# Accurate determination of  $\alpha$  and  $\Delta$  exponents in critical binary fluids by refractive-index measurements

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The critical behavior of the refractive index  $n$  of three critical mixtures, nitroethane-isooctane (N-I), isobutyric-acid-water (I-W), and triethylamine-water (T-W), has been investigated using a very sensitive and reliable interferometric method which enables gravity effects to be ignored. An accuracy  $\delta n/n \simeq 4 \times 10^{-7}$  has been obtained in the range  $T - T_c = 5 \times 10^{-4} - 20^{\circ}$ C [t =  $(T - T_c)/T_c$  $= 1.7 \times 10^{-6} - 7 \times 10^{-2}$ . All anomalies can be interpreted in terms of density or specific-heat anomalies. Thus, for the first time in binary fluids, the  $\alpha$  exponent could be accurately determined, and was found very close to the renormalization-group (RG) value ( $\alpha_{RG} = 0.110 + 0.003$  for  $n = 1$  and  $d = 3$ , to be compared with  $\alpha_{N-1} = 0.10 \pm 0.02$ ,  $\alpha_{1-W} = 0.12 \pm 0.05$ , and  $\alpha_{T-W} = 0.113 \pm 0.005$ ). In the T-W system, the nonanalytical corrections are sufficiently high to allow, also for the first time, the critical exponent  $\Delta$  to be determined accurately. The experimental value  $\Delta_{T,W} = 0.50 + 0.03$  is in good agreement with the value obtained by RG theory ( $\Delta = 0.493 \pm 0.007$ ). The amplitude values of the diverging part are in agreement with the two-scale-factor universality and allow the universal constant  $X = (R_t^*)^3$  to be determined, here also in agreement with the RG values  $[X_{1\cdot w} = (2.0 \pm 0.2) \times 10^{-2}, X_{\text{RG}} \approx 1.76 \times 10^{-2}]$ . In the systems studied here, the regular variation was found to be close to ideal variation, suggesting that the mixing anomaly is due mainly to the critical behavior of the specific heat. In addition, we review the results for other systems which have been studied by density, volume, thermal diffusivity, and specific-heat measurements. Although the data are not generally very accurate or reliable, they all support the same general conclusions derived from the analysis of the present data.

Near the second-order phase transition of pure fluids<sup>1</sup> or binary mixtures,<sup>2</sup> the behavior of the specific heat at constant volume  $V$  (fluid) or at constant pressure  $p$  and composition  $x$  (mixt.) exhibits a power law divergence governed by the exponent  $\alpha \approx 0.1$ . For a binary fluid at critical composition  $(x<sub>c</sub>)$ ,

$$
C_{p, x_c} = C_{p, x_c}^{\text{reg}} + C_{p, x_c}^0 |t|^{-\alpha}
$$
,

with  $t = T/T_c - 1$  and  $T_c$  the absolute critical temperature. The amplitude  $C^0_{\rho,\kappa_c}$  can be related to the amplitude of the correlation length  $\xi = \xi_0 |t|^{-\nu}$ by  $C_{p, x_c}^0 = k_B X / \alpha \rho_c \xi_0^3$ , where  $X \approx 10^{-2}$  is a universal constant,  $k_{\scriptscriptstyle R}$  is the Boltzmann constant, and  $\rho_c$ the critical density.

Direct measurements of  $C_{p, x_c}$  have not yet provided an unambiguous determination of the parameters  $\alpha$ ,  $C_{p, x_c}^{\text{reg}}, C_{p, x_c}^0$  because of experimental difficulties associated with gravity effects and/or temperature instabilities.

An alternative method involves the measurement of the density variation  $\rho$  or the volume variation V near  $T_c$ , since both are connected to  $C_{p, x_c}$ through the well-known relationship

$$
C_{b, x_c} - C_{v, x_c} = T_c V \alpha_{b, x_c} \left( \frac{\partial P}{\partial T_c} \right)_{v, x_c}
$$

where

$$
\alpha_{\rho, x_c} = \frac{1}{V} \left( \frac{\partial V}{\partial T} \right)_{\rho, x_c} = -\frac{1}{\rho} \left( \frac{\partial \rho}{\partial T} \right)_{\rho, x_c}
$$

The anomaly in  $C_{\rho, x_c}$  is small, so the change in the behavior of  $\rho$  or  $V$  is weak, and until now the best determination gave rise to an uncertainty on  $\alpha$  of  $\pm$  70%.<sup>3</sup> Moreover gravity effects are generally present since the measurement is integrated over all the sample height  $(V \text{ measure}$ ment) or over the buoy height ( $\rho$  measurements).

We have developed recently a new method<sup>4</sup> which reduces gravity effects to a level where they are no longer detected. The principle consists in measuring the refractive index variation at constant height near  $T_c$  by a high precision interferometer method. These variations are connected to the volume change and to local-field effects. This last phenomenon, sensitive only to the shortranged part of the correlation function, has been shown to be negligibly small,<sup>5</sup> so the refractive index behavior exactly reflects the density anomaly.

Three critical mixtures have been investigated. The first was nitroethane-isooctane, chosen because  $(\partial T_c/\partial p)_{v, x_c}$  is very low and  $\xi_0$  not too small, so that the local-field effects alone should be evidenced. This first study has been reported.<sup>5</sup> The second mixture was isobutyric acid-water, which

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is interesting due to the matching in density of the two components and the high value of  $(\partial T_c/\partial p)_{v, x_c}$ . Moreover careful volume measurements<sup>6</sup> have already been performed on this mixture. The third is triethylamine water where the  $C_{\nu}$  anomaly is very high. Careful  $C_p$  measurements in this mixture have also been published.<sup>7</sup> This last mixture shows such a large anomaly that important nonanalytical corrections were evidenced. Thus, in addition to the determination of  $\alpha$ ,  $C_{\rho, x_c}^0$ , and  $X$ , we will analyze the influence of such corrections.

## I. THEORETICAL BACKGROUND

As shown in Ref. 5, the variation of the refractive index  $n$  can be written in the following form

$$
\Delta n = \left(\frac{\partial n}{\partial T}\right)^{\text{reg}}_{\rho, x_c} (T - T_c) + \Delta p_c + \Delta F_c.
$$

The first term represents the regular thermal variation. The second term is connected to the density anomaly and the third to the effects of local field. We showed in Ref. 5 that this last effect was negligibly small in a system (N-I) where  $\Delta \rho_c$  being very weak, it should have been the only observable anomaly. We will therefore neglect it in the following.

A simplifying hypothesis leads to the assumption that the whole deviation to the volume additivity is due to the anomaly of  $C_{p, x_c}$  itself, i.e., the regular value is assumed to be the ideal value.  $(\partial n/\partial T)^{id}_{\rho, x_c}$  can be evaluated following the derivation of the generalized Lorentz-Lorentz (L-L) formula

$$
(n^2-1)/(n^2+2) = \frac{4}{3} \pi (\alpha_1 N_1 \phi_1 + \alpha_2 N_2 \phi_2)[1/(1-F)],
$$

where  $\alpha$ , is the polarizability of molecule i considered alone.  $N_i$  is the number density of pure component  $i$  at the considered temperature and pressure.  $\phi_i$ , is the volume fraction of the component i.

 $F \approx 1\%$  is a correction due to local-fields effects independent of the critical behavior<sup>5</sup> and due to electronic overlaps, distortion of the molecular framework, and short-range part of the density correlations  $1-1$ ,  $2-2$ , and  $1-2.^8$ 

The derivative at constant pressure and composition can be expressed as

$$
\left(\frac{\partial n}{\partial T}\right)_{\rho,\phi}^{id} = (n^2 - 1)\left(\frac{n^2 + 2}{6 n}\right)
$$

$$
\times \sum_{i=1}^{2} \frac{\phi_i}{(n_i^2 - 1)\left((n_i^2 + 2)/6 n_i\right)} \left(\frac{\partial n_i}{\partial T}\right)_{\rho,\phi}
$$

$$
+ O(F_{12}).
$$

 $\mathcal{O}(F_{12})$  represents the influence of the cross-cor-

relations 1-2 alone. In most cases, the total influence of  $F$  modifies the L-L derivatives  $(8n/8T)_b^{\text{L-L}}$  of pure fluids by about 6%.<sup>8</sup> Neglecting  $\mathcal{O}(F_{12})$  which is a part of F is certainly a much better approximation.

The critical part  $\Delta \rho_c$  is proportional to  $\alpha_{p, x_c}$ through the L-L relation, thus

$$
\Delta \rho_c = - (n^2 - 1) \left[ (n^2 + 2/6 n) \right] \alpha_{p, x_c}^0
$$
  
 
$$
\times \left[ T_c^{\alpha} / (1 - \alpha) \right] (T - T_c)^{1-\alpha},
$$

where  $\alpha_{p, x_c}^0$  is a function of  $C_{p, x_c}^0$ , and

$$
\Delta \rho_c = - (n^2 - 1) \left( \frac{n^2 + 2}{6 n} \right) \frac{T_c^{\alpha - 1}}{1 - \alpha} \left( \frac{\partial T_c}{\partial p} \right)_{V_r x_c} \rho_c C_{p, x_c}^0
$$

$$
\times (T - T_c)^{1 - \alpha}
$$

$$
= R (T - T_c)^{1 - \alpha}.
$$

This is the asymptotic behavior for  $T - T_c \rightarrow 0$ , but for larger values of  $T - T_c$ , nonanalytical corrections<sup>9, 10</sup> can be relevant, and  $\Delta \rho_c$  must be written

$$
\Delta \rho_c = R (T - T_c)^{1-\alpha}
$$
  
 
$$
\times [1 + b_1 (T - T_c)^{\alpha} + b_2 (T - T_c)^{2\alpha} \dots],
$$

where the exponent  $\Delta$  is expected<sup>11</sup> to be 0.50 and  $b_1, b_2, \ldots$  depend on the system.

Finally the  $(T - T_c)$  variation of *n* can be written as

$$
\Delta n = \left(\frac{\partial n}{\partial T}\right)_{p,\phi}^{\text{reg}} (T - T_c)
$$
  
+
$$
R(T - T_c)^{1-\alpha} \left(1 + \sum_j b_j (T - T_c)^{j\Delta}\right).
$$
 (1)

From the  $\Delta n$  variations versus  $T - T_c$  four kinds of investigations can therefore be considered: (i) test of formula (1), i.e., of the  $\Delta n$  behavior versus  $T - T_c$ ; (ii) determination of the exponent  $\alpha$ ; (iii) determination of the diverging part R and comparison with the value calculated from the amplitude relations; (iv) study of nonanalytical corrections and determination of the amplitudes  $b_i$ and the exponent  $\Delta$ .

### II. EXPERIMENTAL-

All mixtures were prepared in cylindrical quartz cells of radius 1 cm and length  $l = 5.000 \pm 0.002$ cm. The system nitroethane-isoctane (N-I) was prepared from distilled nitroethane whose purity was better than 99.5% as verified by chromatography and NMR measurements. The isooctane used had the quality "reference substance for gas chromatography." The isooctane mass fraction was  $C_1 = 0.5344 \pm 0.0008$ , close to the critical value  $0.5350 \pm 0.0006$ .<sup>12</sup> The mixture was frozen in ue  $0.5350 \pm 0.0006$ .<sup>12</sup> The mixture was frozen in

liquid nitrogen and sealed under vacuum. For the system isobutyric acid-water (I-W), isobutyri acid was of purity better than  $99\%$ . The quality of the water used corresponds to an Ohmic resistance of  $18 \times 10^6 \Omega$ . The experimental mass fraction of the acid was  $C_A = 0.3889 \pm 0.0005$ , close to the critical mass fraction  $0.3885.^{13}$  The. mixture triethylamine-water (T-W) was prepared from triethylamine of purity better than  $99.5\%$ and water of  $18 \times 10^6 \Omega$  resistivity. The experimental mass fraction of triethylamine was  $C_{\tau}$  $= 0.321 \pm 0.001$ ; the critical value is 0.321 following.<sup>14</sup> Since these two mixtures contain water  $321 \pm 0$ <br>.ng.<sup>14</sup> we sealed the corresponding cells at atmospheric pressure and at O'C where the vapor pressures of the mixtures are very low.

The cell was placed into a thermally stabilized copper oven, itself enclosed inside an air regulated box, the whole giving a stability of  $\pm 5$  $\times 10^{-4}$ °C over more than one week and better on shorter periods as verified by a quartz thermometer.

The experimental setup (Fig. 1) is the same as in Ref. 4 and is essentially composed of an interferometer which is the cell itself illuminated by a laser beam  $(\lambda = 6328 \text{ Å}, 6 \text{ mW power})$ . It is nearly insensitive to vibrations and temperature gradients outside the sample. For systems such as I-W or T-W where turbidity is important near  $T_c$ , we unbalanced the intensity of the interfering beams in order to maintain good contrast near the critical point. Data could thus be obtained as close as  $5 \times 10^{-4}$  to  $T_c$ .

The final uncertainty, due to the frequency drifts of the laser and the thermal stabilization accuracy is about 0.1 fringe, i.e.,  $\delta n \approx 7 \times 10^{-7}$ .

The thermal expansion of the quartz cell  $\lceil$  linear thermal expansion coefficient  $(1/l)(\partial l/\partial T)$  $=4.0 \times 10^{-7}$  °C<sup>-1</sup>]<sup>15</sup> corresponds to a correction of  $\Delta n / \Delta T = +6.0 \times 10^{-7}$ °C<sup>-1</sup>.

Since the laser beam diameter is small  $(0.5)$ mm), the refractive index is measured at a given height, at a position nearly in the middle of the sample where the meniscus is situated in the inhomogeneous phase. Therefore the composition remains critical even if gravity induces a density gradient.<sup>16</sup> These composition gradients should deviate the beam in the sample and somewhat disturb the fringes, this however was never visible, possibly because the time necessary to form a density gradient is very long.

We always started from a thermal equilibrium state  $T_i$  to reach another thermal equilibrium state  $T_f$ . About one hour was sufficient to reach this equilibrium, since the thermal diffusivity of a binary mixture exhibits only a weak anomal<br>near  $T_{\alpha}$ <sup>17</sup> The fringes were recorded by a ph near  $T_c$ .<sup>17</sup> The fringes were recorded by a photodiode between the two temperature levels in order to estimate the integer number of the interference order, and the fractional part was directly measured on the photodiode plane. The refractive index variation  $\Delta n$  was inferred from the corresponding interference order number.  $\Delta p$  by the relation  $\Delta n = (\lambda_0/2l) \Delta p = 6.328 \times 10^{-6} \Delta p$  (see Table I-III).

For systems I-W and  $T-W$ , the transition is well determined by the dissappearance of the transmitted beam. The critical temperature was determined for the system N-I (where the matching in refractive indices cancels the turbidity) by the sudden inversion of the fringes as reported in Ref. 12, due to the composition variation when the



FIG. 1. Experimental setup.



TABLE I. Refractive index data  $(n = 6.328 \times 10^{-6}P)$  for nitroethane-isooctane  $(T_c = 29.8480 \degree C)$ .

phase separation process takes place. The measured critical temperature was (weakly) related to the temperature  $T_a$  of the intermediate thermalized housing, showing a small temperature gradient between the sample and the thermometer. For the same  $T_a$  we detected no  $T_c$  drifts during the duration of each of the experiments (3 months each). In order to take into account a possible nonlinearity of the thermometer or other unknown sources of uncertainties, we also studied separately each component of these mixtures in an identical cell. The same temperature region as for the mixtures was investigated, in the same logarithmic scale, in order to detect possible systematic distortions.

In order to analyze the data, we use a program<br>statistical refining due to Tournarie.<sup>18</sup> To of statistical refining due to Tournarie. To estimate the quality of the fit we considered the deviation between the calculated and measured values, and the distribution of errors measured by a quality coefficient  $Q$ . This parameters equals one if the errors are randomly distributed, but is drastically lowered if a systematical distortion occurs. Also the mean error  $\sigma(n)$  can be determined.

#### III. RESULTS

### A. Components (Table IV)

As shown in Table IV all components (except water) exhibit a simple linear relationship between  $\Delta n$  and T so that

$$
\Delta n = \left(\frac{\partial n}{\partial T}\right)_{p} (T - T_c).
$$

Only for water must we add a polynomial development

$$
\Delta n = \frac{\partial n}{\partial T} \int_{P} [1 + W_1(T - T_c) + W_2(T - T_c)^2](T - T_c).
$$

In the range  $27^{\circ}-38^{\circ}$ C (I-W mixture),  $W_2 = 0$ , whereas in the range  $2^{\circ}-18^{\circ}$ C (T-W mixture),  $W_2$  is no longer negligible. These values are in agreement with previous published data.<sup>19</sup> agreement with previous published data.

From these measurements, the value of the ideal (regular) contribution can be estimated and leads to

N - I: 
$$
\left(\frac{\partial n}{\partial T}\right)_{p, \phi}^{id} = -(4.779 \pm 0.010) \times 10^{-4} \text{ K}^{-1}
$$
  
\nI – W:  $\left(\frac{\partial n}{\partial T}\right)_{p, \phi}^{id} = [-2.270 - 8.80 \times 10^{-3} (T - T_c)]$   
\n $\times 10^{-4} \text{ K}^{-1}$   
\nT – W:  $\left(\frac{\partial n}{\partial T}\right)_{p, \phi}^{id} = [-2.367 + 1.08 \times 10^{-2} (T - T_c)]$   
\n $+ 1.50 \times 10^{-4} (T - T_c)^2] \times 10^{-4} \text{ K}^{-1}$ 

TABLE II. Refractive index data  $(n = 6.328 \times 10^{-6}P)$  for isobutyric-acid-water  $(T_c = 26.7930 \degree C)$ .



$T(^{\circ}C)$ , $P$	$T({}^{\circ}C)$ , $P$	$T(^{\circ}C)$ , P	$T({}^{\circ}C)$ , $P$	$T(^{\circ}C)$ , P
03,1230, 000,000	06.8125, 200.480	11.9745, 498.930	16.1015, 007.540	18.3525, 036.850
04.0455, 049.790	06.9055, 205.590	12.1595, 510.170	16.1925, 014.620	18.0700, 000.000
04.9695, 099.900	07.0895, 215.700	12.2510, 515.870	16.2845, 021.690	18.0820, 001.360
05.9825, 154.950	07.0915, 215.900	12.3435, 521.570	16.3775, 028.880	18.1182, 005.640
06.9975, 210.630	07.1845, 221.100	12,4355, 527, 260	16.5605, 043.460	18.1384, 008.000
08.0025, 267.100	07.1365, 219.000	12.6215, 538.790	16.6525, 050.430	18.1587, 010.630
09.0215, 325.090	07.1815, 221.250	12.7135, 544.610	16.7455, 058.000	18.1787, 012.960
10.0325, 383.600	07.1735, 221.100	12,8055, 550.370	16.8375, 065.670	18.2188, 018.000
10.9595, 437.800	07.2655, 226.150	12.8980, 556.170	17.0225, 081.270	18.2386, 020.580
12.0670, 504.550	07.3585, 231.200	12.9905, 562.000	17.1145, 089.230	18.2702, 024.750
12.5285, 533.000	07.4495, 236.230	13.1755, 573.750	17.2065, 097.220	18.2778, 025.870
13.0825, 567.880	07.5425, 241.330	13.2685, 579.960	17.3915, 113.690	18.2908, 027.610
13.5455, 597.730	07.6345, 246.440	13.3605, 585.980	17.4835, 122.230	18.3077, 030.040
14.0075, 627.830	07.7265, 251.670	13.4525, 591.820	17.6680, 139.810	18.3236, 032.430
14.5005, 661.090	07.8180, 256.800	13.6380, 603.710	17.7135, 144.230	18.3285, 033.170
15,0565, 699.620	07.9105, 262.000	13.7305, 609.700	17.7605, 148.850	18.3343, 034.000
15.5145, 732.640	08.0955, 272.570	13,8230, 615,750	17.8525, 158.230	18.3353, 034.170
16.0685, 773.960	08.1865, 277.710	13.9145, 621.770	17.9445, 168.000	18.3363, 034.290
03.2155, 004.930	08.2795, 282.880	14.1025, 634.040	18.0375, 178.077	18.3385, 034.730
03.3080, 010.070	08.3725, 288.100	14.1735, 639.420	17.9905, 172.920	18,3395, 034,750
03.4005, 015.110	08.4655, 293.290	14.0820, 633.000	17.9857, 172.310	18.3405, 035.000
03.4925, 020.150	08.5565, 298.430	14.1315, 636.200	18,0021, 174.220	18.3412, 035.130
03.5855, 025.120	08.6485, 303.640	14.2235, 642.250	18.0233, 176.620	18.3441, 035.390
03.6755, 030.040	08.7415, 308.930	14.3165, 648.550	18.0695, 181.850	18.3442, 035.570
03,7690, 035,000	08,8335, 314,100	14,4085, 654,870	18.0897, 184.130	18.3450, 035.700
03.8615, 039.860	08.9275, 319.360	14.5930, 667.350	18.1103, 186.641	18,3462, 035,860
03.9535, 044.710	09.1125, 330.280	14.6865, 673.830	18.1318, 189.000	18.3472, 036.050
04.1375, 054.790	09.2045, 335.620	14.7795, 680.260	18.1508, 191.420	18.3482, 036.180
04.2305, 059.790	09.2975, 340.900	14.8715, 686.630	18.1699, 193.640	18.3501, 036.550
04.3225, 064.760	09.3905, 346.190	14.9635, 693.110	18.1892, 196.170	18.3516, 036.750
04,4145, 069,700	09.4820, 351.450	15.1485, 706.170	18.2082, 198.550	$\cdots$
04.5125, 075.240	09.5745, 356.810	15.2405, 712.870	18.2275, 200.940	
04.6055, 080.200	09.6645, 362.140	15.3295, 719.090	18.2460, 203.430	
04.6935, 085.150	09.7565, 367.470	15.4225, 725.730	18.2646, 206.000	
04.7855, 090.000	09.8475, 372.880	15,6065, 739,270	18.2835, 208.670	
04.8775, 094.950	09.9395, 378.280	15.6995, 746.140	18.3019, 211.330	
05.0615, 104.850	10.1245, 388.950	15.7920, 753.000	18.3032, 211.670	
05,0625, 104,950	10.2185, 394.450	15.8840, 760.000	18.3205, 214.430	
05.1545, 109.900	10.3105, 399.820 10.4035, 405.180	15.9765, 766.910 16.1605, 781.000	18,3306, 215.960	
05.2475, 114.850 05.3395, 119.900	10.4975, 410.550	16.2525, 788.090	18,3382, 217,170	
05.4315, 124.850	10.5905, 415.960	16.3445, 795.350	18,3423, 218,000 18.3447, 218.500	
05.5245, 129.850	10.6825, 421.320	16.4365, 802.610	18.3472, 218.840	
05.6155, 134.800	10.7745, 426.730	16.5295, 810.090	18.3498, 219.000	
05.7085, 139.770	10.8665, 432.270	16.6215, 817.550	$\cdots$	
05.8005, 144.780	11,0515, 443,310	$\cdots$	18.0996, 003.450	
05.8925, 149.800	11.1445, 448.860	16.0105, 000.620	18,1987, 015.450	
06.0745, 159.950	11.2365, 454.360	16.4690, 036.130	18,2553, 022,750	
06.1675, 165.000	11.3295, 459.820	16.9295, 073.420	18,2994, 028,870	
06.2595, 170.000	11,4205, 465,360	17.2985, 105.380	18,3155, 031,180	
06.3525, 175.200	11.5125, 470.840	17.5755, 130.920	18.3325, 033.690	
06.4445, 180.200	11.6065, 476.400	17,8055, 153,460	18,3374, 034,550	
06.5365, 185.260	11.6985, 482.040	17.8985, 163.000	18.3422, 035.330	
06.6295, 190.290	11.7895, 487.640	17.9905, 173.000	18.3488, 036.360	
06.7215, 195.390	11.8825, 493.360	18.0471, 179.270	18.3509, 036.700	

TABLE III. Refractive index data  $(n=6.328\times10^{-6}P)$  for triethylamine-water  $(T_c=18.3535\textdegree C)$ .

TABLE IV. Refractive index variation of components fitted to

$\Delta_n = (\partial_n / \partial T)_p [1 + W_1 (T - T_c) + W_2 (T - T_c)^2] (T - T_c).$	

For absolute values the correction due to the cell expansion is needed, i.e., add +6.0 $\times10^{-7}$  to  $(\partial n/\partial T)_p$ .



We used the values of Table V. For reasons explained below, it is useful to calculate the uncertainty on the  $(\partial n/\partial T)^{id}_{\rho,\phi}$  value in the N-I system.

## B. The critical mixtures

We tried to fit the data to the following  $\Delta n$  variation (Fig. 2):

$$
\Delta n = R \left| T - T_c \right|^{1-\alpha} \left[ 1 + b_1 \right| T - T_c \left| ^{\Delta} + b_2 \right| T - T_c \left| ^{2\Delta} + \cdots \right]
$$
  
+ 
$$
+ A \left| T - T_c \right| + B \left| T - T_c \right|^{2} + C \left| T - T_c \right|^{3}.
$$

The first term represents the  $\alpha$  behavior modified at large  $T - T_c$  by nonanalytical corrections, while the second term concerns the regular term. The

principle of the fits is the following: (a) Table VI: all parameters  $R, \alpha, b_1, b_2, \ldots \Delta, A, B, C$  are considered as adjustable parameters. Of course they are not all relevant, and we progressively reduced the number of free parameters until the quality of the fit decreased, so determining the actually significant parameters. Practically, it is difficult to separate the behavior with exponents  $2\Delta - \alpha$ ,  $3\Delta - \alpha$ ... from the regular behavior. So in any case only the single term  $b_1$  will be determined,  $B$  and  $C$  integrating the effects of both the regular terms and the nonanalytical corrections of higher order. (b) Table VII: the correlation between  $R$  and  $A$  is very strong since  $\alpha$  is small. So any extra information on  $A$  greatly increases the accuracy on R and  $\alpha$ . The simplest way is to

TABLE V.

Fluid	n $(6328 \text{ Å})$	ρ $(g \times \text{cm}^{-3})^b$	vol. fraction of
Nitroethane	1,3900 $(30 °C)$ <sup>a</sup>	$1.0404~(30^{\circ}C)$	$0.368 \pm 0.001$
Isooctane	1.3885 (30 °C) <sup>a</sup>	0.6937(30°C)	$\cdots$
Isobutyric	1,3930 $(20 °C)^b$	0.9490(20°C)	$0.4013 \pm 0.0005$
acid			
Water	1.3330 $(20^{\circ}C)^{b}$	0.9982(20°C)	$\cdots$
Triethylamine	1.4000 $(20 °C)^b$	0.7230(20°C)	$0.3950 \pm 0.001$
$N-I$	$1.3890(30°C)^c$	$\cdots$	$\cdots$
$A-W$	$1.3568$ $\degree$	$\cdots$	$\cdots$
$T-W$	$1.3591$ $^{\circ}$	$\cdots$	$\cdots$

<sup>a</sup> References 8 and 12.

Reference 15. '

From the L-L formula.

consider that the regular value is not far from the ideal value, i.e., evaluate  $A$  ( $B$  and  $C$  are connected to nonanalytical corrections as stressed in (a), which prevents them from being evaluated). Thus, for the N-I system where each component exhibits almost the same value of  $A$ , we could impose within  $\pm 0.2\%$  the regular term. For the T - W mixture, we imposed A within  $\pm 13\%$ , whereas for the  $I - W$  system we could not increase the accuracy, due perhaps to weak nonanalytical corrections, high enough to influence the regular behavior, but not sufficiently large to be accu-

rately determined. (c) Table VIII: in order to test the amplitude relations, the best way is to impose both exponents  $\Delta$ ,  $\alpha$  (and the regular term A for the  $N - I$  and  $T - W$  systems). (d) (Table IX): when nonanalytical corrections are necessary  $(T - W$  system), the accuracy on  $\Delta$  is greatly increased when both  $\alpha$  and  $A$  are imposed. In the same way, the determination of  $b_1, b_2, \ldots$  is more accurate if  $\alpha$ ,  $\Delta$ ,  $A$ ,  $B$  and  $C$  are imposed in the known limits.

Concerning the N-I mixture, the quality of the fit and the mean error enable no anomaly to be



FIG. 2. Reduced deviation (semilog plot) of the experimental refractive index  $n$ , or the interference order  $p$ , with respect to  $T - T_c$ , when fitting to: (a) the Table VII formula and with the nitroethaneisooctane system; (b) the Table VI formula and with the isobutyric acid-water system; (c) the Table VII formula and with the triethylamine-water system.



FIG. 2. {Continued)

observed, and only A can be determined. However, this regular term (when  $R = 0$  is assumed) is not in agreement with the ideal value whereas the volume additivity is verified within  $1\%^{12}$  and that each components exhibits almost the same  $(m_x/\vartheta T)_b$  value. When this ideal value was imposed, the value  $\alpha = 0.10 \pm 0.02$  was obtained. An amplitude  $R \approx 0.04$  times the regular parameter was found, whereas no corrections were necessary.

For the I-W mixture, it is not possible to accurately describe the results using only the regular terms. The exponent value  $\alpha = 0.12 \pm 0.05$  was found. The amplitude  $R \approx 0.3$ A.

Finally the T-W mixture allows both the  $\alpha$  divergence and nonanalytical corrections to be determined. The value  $\alpha = 0.11 \pm 0.02$  was obtained when only  $\Delta$  was fixed. With A estimated within  $\pm 13\%$ , we get  $\alpha = 0.113 \pm 0.003$ , with the corresponding amplitude  $R \approx 2.7$  times the regular term A. Concerning nonanalytical corrections, we found  $\Delta = 0.50 \pm 0.03$ , whereas nonanalytical corrections up to fourth order could be estimated when all the regular terms were fixed.

All these values are clearly much more accurate than those already reported. So we will compare our results as well as other data to the theoretical prediction of renormalization-group theory concerning the value of the exponent  $\alpha$  and the exponent  $\Delta$ , and the amplitude relation concerning  $R<sub>o</sub>$ 

#### VI. DISCUSSION

The renormalization-group calculation, in the case of a scalar order parameter  $n = 1$  and dimensionality  $d = 3$  leads to a value  $\alpha_{R-G} = 0.110$  $\pm 0.003$ , in close agreement with our results (Table X), since for the T-W system a value of 0.113  $\pm 0.005$  was found. Thus, for the first time in binary fluids, an accurate experimental value of  $\alpha$  has been found, which is in better agreement with the renormalization group than with the series expansion value  $[\alpha = 0.125 \pm 0.020$  (Ref. 37)].

Another interesting feature concerns the  $\Delta$  value, also found to be in close agreement with the renormalization-group result as can be seen in Table XI. Another  $\Delta$  value is reported from the viscosity behavior of ethane near  $T_c$ , but it involves rather the dynamic exponents. Only few other experiments in fluid mixtures are concerned with experiments in fluid mixtures are concerned with<br>nonanalytical corrections,<sup>7,38,39</sup> and both the magnitude of the correction and the accuracy of the experiment does not allow the  $\Delta$  exponent to be obtained.

Concerning the universal amplitude relations between  $\xi_0$  and the diverging part of  $C^0_{p,\chi_p}$ , we compared (Table XII), the experimental values using  $C_{\lambda, x_0}^0$  data deduced from our R values and other data already published (unfortunately in this last case the uncertainty on  $C^0_{\rho, x_c}$  is often poor or unknown) with the dimensionless quantity  $X$  $=(\alpha \rho_c \xi_0^3/k_B) C_{\rho, x_c}^0$ . The  $\xi_0$  values enter to the third



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 $\frac{19}{2}$ 



TABLE VIII. Determination of the anomaly amplitude R. Fit to:

 $\begin{split} \Delta &= R\,|\,T-T_c\,|^{1-\alpha}\,[1+b_1]\,T-T_c\,|^{\Delta}\,] \\ + A\,|\,T-T_c\,|^{1+\Delta}\,|T-T_c\,|^{\,2}+C\,|\,T-T_c\,|^{\,3} \quad. \end{split}$ 

 $\frac{19}{2}$ 

## ACCURATE DETERMINATION OF  $\alpha$  AND  $\Delta$  EXPONENTS IN...

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 $i$  Reference 22.  $<sup>j</sup>$  Reference 23.</sup> Reference 24.

TABLE X. Measurements of the critical exponent  $\alpha$  by different methods. The expected theoretical value by renormalization-group theory is (Ref. 11)  $\alpha_{\text{RG}} = 0.110 \pm 0.003$  and by series expansion (Ref. 37)  $\alpha_{\rm SE} = 0.125 \pm 0.020$ .

<sup>b</sup> Table VI.

<sup>c</sup> Reference 6.

<sup>d</sup> Reference 20.

<sup>e</sup> Reference 7.

 $<sup>f</sup>$  Reference 3.</sup>

power, so even a small uncertainty on this quantity gives rise to a large uncertainty on  $X$ . Nevertheless, for two systems, isobutyric acid-water and carbon tetrechloride-perfluromethyl cyclohexane a value of 2.0 is found, in reasonable agreement with the renormalization calculation  $X = (R_t^*)^3 \approx 1.76 \times 10^{-2}$  [Ref. 1(b)] and the series expansion result  $X \simeq 1.60 \times 10^{-2}$  [Ref. 1(a)].

Finally, one can note that for the three mixtures that we have studied here and which show weak,

TABLE XI. Values of the nonanalytical correction exponent  $\Delta$  in several systems. The value  $\Delta_{RG} = 0.493$  $\pm 0.007$  is inferred from the renormalization-group calculation (Ref. 11), and  $\Delta_{SE} = 0.6 \pm 0.1$  from the series expansion (Ref. 37}.



From Table IX with  $\alpha$  imposed

 $<sup>b</sup>$  References 30 and 35.</sup>

<sup>c</sup> Reference 36.

mean and high-amplitude anomalies, respectively, the regular part was always found to be close to the calculated ideal value. In Table XIII we report also others systems where the  $V$ ,  $\rho$  and  $\Lambda/Cp$  variations have been studied. They all support the same conclusion, namely that the anomaly of volume mixing seems to be almost fully related to the critical behavior of the specific heat.

## V. CONCLUSION

Very accurate data of the critical refractive index variations of three mixtures, nitroethaneisooctane, isobutyric acid-water, triethylaminewater have been reported. All the observed anomalies could be explained in relation to the density behavior alone, which itself is related to the specific-heat anomaly.

For the first time accurate values of the exponent  $\alpha$  have been obtained, with values 0.10  $\pm 0.02$ ,  $0.12 \pm 0.05$ ,  $0.113 \pm 0.005$ , respectively, which are very close to the predictions of renormalization-group theory  $(\alpha_{\text{RG}} = 0.110 \pm 0.003)$ with  $n = 1$  and  $d = 3$ ). Moreover the amplitude of this anomaly can be used, together with the correlation length  $\xi_0$ , to check the two scale factor

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TABLE XII. Relation between the amplitude of the diverging part of the specific heat  $C_{\rho,x_c}^0$ obtained by different measurement methods and the amplitude  $\xi_0$  of the correlation length. X is expected to be 1.76  $\times$ 10<sup>-2</sup> from renormalization-group calculation [Ref. 1(a)] and 1.60  $\times$ 10<sup>-2</sup> from series expansion  $[Ref. 1(b)].$ 



<sup>a</sup> From Table VIII.

 $<sup>b</sup>$  Reference 6.</sup>

 $c$  Reference 26.

 $d$  Reference 20.

<sup>e</sup> Reference 27.

f Reference 7.

<sup>g</sup> Reference 3.

h Reference 21.

 $i$  References 17 and 30.

 $^j$  Reference 28.

 $k$  Reference 24.

 $1$  References 29-31.

<sup>m</sup>Intensity data of Ref. 32.

universality. The general agreement is good but the errors remain large, due chiefly to the lack of accurate  $\xi_0$  values. In the triethylamine-water mixture, a noticeable influence of nonanalytical corrections was found, allowing the corresponding exponent  $\Delta$  to be measured. The value  $0.50 \pm 0.03$ is nearly the same as that deduced from the renormalization-group calculation  $(0.493 \pm 0.007)$ . Finally, the ideal part of the density behavior near  $T_c$  was found to be close to the measured regular part, strongly suggesting that the volume anomaly of mixing is mainly due to the criticality.

" Reference 7 and our turbidity data.

<sup>o</sup> Reference 33 with correction  $\frac{6}{5}$  as stressed

in Ref. 25.

P References 30 and 34.

**4** Reference 40 with correction  $\frac{6}{5}$  as stressed in Ref. 25.

- <sup>r</sup> Impure sample containing at least 5% water  $(Ref. 30)$ .
- $s$  Reference 30.
- $t$  Reference 41.
- <sup>u</sup> Reference 42.
- <sup>v</sup> Reference 43.
- <sup>w</sup>Reference 44.

These various points have been compared to other data found in the literature which concern volume, density, or specific-heat measurement, for the same mixtures and for other systems. Although the accuracy on most of these parameters is not as high as that obtained from refractive index measurements, they nevertheless support the same conclusions.

Note added in proof. From Ref. 45 it appears that a new analysis of the critical exponents of the Ising model by high-temperature series expansion, using a modified ratio method, gives  $\alpha_{SE} = 0.110$ .



TABLE XIII. Comparison between the experimental regular part of the coefficient of thermal expansion and the ideal value.

<sup>d</sup> Reference 6.

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Table VIII. Ratio  $B$   $^\mathrm{reg}/B$