | 2.80 |
| Carbon 1–3 $\alpha_S=78.0$ | | | | | | | |
| 0.5 | 19 | 19.3 | 4.70 | 42.8 | 33.5 | 50.0 | 70.0 |
| 0.5 | 20 | 15.6 | 5.70 | 9.2 | 32.2 | 48.0 | 65.0 |
| 1.0 | 19 | 20.2 | 4.00 | 60.6 | 35.0 | 55.0 | 75.0 |
| 1.0 | 20 | 18.9 | 4.80 | 13.0 | 34.5 | 55.0 | 70.0 |
| 1.0 | 21 | 15.3 | 5.90 | 2.8 | 32.5 | 48.2 | 65.0 |
| 2.0 | 19 | 20.6 | 3.40 | 85.7 | 38.0 | 55.0 | 80.0 |
| 2.0 | 20 | 20.0 | 4.10 | 18.4 | 36.5 | 55.0 | 75.0 |
| 2.0 | 21 | 18.7 | 4.90 | 3.9 | 35.2 | 55.0 | 70.0 |
| 2.0 | 22 | 14.8 | 6.00 | 0.8 | 32.7 | 48.2 | 65.0 |
| Carbon 1–4 $\alpha_S=120$ | | | | | | | |
| 0.5 | 19 | | 7.60 | 40.6 | 4.07 | 7.6 | 73.2 |
| 0.5 | 20 | | 9.20 | 8.7 | 7.00 | 54.3 | 86.2 |
| 1.0 | 19 | | 6.40 | 57.4 | 3.00 | 5.3 | 9.1 |
| 1.0 | 20 | | 7.70 | 12.3 | 5.02 | 10.1 | 81.3 |
| 1.0 | 21 | | 9.40 | 2.6 | 7.75 | 57.5 | 87.8 |
| 2.0 | 19 | | 5.40 | 81.2 | 2.40 | 3.9 | 6.2 |
| 2.0 | 20 | | 6.50 | 17.5 | 3.75 | 6.8 | 14.3 |
| 2.0 | 21 | | 7.90 | 3.7 | 5.70 | 12.3 | 84.6 |
| Carbon 1–5 $\alpha_S=190$ | | | | | | | |
| 0.5 | 19 | 26.7 | 11.0 | 39.6 | 90.5 | 143.7 | 198.7 |
| 0.5 | 20 | 24.7 | 14.0 | 8.5 | 87.0 | 136.2 | 185.6 |
| 1.0 | 19 | 27.5 | 9.5 | 56.1 | 93.1 | 147.5 | 205.0 |
| 1.0 | 20 | 26.5 | 12.0 | 12.0 | 92.2 | 143.7 | 198.7 |
| 1.0 | 21 | 24.1 | 14.0 | 2.6 | 82.6 | 130.0 | 176.2 |
| 2.0 | 19 | 27.8 | 8.0 | 79.3 | 93.1 | 151.8 | 210.0 |
| 2.0 | 20 | 27.6 | 9.7 | 17.1 | 94.8 | 150.0 | 207.5 |
| 2.0 | 21 | 26.5 | 12.0 | 3.6 | 90.5 | 141.2 | 195.0 |
| Carbon 1–6 $\alpha_S=260$ | | | | | | | |
| 2.0 | 18 | | 9.30 | 363.6 | 5.1 | 9.2 | 16.3 |
| 2.0 | 19 | | 11.0 | 78.3 | 8.5 | 17.2 | 200.6 |
| 2.0 | 20 | | 14.0 | 16.8 | 12.7 | 143.7 | 225.0 |
| Carbon 1–7 $\alpha_S=350$ | | | | | | | |
| 2.0 | 18 | 38.9 | 12.0 | 360.9 | 189.3 | 307.5 | 430.0 |
| 2.0 | 19 | 40.0 | 15.0 | 77.7 | 200.6 | 312.5 | 435.0 |
| 2.0 | 20 | 39.1 | 18.0 | 16.7 | 195.0 | 305.0 | 420.0 |
| Oxygen 1–2 $\alpha_S=12.0$ | | | | | | | |
| 1.0 | 21 | | 0.90 | 1.62 | 0.147 | 0.256 | 0.397 |
| 2.0 | 21 | | 0.73 | 2.30 | 0.110 | 0.190 | 0.293 |
| 2.0 | 22 | | 0.88 | 0.49 | 0.190 | 0.335 | 0.527 |
| 4.0 | 21 | | 0.61 | 3.25 | 0.082 | 0.141 | 0.216 |
| 4.0 | 22 | | 0.74 | 0.70 | 0.143 | 0.249 | 0.385 |
| 4.0 | 23 | | 0.92 | 0.15 | 0.238 | 0.420 | 0.675 |
| Oxygen 1–3 $\alpha_S=18.0$ | | | | | | | |
| 1.0 | 21 | 3.32 | 1.40 | 1.37 | 7.50 | 12.0 | 16.0 |
| 2.0 | 21 | 4.17 | 1.20 | 1.94 | 8.31 | 14.0 | 18.0 |
| 2.0 | 22 | 3.25 | 1.40 | 0.42 | 7.50 | 12.0 | 16.0 |
| 4.0 | 21 | 4.44 | 0.98 | 2.74 | 9.12 | 16.0 | 20.0 |
| 4.0 | 22 | 4.16 | 1.20 | 0.59 | 8.31 | 14.0 | 18.0 |
| 4.0 | 23 | 3.17 | 1.40 | 0.13 | 7.31 | 12.0 | 16.0 |

TABLE III. (*Continued*)

| $10^{-6}T$ (K) | $\log_{10}N_e$ | $\alpha_m$ | $\alpha_C$ | $\alpha_D$ | $\alpha_{1/2}$ | $\alpha_{1/4}$ | $\alpha_{1/8}$ |
|---|---|---|---|---|---|---|---|
| Oxygen 1–4 $\alpha_S = 29.0$ | | | | | | | |
| 1.0 | 20 | | 1.80 | 6.03 | 0.98 | 1.85 | 19.5 |
| 1.0 | 21 | | 2.20 | 1.30 | 1.62 | 14.00 | 21.5 |
| 2.0 | 20 | | 1.50 | 8.53 | 0.73 | 1.30 | 2.2 |
| 2.0 | 21 | | 1.90 | 1.84 | 1.16 | 2.33 | 21.0 |
| 2.0 | 22 | | 2.60 | 0.40 | 1.62 | 13.50 | 21.0 |
| 4.0 | 20 | | 1.30 | 12.06 | 0.55 | 0.96 | 1.5 |
| 4.0 | 21 | | 1.60 | 2.60 | 0.87 | 1.59 | 3.1 |
| 4.0 | 22 | | 1.90 | 0.56 | 1.23 | 2.51 | 21.0 |
| Oxygen 1–5 $\alpha_S = 44.0$ | | | | | | | |
| 1.0 | 19 | 6.55 | 2.30 | 27.33 | 22.2 | 34.7 | 49.0 |
| 1.0 | 20 | 6.45 | 2.70 | 5.89 | 21.5 | 33.7 | 46.5 |
| 1.0 | 21 | 5.80 | 3.30 | 1.27 | 18.9 | 30.2 | 41.0 |
| 2.0 | 19 | 6.54 | 1.90 | 38.66 | 22.7 | 36.0 | 51.2 |
| 2.0 | 20 | 6.56 | 2.30 | 8.33 | 22.7 | 35.5 | 49.7 |
| 2.0 | 21 | 6.40 | 2.80 | 1.79 | 21.5 | 33.5 | 46.0 |
| 4.0 | 19 | 6.54 | 1.60 | 54.67 | 22.7 | 36.5 | 53.0 |
| 4.0 | 20 | 6.59 | 1.90 | 11.78 | 23.5 | 37.0 | 52.2 |
| 4.0 | 21 | 6.62 | 2.30 | 2.54 | 22.7 | 35.7 | 49.7 |
| Oxygen 1–6 $\alpha_S = 62.0$ | | | | | | | |
| 4.0 | 19 | | 2.20 | 53.9 | 1.23 | 2.20 | 3.81 |
| 4.0 | 20 | | 2.70 | 11.6 | 1.94 | 3.80 | 48.00 |
| Oxygen 1–7 $\alpha_S = 83.0$ | | | | | | | |
| 4.0 | 19 | 9.60 | 3.00 | 53.5 | 49.0 | 77.5 | 120.0 |
| 4.0 | 20 | 9.70 | 3.60 | 11.5 | 49.0 | 76.2 | 120.0 |
| Neon 1–2 $\alpha_S = 3.8$ | | | | | | | |
| 1.0 | 21 | | 0.28 | 0.93 | 0.040 | 0.070 | 0.109 |
| 2.0 | 21 | | 0.24 | 1.31 | 0.030 | 0.052 | 0.080 |
| 2.0 | 22 | | 0.29 | 0.28 | 0.053 | 0.093 | 0.145 |
| 4.0 | 21 | | 0.20 | 1.85 | 0.023 | 0.037 | 0.059 |
| 4.0 | 22 | | 0.24 | 0.40 | 0.039 | 0.069 | 0.106 |
| 4.0 | 23 | | 0.30 | 0.09 | 0.067 | 0.119 | 0.189 |
| Neon 1–3 $\alpha_S = 6.0$ | | | | | | | |
| 1.0 | 20 | 1.19 | 0.38 | 3.62 | 2.71 | 3.92 | 5.27 |
| 1.0 | 21 | 0.97 | 0.46 | 0.78 | 2.26 | 3.27 | 4.38 |
| 2.0 | 20 | 1.39 | 0.32 | 5.12 | 3.12 | 4.53 | 6.10 |
| 2.0 | 21 | 1.18 | 0.38 | 1.10 | 2.71 | 3.91 | 5.22 |
| 2.0 | 22 | 0.97 | 0.46 | 0.24 | 2.26 | 3.27 | 4.38 |
| 4.0 | 20 | 1.54 | 0.27 | 7.25 | 3.46 | 5.00 | 6.72 |
| 4.0 | 21 | 1.39 | 0.32 | 1.56 | 3.12 | 4.53 | 6.07 |
| 4.0 | 22 | 1.18 | 0.39 | 0.34 | 2.71 | 3.92 | 5.27 |
| 4.0 | 23 | 0.95 | 0.47 | 0.07 | 2.22 | 3.21 | 4.31 |
| Neon 1–4 $\alpha_S = 9.7$ | | | | | | | |
| 1.0 | 19 | | 0.50 | 15.94 | 0.156 | 0.274 | 0.43 |
| 1.0 | 20 | | 0.60 | 3.43 | 0.274 | 0.506 | 4.85 |
| 1.0 | 21 | | 0.73 | 0.74 | 0.471 | 3.525 | 5.82 |
| 2.0 | 19 | | 0.42 | 22.55 | 0.116 | 0.202 | 0.31 |
| 2.0 | 20 | | 0.51 | 4.86 | 0.202 | 0.360 | 0.60 |
| 2.0 | 21 | | 0.61 | 1.05 | 0.337 | 0.656 | 5.55 |
| 2.0 | 22 | | 0.74 | 0.23 | 0.519 | 3.700 | 5.90 |
| 4.0 | 19 | | 0.35 | 31.89 | 0.087 | 0.150 | 0.23 |
| 4.0 | 20 | | 0.42 | 6.87 | 0.152 | 0.267 | 0.41 |
| 4.0 | 21 | | 0.52 | 1.48 | 0.252 | 0.452 | 0.81 |
| 4.0 | 22 | | 0.62 | 0.32 | 0.375 | 0.760 | 5.82 |
| Neon 1–5 $\alpha_S = 14.0$ | | | | | | | |
| 1.0 | 19 | 2.09 | 0.74 | 15.57 | 6.56 | 11.5 | 16.0 |
| 1.0 | 20 | 2.04 | 0.90 | 3.35 | 6.50 | 10.7 | 14.5 |
| 1.0 | 21 | 1.76 | 1.10 | 0.72 | 5.93 | 9.6 | 12.2 |
| 2.0 | 19 | 2.10 | 0.62 | 22.02 | 6.93 | 12.5 | 17.5 |
| 2.0 | 20 | 2.11 | 0.75 | 4.74 | 6.93 | 11.7 | 16.2 |

TABLE III. (*Continued*)

| $10^{-6}T$ (K) | $\log_{10}N_e$ | $\alpha_m$ | $\alpha_C$ | $\alpha_D$ | $\alpha_{1/2}$ | $\alpha_{1/4}$ | $\alpha_{1/8}$ |
|---|---|---|---|---|---|---|---|
| 2.0 | 21 | 2.02 | 0.91 | 1.02 | 6.62 | 10.7 | 14.2 |
| 4.0 | 19 | 2.11 | 0.52 | 31.14 | 7.32 | 13.2 | 18.5 |
| 4.0 | 20 | 2.14 | 0.63 | 6.71 | 7.32 | 12.5 | 17.7 |
| 4.0 | 21 | 2.12 | 0.77 | 1.45 | 7.16 | 11.7 | 16.2 |
| Neon 1–6 $\alpha_S = 20.0$ | | | | | | | |
| 4.0 | 19 | | 0.73 | 30.75 | 0.344 | 0.60 | 0.99 |
| 4.0 | 20 | | 0.89 | 6.62 | 0.571 | 1.06 | 16.10 |
| 4.0 | 21 | | 1.10 | 1.43 | 0.800 | 10.41 | 17.40 |
| Neon 1–7 $\alpha_S = 27.0$ | | | | | | | |
| 4.0 | 19 | 3.19 | 0.98 | 30.52 | 15.0 | 26.7 | 36.1 |
| 4.0 | 20 | 3.19 | 1.20 | 6.57 | 15.5 | 26.0 | 35.0 |
| 4.0 | 21 | 3.06 | 1.40 | 1.42 | 14.7 | 23.7 | 32.3 |
| Magnesium 1–2 $\alpha_S = 1.5$ | | | | | | | |
| 2.0 | 22 | | 0.12 | 0.18 | 0.018 | 0.032 | 0.050 |
| 4.0 | 22 | | 0.10 | 0.25 | 0.014 | 0.024 | 0.037 |
| 4.0 | 23 | | 0.12 | 0.05 | 0.024 | 0.041 | 0.066 |
| 8.0 | 22 | | 0.08 | 0.36 | 0.010 | 0.018 | 0.027 |
| 8.0 | 23 | | 0.10 | 0.08 | 0.018 | 0.031 | 0.048 |
| 8.0 | 24 | | 0.12 | 0.02 | 0.029 | 0.051 | 0.083 |
| Magnesium 1–3 $\alpha_S = 2.4$ | | | | | | | |
| 2.0 | 21 | 0.46 | 0.15 | 0.70 | 1.06 | 1.52 | 2.03 |
| 2.0 | 22 | 0.36 | 0.19 | 0.15 | 0.84 | 1.22 | 1.63 |
| 4.0 | 21 | 0.57 | 0.13 | 0.99 | 1.26 | 1.80 | 2.42 |
| 4.0 | 22 | 0.46 | 0.16 | 0.21 | 1.05 | 1.52 | 2.02 |
| 4.0 | 23 | 0.36 | 0.19 | 0.05 | 0.83 | 1.21 | 1.61 |
| 8.0 | 21 | 0.62 | 0.11 | 1.40 | 1.42 | 2.03 | 2.75 |
| 8.0 | 22 | 0.56 | 0.13 | 0.30 | 1.25 | 1.80 | 2.41 |
| 8.0 | 23 | 0.45 | 0.16 | 0.06 | 1.05 | 1.50 | 2.01 |
| Magnesium 1–4 $\alpha_S = 3.9$ | | | | | | | |
| 2.0 | 20 | | 0.20 | 3.08 | 0.072 | 0.126 | 0.202 |
| 2.0 | 21 | | 0.25 | 0.66 | 0.121 | 0.229 | 2.075 |
| 2.0 | 22 | | 0.30 | 0.14 | 0.194 | 1.415 | 2.250 |
| 4.0 | 20 | | 0.17 | 4.35 | 0.053 | 0.093 | 0.145 |
| 4.0 | 21 | | 0.21 | 0.24 | 0.090 | 0.160 | 0.272 |
| 4.0 | 22 | | 0.25 | 0.20 | 0.140 | 0.278 | 2.237 |
| 4.0 | 23 | | 0.30 | 0.04 | 0.180 | 1.340 | 2.137 |
| 8.0 | 20 | | 0.14 | 6.15 | 0.040 | 0.069 | 0.106 |
| 8.0 | 21 | | 0.17 | 1.33 | 0.067 | 0.119 | 0.187 |
| 8.0 | 22 | | 0.21 | 0.29 | 0.106 | 0.191 | 0.357 |
| 8.0 | 23 | | 0.26 | 0.06 | 0.139 | 0.280 | 2.212 |
| Magnesium 1–5 $\alpha_S = 5.8$ | | | | | | | |
| 2.0 | 20 | 0.87 | 0.30 | 3.00 | 2.78 | 4.57 | 6.40 |
| 2.0 | 21 | 0.82 | 0.37 | 0.65 | 2.59 | 4.20 | 5.71 |
| 2.0 | 22 | 0.63 | 0.44 | 0.14 | 2.11 | 3.45 | 4.65 |
| 4.0 | 20 | 0.87 | 0.25 | 4.25 | 2.89 | 4.87 | 7.00 |
| 4.0 | 21 | 0.87 | 0.31 | 0.92 | 2.81 | 4.60 | 6.45 |
| 4.0 | 22 | 0.82 | 0.37 | 0.20 | 2.53 | 4.10 | 5.60 |
| 8.0 | 20 | 0.88 | 0.21 | 6.01 | 2.98 | 5.22 | 7.55 |
| 8.0 | 21 | 0.87 | 0.26 | 1.29 | 2.95 | 4.95 | 7.05 |
| 8.0 | 22 | 0.86 | 0.31 | 0.28 | 2.80 | 4.57 | 6.35 |
| Magnesium 1–6 $\alpha_S = 8.1$ | | | | | | | |
| 8.0 | 19 | | 0.25 | 27.53 | 0.15 | 0.25 | 0.30 |
| 8.0 | 20 | | 0.30 | 5.93 | 0.20 | 0.30 | 0.45 |
| 8.0 | 21 | | 0.36 | 1.28 | 0.25 | 0.45 | 6.10 |
| 8.0 | 22 | | 0.44 | 0.28 | 0.30 | 0.60 | 6.15 |
| Magnesium 1–7 $\alpha_S = 11.0$ | | | | | | | |
| 8.0 | 19 | 1.30 | 0.33 | 27.32 | 6.90 | 11.5 | 16.0 |
| 8.0 | 20 | 1.29 | 0.40 | 5.89 | 6.75 | 11.1 | 15.3 |
| 8.0 | 21 | 1.26 | 0.49 | 1.27 | 6.45 | 10.3 | 14.3 |

TABLE III. (*Continued*)

| $10^{-6}T$ (K) | $\log_{10}N_e$ | $\alpha_m$ | $\alpha_C$ | $\alpha_D$ | $\alpha_{1/2}$ | $\alpha_{1/4}$ | $\alpha_{1/8}$ |
|---|---|---|---|---|---|---|---|
| Aluminum 1–2 $\alpha_S=1.0$ | | | | | | | |
| 12.0 | 24 | | 0.07 | 0.02 | 0.016 | 0.028 | 0.043 |
| Aluminum 1–3 $\alpha_S=1.6$ | | | | | | | |
| 12.0 | 22 | 0.40 | 0.08 | 0.30 | 0.890 | 1.28 | 1.74 |
| 12.0 | 23 | 0.32 | 0.10 | 0.06 | 0.740 | 1.08 | 1.45 |
| 12.0 | 24 | 0.26 | 0.12 | 0.01 | 0.539 | 0.78 | 1.05 |
| Aluminum 1–4 $\alpha_S=2.6$ | | | | | | | |
| 12.0 | 22 | | 0.13 | 0.28 | 0.058 | 0.104 | 0.167 |
| 12.0 | 23 | | 0.16 | 0.06 | 0.080 | 0.152 | 1.365 |
| Aluminum 1–5 $\alpha_S=3.9$ | | | | | | | |
| 12.0 | 20 | 0.75 | 0.13 | 5.95 | 2.33 | 4.04 | 5.60 |
| 12.0 | 21 | 0.68 | 0.16 | 1.28 | 2.18 | 3.74 | 5.21 |
| 12.0 | 22 | 0.62 | 0.19 | 0.28 | 1.99 | 3.37 | 4.67 |
| Aluminum 1–6 $\alpha_S=5.4$ | | | | | | | |
| 12.0 | 21 | | 0.22 | 1.27 | 0.127 | 0.227 | 0.45 |
| 12.0 | 22 | | 0.26 | 0.27 | 0.162 | 0.320 | 3.99 |
| Aluminum 1–7 $\alpha_S=7.3$ | | | | | | | |
| 12.0 | 20 | 1.04 | 0.24 | 5.83 | 4.67 | 7.90 | 11.0 |
| 12.0 | 21 | 0.98 | 0.30 | 1.26 | 4.45 | 7.45 | 10.3 |
| 12.0 | 22 | 0.87 | 0.36 | 0.27 | 3.93 | 6.55 | 9.0 |
| Silicon 1–2 $\alpha_S=0.7$ | | | | | | | |
| 5.0 | 23 | | 0.05 | 0.04 | 0.009 | 0.015 | 0.024 |
| 10.0 | 23 | | 0.04 | 0.06 | 0.007 | 0.011 | 0.017 |
| 10.0 | 24 | | 0.05 | 0.01 | 0.011 | 0.019 | 0.031 |
| 20.0 | 23 | | 0.04 | 0.08 | 0.005 | 0.009 | 0.013 |
| 20.0 | 24 | | 0.04 | 0.02 | 0.009 | 0.014 | 0.023 |
| Silicon 1–3 $\alpha_S=1.1$ | | | | | | | |
| 5.0 | 22 | 0.22 | 0.07 | 0.16 | 0.509 | 0.732 | 0.979 |
| 5.0 | 23 | 0.17 | 0.08 | 0.04 | 0.401 | 0.575 | 0.762 |
| 10.0 | 22 | 0.26 | 0.06 | 0.23 | 0.592 | 0.855 | 1.155 |
| 10.0 | 23 | 0.22 | 0.07 | 0.05 | 0.506 | 0.727 | 0.967 |
| 20.0 | 22 | 0.29 | 0.05 | 0.33 | 0.669 | 0.967 | 1.300 |
| 20.0 | 23 | 0.26 | 0.06 | 0.07 | 0.587 | 0.850 | 1.145 |
| Silicon 1–4 $\alpha_S=1.8$ | | | | | | | |
| 5.0 | 21 | | 0.09 | 0.72 | 0.034 | 0.060 | 0.096 |
| 5.0 | 22 | | 0.11 | 0.15 | 0.054 | 0.104 | 0.955 |
| 5.0 | 23 | | 0.13 | 0.03 | 0.075 | 0.580 | 0.980 |
| 10.0 | 21 | | 0.08 | 1.01 | 0.026 | 0.044 | 0.069 |
| 10.0 | 22 | | 0.09 | 0.22 | 0.041 | 0.073 | 0.122 |
| 10.0 | 23 | | 0.11 | 0.05 | 0.058 | 0.111 | 0.985 |
| 20.0 | 21 | | 0.07 | 1.43 | 0.020 | 0.034 | 0.052 |
| 20.0 | 22 | | 0.08 | 0.31 | 0.032 | 0.055 | 0.087 |
| 20.0 | 23 | | 0.09 | 0.07 | 0.045 | 0.081 | 0.137 |
| Silicon 1–5 $\alpha_S=2.7$ | | | | | | | |
| 5.0 | 20 | 0.44 | 0.11 | 3.25 | 1.48 | 2.51 | 3.53 |
| 5.0 | 21 | 0.41 | 0.14 | 0.70 | 1.34 | 2.26 | 3.15 |
| 5.0 | 22 | 0.36 | 0.16 | 0.15 | 1.16 | 1.96 | 2.71 |
| 10.0 | 20 | 0.49 | 0.09 | 4.60 | 1.59 | 2.73 | 3.84 |
| 10.0 | 21 | 0.45 | 0.11 | 0.99 | 1.49 | 2.51 | 3.52 |
| 10.0 | 22 | 0.41 | 0.14 | 0.21 | 1.34 | 2.25 | 3.12 |
| 20.0 | 20 | 0.54 | 0.08 | 6.50 | 1.68 | 2.91 | 4.08 |
| 20.0 | 21 | 0.49 | 0.09 | 1.40 | 1.59 | 2.73 | 3.84 |
| 20.0 | 22 | 0.45 | 0.12 | 0.30 | 1.49 | 2.50 | 3.51 |
| Silicon 1–6 $\alpha_S=3.8$ | | | | | | | |
| 20.0 | 20 | | 0.11 | 6.42 | 0.044 | 0.076 | 0.117 |
| 20.0 | 21 | | 0.13 | 1.38 | 0.069 | 0.122 | 0.197 |
| Silicon 1–7 $\alpha_S=5.0$ | | | | | | | |
| 20.0 | 20 | 0.75 | 0.15 | 6.37 | 3.35 | 5.75 | 7.93 |
| 20.0 | 21 | 0.70 | 0.18 | 1.37 | 3.22 | 5.42 | 7.52 |

ion broadening is nearly constant along the isoelectronic sequence if the electron density increases as $z^6$. This tends to be the case under typical experimental conditions for Stark-broadening experiments[26-28] on hydrogenic-ion lines in the Lyman series.

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