Atomic spectral lines in helium containing a quark in the nucleus

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The authors have used accurate theoretical values for the energies of low-lying states of the isoelectronic series of helium, together with $1/Z$-expansion theory, to compute to six-figure accuracy some of the spectral lines to be expected if a quark were to be embedded in the nucleus of a helium atom. These wavelengths may be of interest to searches for quark-He lines in laboratory or astrophysical spectra or for single-atom detection of such quark atoms.

The claim[1] for the observation of a free fractional charge has revived interest in the search for free quarks. One method[2] of looking for them is through atomic experiments (for example, single-atom detection); a free quark in the nucleus of an atom will affect all the spectral transitions. The dominant effect is the change in the nuclear charge with a subsidiary (and far smaller) correction due to the change in the mass. For quark-hydrogen the spectral lines can be computed to very high precision but for other atoms such numbers are less immediately obtained. It has been pointed out[3,4] that experimental data on isoelectronic series afford a straightforward way of getting these numbers. A recent paper[4] has demonstrated the success of this procedure for even very heavy atoms ($Z \simeq 80$) where accuracies of 1 in 10^4 can be obtained. The motivation for this paper is that there has been some speculation that the fractional charges seen in the experiment of Ref. 1 are somehow associated with helium. As a result, there are ongoing efforts to search for such quark-helium spectral lines both in laboratory[5] and astrophysical[6] spectra. Since there are available[7,8] very accurate variational calculations on the ground and low-lying excited states of helium and its isoelectronic sequence ($Z \leqslant 10$), it would seem possible to get very high precision data on quark-helium through a similar isoelectronic fit to $1/Z$ theory.[9,10] We present here the results of such a study which may be of interest to these and other searches for quark-helium. We note that helium is the only other atom besides hydrogen for which such accurate values (1 in 10^6 or better) for the energies are now available.

Our method is a straightforward application of $1/Z$ expansions to the (calculated) data available on the isoelectronic series of helium. These results[7] present the nonrelativistic energies and the mass-polarization and relativistic corrrections for the singlet and triplet S and P states for n values up to 5 and Z up to 10. According to $1/Z$ theory, the relativistic energies are given by an expansion in powers of $1/Z$ beginning with Z^4, whereas the nonrelativistic energies have Z^2 as the leading term. In fitting the numbers for $Z=2-10$ to such expansions, one must, however, filter out any dependences on the nuclear mass M because M does not vary smoothly with Z. Accad *et al.*[8] have presented such values for the nonrelativistic energies, $E_{\rm nr}$ [they actually give $\epsilon \equiv (-E_{\rm nr})^{1/2}$], but for the other corrections the available results are in $\rm cm^{-1}$ with the M incorporated into them. Therefore, we divided the relativistic energies, E_J, by $\alpha^2 R_M$ drawn from Table XXIII of Ref. 7 and the mass-polarization correction, ϵ_M, by $\alpha^2 R_M (2m/M)R$ from the same table before fitting to the expansions. Here R is the Rydberg constant and R_M the corresponding reduced Rydberg for the nucleus in question. The authors of Ref. 8 noted that the nonrelativistic values ϵ can be reproduced by the expression

$$Z(a_0 + a_1/Z + \cdots + a_6/Z^6).$$

The theory of $1/Z$ expansions, on the other hand, gives

$$E_{\rm nr} = Z^2 \sum_{k=0} \epsilon_k Z^{-k}, \qquad (1)$$

and the first two coefficients ϵ_0 and ϵ_1 are readily obtained. Thus, for instance, for the He isoelectronic sequence, $\epsilon_0 = -(1+1/n^2)R$, where n is the principal quantum number of the excited electron, and an analytical formula for ϵ_1 is available[11] from which we computed the necessary numbers for the states of interest (for the P states these have already been computed in Ref. 10). Thus $a_0 = (-\epsilon_0)^{1/2}$, $a_1 = a_0\epsilon_1/2\epsilon_0$, and our fit to the nonrelativistic values was actually a five-parameter fit. Table I summarizes the values of these coefficients. With these parameters, we fit the nonrelativistic energies in Ref. 8 to the ten-figure accuracy to which they are quoted. We can then compute the corresponding nonrelativistic energies for quark-He. Similarly the mass-polarization correction ϵ_M was fitted, after removing the M dependence as noted above, to an expansion of the form

$$Z^2(b_0 + b_1/Z + \cdots + b_3/Z^3)$$

and the relativistic correction[12] E_J to an expansion

$$Z^4(c_0 + c_1/Z + \cdots + c_3/Z^3)\,.$$

To obtain the total energy, E_{tot}, in cm^{-1} from the above calculations, we made the following assumptions for the mass of the quark. We took $m_q = \frac{1}{10} m_p$ and 40 m_p as two extreme values and the entries in the top and bottom rows in Table II are, respectively, the energies with these masses for the different quark charges. Current confinement theories talk of quark masses of about one-tenth of the proton mass. However, for our purposes where presumably there is an unconfined quark, these values may really not be relevant and for this reason, we also considered as another extreme the value of 40 m_p as characteristic of the picture ten years ago when the nonobservation of quarks was attributed to their being very massive. These two extreme choices should, therefore, encompass different theories about quarks and we note that they make for a relatively small change in the entries in Table II.

The above procedure yielded all the S and P states with $n \leq 5$. Some extension to larger n values up to 10 for the P states was possible because Ref. 10 gives the ϵ_k values for these states for $k \leq 3$. Using these, we computed the nonrelativistic energies for n up to 5. Comparison with the accurate numbers of Ref. 8 showed that, as expected, the results agreed to several significant figures. Up to five significant figures of the accurate energies were reproduced, indicating that the higher ϵ_k come in only beyond that. Also, the residual error showed a smooth dependence with n. We used this fact to fit these residuals for $n = 2$–5 to a polynomial

$$d_0/n^2 + d_1/n^3 + \cdots + d_3/n^5\,,$$

and, thereby obtained the residuals for $n = 6$–10. Using the ϵ_k, $k \leq 3$, for $n = 6$–10 and adding on the extrapolated residuals gave us energy values for these higher states. Though not on par with our results for $n \leq 5$, we are confident that the values

TABLE I. Coefficients of the $1/Z$ expansion for the nonrelativistic energy (in atomic units) of the helium isoelectronic sequence: $(-E_{\text{nr}})^{1/2} = Z(a_0 + a_1/Z + \cdots + a_6/Z^6)$.

State	a_0	a_1	a_2	a_3	a_4	a_5	a_6
2^1S	0.790 569 415 0	−0.146 618 623 0	0.058 832 137 4	0.004 901 988 6	0.000 386 164 5	−0.007 450 690 6	0.005 439 099 2
2^3S	0.790 569 415 0	−0.118 856 493 4	0.021 052 023 6	0.006 194 057 9	0.003 238 762 4	0.000 192 332 6	0.002 947 872 7
3^1S	0.745 355 992 5	−0.070 607 285 7	0.029 609 510 5	0.003 428 090 3	0.002 241 710 7	−0.007 648 363 4	0.004 538 283 4
3^3S	0.745 355 992 5	−0.062 868 940 0	0.019 028 674 8	0.003 023 325 4	0.001 139 182 3	0.000 174 197 2	0.000 873 381 1
4^1S	0.728 868 986 9	−0.041 141 080 4	0.018 130 931 1	0.001 871 270 5	−0.000 986 960 1	0.001 580 030 3	−0.002 257 867 1
4^3S	0.728 868 986 9	−0.037 931 127 4	0.013 626 162 7	0.001 461 749 3	0.000 489 201 4	0.000 101 246 9	0.000 361 149 0
5^1S	0.721 110 255 1	−0.026 829 245 2	0.012 234 940 2	0.000 323 921 4	0.002 798 961 5	−0.006 634 647 2	0.004 840 947 0
5^3S	0.721 110 255 1	−0.025 197 277 4	0.009 854 393 0	0.000 784 961 1	0.000 250 312 7	0.000 079 353 3	0.000 163 961 3
2^1P	0.790 569 415 0	−0.164 355 536 0	0.082 225 701 9	0.000 655 836 9	−0.008 329 918 2	0.003 083 257 5	0.004 145 041 3
2^3P	0.790 569 415 0	−0.142 762 756 0	0.033 267 939 4	0.016 692 131 5	0.007 466 518 2	0.008 653 479 1	−0.003 892 819 6
3^1P	0.745 355 992 5	−0.076 042 552 2	0.036 874 133 8	0.003 959 277 8	−0.003 234 075 0	−0.004 429 984 4	0.005 215 011 1
3^3P	0.745 355 992 5	−0.069 962 423 5	0.024 793 249 8	0.004 601 490 7	0.001 664 054 3	0.001 548 330 2	0.000 000 843 4
4^1P	0.728 868 986 9	−0.043 464 924 5	0.021 328 262 8	0.001 735 861 3	−0.001 026 915 3	−0.002 646 865 3	0.002 513 841 1
4^3P	0.728 868 986 9	−0.040 934 726 3	0.016 441 991 0	0.001 839 562 4	0.000 579 249 7	0.000 598 102 9	0.000 068 988 8
5^1P	0.721 110 255 1	−0.028 026 834 3	0.013 871 914 7	0.000 802 297 1	−0.000 168 837 7	−0.002 129 532 4	0.001 851 611 9
5^3P	0.721 110 255 1	−0.026 738 619 2	0.011 404 702 3	0.000 888 970 4	0.000 279 231 8	0.000 284 956 5	0.000 053 810 2

TABLE II. Energy levels (in cm^{-1}) of quark-He measured from the He^+ ($n=1$) limit. For each state and each quark charge, the top entry is for $m_q=\frac{1}{10}m_p$ and the bottom entry for $m_q=40\ m_p$.

State	$q=+\frac{2}{3}$	$q=+\frac{1}{3}$	$q=-\frac{1}{3}$
2^1S	83 746.30	54 863.16	15 232.23
	83 756.93	54 870.18	15 234.23
2^3S	96 039.55	64 171.71	18 938.75
	96 051.58	64 179.79	18 941.16
3^1S	35 972.91	23 356.11	6 231.660
	35 977.41	23 359.04	6 232.459
3^3S	39 179.78	25 760.87	7 138.897
	39 184.63	25 764.07	7 139.790
4^1S	19 918.03	12 881.80	3 369.983
	19 920.50	12 883.40	3 370.414
4^3S	21 201.79	13 839.75	3 729.234
	21 204.41	13 841.46	3 729.699
5^1S	12 629.32	8 148.763	2 118.427
	12 630.88	8 149.772	2 118.704
5^3S	13 270.01	8 626.171	2 287.443
	13 271.64	8 627.231	2 287.725
2^1P	75 209.58	48 189.06	12 140.30
	75 223.07	48 197.45	12 142.22
2^3P	80 748.74	51 866.39	12 932.93
	80 753.03	51 869.32	12 933.92
3^1P	33 555.72	21 484.21	5 397.018
	33 561.10	21 487.60	5 397.818
3^3P	35 230.94	22 615.80	5 652.981
	35 233.76	22 617.63	5 653.495
4^1P	18 916.97	12 109.44	3 037.390
	18 919.82	12 111.24	3 037.821
4^3P	19 623.55	12 589.88	3 148.334
	19 626.35	12 591.04	3 148.645
5^1P	12 123.53	7 760.057	1 945.705
	12 125.29	7 761.175	1 945.962
5^3P	12 485.51	8 006.441	2 002.314
	12 486.74	8 007.228	2 002.530

TABLE III. Certain Rydberg series in quark-He for $q=-\frac{1}{3}$. All wavelengths are in vacuum and in angstroms.

Transition	$m_q=\frac{1}{10}m_p$	$m_q=40m_p$
$2^1S - 5^1P$	7526.4	7525.4
$- 6^1P$	7226.9	7226.0
$- 7^1P$	7037.2	7036.2
$- 8^1P$	6919.8	6918.9
$- 9^1P$	6843.1	6842.2
-10^1P	6784.0	6783.1
$-\infty^1P$	6565.0	6564.2
$2^3S - 3^3P$	7526.9	7525.8
$- 4^3P$	6333.0	6332.1
$- 5^3P$	5904.4	5903.7
$- 6^3P$	5693.7	5693.0
$- 7^3P$	5577.8	5577.0
$- 8^3P$	5505.1	5504.4
$- 9^3P$	5457.1	5456.4
-10^3P	5420.8	5420.1
$-\infty^3P$	5280.2	5279.5

for the higher n's are accurate to six or seven figures. Therefore, in Tables III and IV, where we have used the entries in Table II to write some Rydberg series in quark-He that lie in the visible-wavelength region, we have also included our values for higher n. These tables are presented only as a representative sample of a few well-defined series in visible wavelengths but we note that several other transitions, including those in the far infrared and ultraviolet, can be obtained from the entries in Table II simply by subtraction of the quoted energies for the states involved in the transition. For completeness, besides the S and P states we have considered above, the only other excited states in He for which energies of com-

TABLE IV. Certain Rydberg series in quark-He for $q=\frac{1}{3}$ and $\frac{2}{3}$. All wavelengths are in vacuum and in angstroms.

Transition	$q=+\frac{2}{3}$, $m_q=\frac{1}{10}m_p$	$q=+\frac{2}{3}$, $m_q=40m_p$	$q=+\frac{1}{3}$, $m_q=\frac{1}{10}m_p$	$q=+\frac{1}{3}$, $m_q=40m_p$
$3^1S- 4^1P$	5863.05	5862.49	...	...
$- 5^1P$	4192.98	4192.49	6411.88	6411.13
$- 6^1P$	3628.28	3627.82	5564.85	5564.15
$- 7^1P$	3357.65	3357.22	5157.67	5157.02
$- 8^1P$	3201.67	3201.27	4921.44	4920.82
$- 9^1P$	3103.04	3102.65	4772.16	4771.55
-10^1P	3035.50	3035.12	4669.06	4668.47
$-\infty^1P$	2779.86	2779.52	4281.53	4280.99
$3^3S- 4^3P$	5113.46	5112.92	7592.44	7591.26
$- 5^3P$	3746.12	3745.61	5632.39	5631.63
$- 6^3P$	3275.26	3274.85	4946.47	4945.85
$- 7^3P$	3044.57	3044.19	4608.26	4607.69
$- 8^3P$	2911.96	2911.60	4413.21	4412.66
$- 9^3P$	2827.90	2827.55	4289.46	4288.92
-10^3P	2770.44	2770.09	4204.29	4203.76
$-\infty^3P$	2552.33	2552.02	3881.85	3881.37

TABLE V. Similar to Table II. Energy levels (in cm^{-1}) of D states of quark-He.

	$q=+\frac{2}{3}$	$q=+\frac{1}{3}$	$q=-\frac{1}{3}$
3^1D	33 895.58	21 696.46	5425.178
	33 899.69	21 699.09	5425.836
3^3D	33 913.50	21 705.49	5425.041
	33 917.62	21 708.13	5425.699
4^1D	19 062.43	12 201.72	3053.111
	19 064.74	12 203.20	3053.482
4^3D	19 073.14	12 207.25	3052.948
	19 075.46	12 208.74	3053.318

parable accuracy are known are the singlet and triplet D states with $n=3$ and 4. The coefficients of a $1/Z$ expansion for these states have been given[13] and, using them, we determine the energies quoted in Table V for quark-He.

Tables II–V may be useful in spectroscopic and single-atom searches[14] for quark-He. As regards the accuracy of our results in Tables II-V, we believe that all the figures quoted are significant except that the last figure may be doubtful for the cases $n>5$ because of the *ad hoc* procedure detailed above for fitting the residuals to a $1/n$ expansion. Also, the entries for quark charge $-\frac{1}{3}$ may be less reliable than those for the positive quarks. This is because of our use of $1/Z$ theory which, as is well-known, is better for interpolating among the known Z values than for the extrapolation down to lower Z that is involved for the case of $-\frac{1}{3}$. To indicate that these results may be as much as one order of magnitude less reliable, the wavelengths for $-\frac{1}{3}$ in Table III are given to one decimal place only. Regarding quantum electrodynamic corrections of order α^3, as noted in Ref. 12, they have been calculated for the states under consideration. Only for the 2S states are they relevant to the accuracy we are working with and the corrections have been included. In all the other cases, these Lamb-shift corrections do not come in till the seventh significant figure. Finally, we make a remark about corrections due to the finite size of the nucleus. In particular, an extreme situation would be the one where the quark of $-\frac{1}{3}$ is only bound by the Coulomb force to the helium nucleus. For the lightest mass assumed of $\frac{1}{10}m_p$, considering the finite size of this charge distribution, we estimate that only for the case of $n=2$ could there by an effect at the level of the last (sixth) significant figure we present. For all the other states and for the case of $m_q=40m_p$, the effect is negligible.

Spectroscopic searches for quark-He lines based on the figures we present should, of course, proceed with the usual caution regarding accidental coincidences as discussed in Ref. 3. Since we give numbers good to a couple of hundredths of an angstrom, any coincidences, particularly of several members of a series (along with the usual expected $1/n^3$ correlation for their intensities) would be suggestive and would then call for the additional checks to strengthen the claim. These include multiplet structure (for the ^3P states we have only presented the energy of the $J=1$ state but the others can also be readily obtained from the available data on the He isoelectronic structure) and oscillator strengths which can also be generated by the $1/Z$ expansion.[10]

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